International Journal of Latest Research in Science and Technology **ISSN (Online):2278-5299**
Volume 9, Issue 3: Page No.14 -20 ,May-June,2020
https://www.mnkjournals.com/journal/ijlrst/index.php

ROLLING ELEMENT BEARING FAULT DIAGNOSIS BASED ON ENVELOPE SPECTRUM OF FRACTIONAL LOWER ORDER COVARIANCE

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Received : 25, June 2020; **Accepted** :29, June 2020 ; **Published** :30, June 2020

Abstract : ***Extracting the signature frequency of the impacts caused by the rolling element bearing fault in the environment of alpha stable distribution noise, the conventional envelope spectrum analysis method based on second-order statistics are not effective. With regard to the difficulty, a new envelope spectrum analysis method is proposed based on fractional lower order covariance. Firstly, the definition and properties of alpha stable distribution are described. The calculation method of fractional lower order statistics is presented. Then, simulation signals are used to compare the effects of the conventional envelope spectrum with the fractional lower order covariance based envelope spectrum in extracting the signature frequency of the impacts. Finally, the conclusion is reached that the fractional lower order covariance based envelope spectrum is valid in extracting the fault signature frequency in both Gaussian and alpha stable distributed background noise.***

Keywords- *R*olling element bearing; fault diagnosis; alpha stable distribution; fractional lower order covariance; envelope spectrum

I. INTRODUCTION

Rolling element bearings are the most widely used standard elements in all kinds of rotating machine, the condition of this key element in operation influences the operating quality of the whole equipment. Thus, it is of extremely important meaning to explore the technologies that can be applied to the diagnosis of the rolling element bearings. Due to the capability of separating the modulating frequency from the carrier frequency, envelope demodulation analysis methods have been widely used in the diagnosis of the rolling element bearings [1-5]. But it must be noted that all those applications are based on the premise that the background noise obey Gaussian distribution, the corresponding tool is second-order statistics. The fact is that there exist non-Gaussian distributed signal around, for instance, thunder, air noise, faults of motor and gearbox and so on. In the case that the background noise is non-Gaussian distributed, the analysis based on second-order statistics is not valid.

The waveforms of the non-Gaussian distributed noise show strong impulsive behaviour, which reflects in the probability density functions (PDF) with heavy tails. It can't be described by Gaussian distribution. In order to give a correct description of the impulsive non-Gaussian signals, models like Mixed Gaussian distribution, Cauchy distribution, Student distribution and alpha stable distribution were brought up. Among all the models, alpha stable distribution is the class of distribution that complies with the generalized central limit theorem and has stability property; besides, alpha stable distribution is in line with the engineering real data. All of these make alpha stable distribution a widely used model. In the field of mathematics, the concept of alpha stable distribution was first proposed by Levy during the research of the generalized central limit theorem in 1925. In the 1980s, alpha stable distribution theory had been widely stressed and developed [6]. However, it was not put into practical use until Nikias and his research team members had systematically established the signal processing frameworks based on the fractional lower order statistics in the 1990s, which extended alpha stable distribution theory to the field of signal processing. With the contributions of Nikias and his fellows, the alpha stable distribution theory was further developed and applied [7-11]. With regard to alpha stable distribution noise model, researchers from home and abroad have proposed many theories and methods, among which methods based on the fractional lower order statistics get extensive attention and wide application in fields like communication signal processing [12-14] and engineering signal processing [15-17] and so on. Aimed at the subject of extracting the signature frequency of the impacts caused by the rolling element bearing fault in the environment of alpha stable distribution noise, this paper proposes a method based on fractional lower order covariance envelope spectrum. In contrast of the conventional method based on auto-correlation function (second-order statistics) envelope spectrum, the truth that the method based on second-order statistics degrades is verified. Furthermore, comparison is made among the envelope spectrum of the three fractional lower order statistics, namely covariation, fractional lower order correlation and fractional lower order covariance. All the processing is conducted on the simulation signal, and the outcomes prove the validity of the proposed method of this paper.

II. ALPHA STABLE DISTRIBUTIONS

Alpha stable distributions are statistical models used to characterize impulsive signal and noise. Unlike most statistical models, the alpha stable distributions do not have closed-form probability density functions (PDF), except for a few known cases.

A. *Definition of Alpha Stable Distributions*

They are conveniently described by the characteristic function:

$$\varphi(t) = exp\{j\mu t - \gamma|t|^{\alpha}[1 + j\beta sign(t)\Psi(t,\alpha)]\} \quad (1)$$

where

$$\Psi(t,\alpha) = \begin{cases} tan\frac{\alpha\pi}{2} & \alpha \neq 1 \\ \frac{2}{\pi}log|t| & \alpha = 1 \end{cases} \quad (2)$$

$$\text{sign}(t) = \begin{cases} 1 & t > 0 \\ 0 & t = 0 \\ -1 & t < 0 \end{cases} \quad (3)$$

And $0 < \alpha \le 2, -1 \le \beta \le 1, \gamma > 0, -\infty < \mu < +\infty$. An alpha stable distribution is completely determined by four parameters: α, β, γ and μ. The concrete meaning of the four parameters are as follows: α is the characteristic exponent, β is the symmetry parameter (the distribution is symmetric α-stable (SαS) when β = 0), γ is the dispersion, and μ is the location parameter. The thick tail in the figure of the PDF of alpha stable distribution is an important property, which makes the alpha stable distribution an appealing model for impulsive signal environments. The smaller the value of α, the thicker the tails, especially when α = 2, the alpha stable distribution equals to Gaussian distribution. Fig.1 is time waveform of the α stable distribution noise and Gaussian distributed noise. Here, for the α stable distribution noise, the four parameters are: $\alpha = 1.2, \beta = 0, \gamma = 0.5, \mu = 0$;For the Gaussian distributed noise, the four parameters are: $\alpha = 2, \beta = 0, \gamma = 0.5, \mu = 0$. In Fig.1, one can visually find that there exists strong impulsive behaviours in the waveform of theαstable distribution noise. Theses impulsive behaviours correspond to the thick tail in the PDF of the α stable distribution, which Gaussian distribution (α = 2) does not have.

B. *Main Characteristics of Alpha Stable Distributions*

CHARACTERISTIC1: IF RANDOM

VARIABLES $X_1 \sim s(\alpha, \beta_1, \gamma_1, \mu_1)$, $X_2 \sim s(\alpha, \beta_2, \gamma_2, \mu_2)$, THEN X_1 AND X_2 ARE INDEPENDENT FROM EACH OTHER, THEN $X_1 + X_2 \sim s(\alpha, \beta, \gamma, \mu)$, WHERE

$$B = \frac{\beta_1\gamma_1^{\alpha} + \beta_2\gamma_2^{\alpha}}{\gamma_1^{\alpha} + \gamma_2^{\alpha}}, \Gamma = (\gamma_1^{\alpha} + \gamma_2^{\alpha})^{\frac{1}{\alpha}}, M = \mu_1 + \mu_2 \quad (4)$$

CHARACTERISTIC2: IF $X \sim S(\alpha, \beta, \gamma, \mu)$, WHEN $0 < A < 2$, FOR P, $0 < P < A$, THEN

$$E[|X|^p] < \infty \quad (5)$$

FOR ANY $P \ge A$, THEN

$$E[|X|^p] = \infty \quad (6)$$

The meaning of this character is alpha stable distributions do not have finite second-order or higher order moments when α < 2. Therefore, the quality of conventional methods based on second-order statistics degrades in dealing with the alpha stable distribution signals. The fractional lower order statistics become the new tools for signal processing.

III.FRACTIONAL LOWER ORDER STATISTICS

If a random variable X is an alpha stable distributed, its theoretical second-order or higher order moments do not exist when $\alpha < 2$. The conventional signal processing methods are mainly based on second-order statistics like autocorrelation and the Fourier transform of autocorrelation. If we continue using the conventional methods in processing the alpha stable distributed signals, we will get wrong results. However, the methods based on fractional lower order statistics lead to satisfactory outcome in dealing with the alpha stable distributed signals. To such methods, we qualify covariation, fractional lower order correlation and fractional lower order covariance.

A. Covariation

For two jointly SαS random variables X and Y with $1 < \alpha \le 2$, their covariation is defined as $[x,y]_\alpha$. Covariation can be computed as follow:

$$[x,y]_\alpha = \frac{E[xy^{\langle p-1\rangle}]}{E[|y|^p]}\gamma_y \qquad 1 \le p < \alpha \quad (7)$$

Where γ_y is the dispersion of y. If y is real number, then

$$y^{\langle p\rangle} = |y|^p sign(y). \quad (8)$$

If y is complex number, then

$$y^{\langle p\rangle} = |y|^{p-1}y^* \quad (9)$$

y^* is the symbol for conjugate.

$E[|y|^p]$ is the fractional lower order moment(FLOM).

$$E[|y|^p] = C_1(p,\alpha)\gamma^{p/\alpha}, 0 < p < \alpha \quad (10)$$

$$C_1(p,\alpha) = \frac{2^{p+1}\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(-\frac{p}{\alpha}\right)}{\alpha\sqrt{\pi}\Gamma\left(-\frac{p}{2}\right)} \quad (11)$$

B. Fractional Lower Order Correlation

For two jointly SαS random variables X and Y with $1 < \alpha \leq 2$, their fractional lower order correlation is

$$R_{xy}^{p} = E\left[xy^{\langle p-1\rangle}\right] \qquad 1 \leq p < \alpha \qquad (12)$$

When $p = 2$, the formula is the conventional correlation function.

C. Fractional Lower Order Covariance

For two jointly SαS random variables X and Y with $0 < \alpha \leq 2$, their fractional lower order covariance is

$$R_{xy}^{FLOC} = E\left[x^{\langle a\rangle} y^{\langle b\rangle}\right] \quad 0 < a \leq \frac{\alpha}{2}, 0 < b \leq \frac{\alpha}{2} \qquad (13)$$

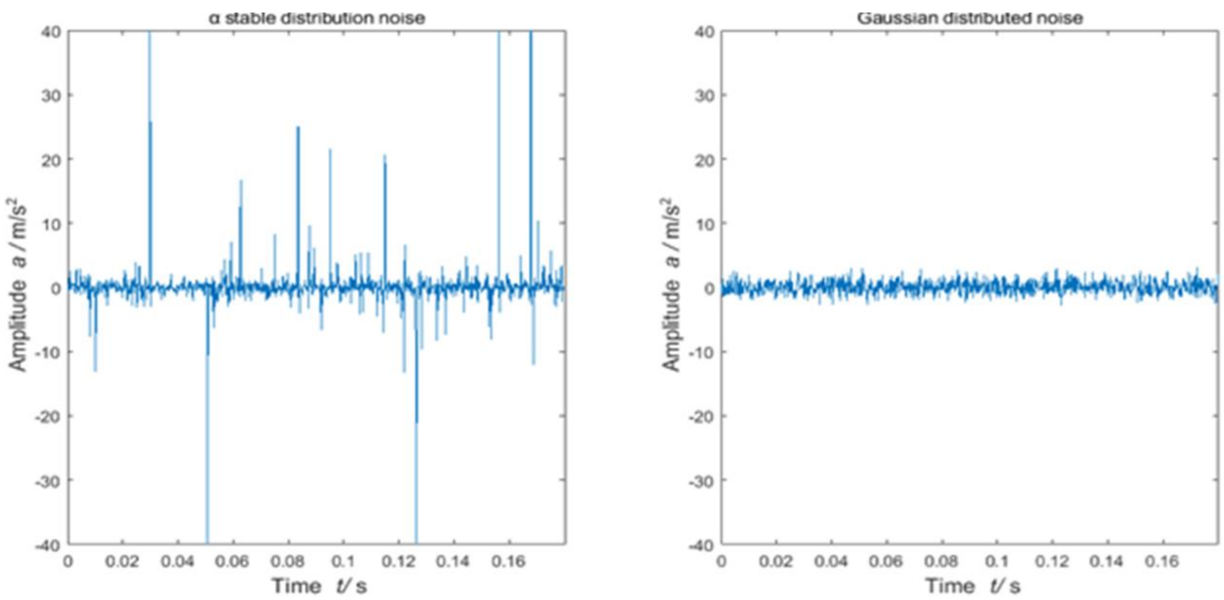

Fig.1 Time waveform of the α stable distribution noise and Gaussian distributed noise

IV. SIMULATED DATA ANALYSIS

According to the theories of conventional signal processing methods with envelope spectrum, the analysis mainly operates as follow:

Calculate auto-correlation function $R_x(\tau)$:

$$R_x(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)\,dt \qquad (14)$$

Conduct Hilbert transform on $R_x(\tau)$ to get its envelope $H[R_x(\tau)]$:

$$H[R_x(\tau)] = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{R_x(m)}{\tau - m} dm = R_x(\tau) \times \frac{1}{\pi\tau} \qquad (15)$$

Conduct Fourier transform on $H[R_x(\tau)]$ to get envelope spectrum:

$$\int_{-\infty}^{\infty} H[R_x(\tau)]\, e^{-i2\pi f\tau} d\tau = \int_{-\infty}^{\infty} R_x(\tau) \times \frac{1}{\pi\tau} e^{-i2\pi f\tau} d\tau \qquad (16)$$

When signal X is alpha stable distributed, rather than Gaussian distributed, second-order statistics do not exist, thus the method based on auto-correlation function degrades, may even lead to wrong answer. From previous analysis, the fractional lower order statistics (covariation, fractional lower order correlation and fractional lower order covariance) are the effective tools for the alpha stable distributed signal. Similarly, the envelope spectrum analyses based on factional lower order statistics are as follows:

(1) Calculate covariation, fractional lower order correlation and fractional lower order covariance of signal x(t);

(2) Calculate covariation envelope spectrum, fractional lower order correlation envelope spectrum and fractional lower order covariance envelope spectrum of signal x(t).

In order to verify the validity of the proposed method, conduct the following simulation analysis:

$$x(t) = s(t) + n(t) \qquad (17)$$

In this formula, s(t) is the simulated impact signal. The formula of s(t) is:

$$s(t) = \sum_t A_i\, h(t - iT) \qquad (18)$$

$$h(t) = e^{-2\pi\zeta f_n t} \sin(2\pi f_d t) \qquad (19)$$

Where, A_i is amplitude, which reflects the degree of the fault; h(t) is exponentially decayed sine signal; i is positive integer ; T is the period of the impact; ζ is the damping ratio; f_n is nature frequency; f_d is free frequency, $f_d = f_n\sqrt{1-\zeta^2}$ .Here, $A_i = 5$, $T = 0.01s$, $\zeta = 0.05$, $f_n = 2000Hz$,sampling frequency: $f_s = 6000Hz$,total sampling time is 0.18s.

The time waveform of s(t) is shown in Fig.2;

n(t) is the background noise. In this paper, researches are done for n(t) being alpha stable distribution and being Gaussian distributed respectively.

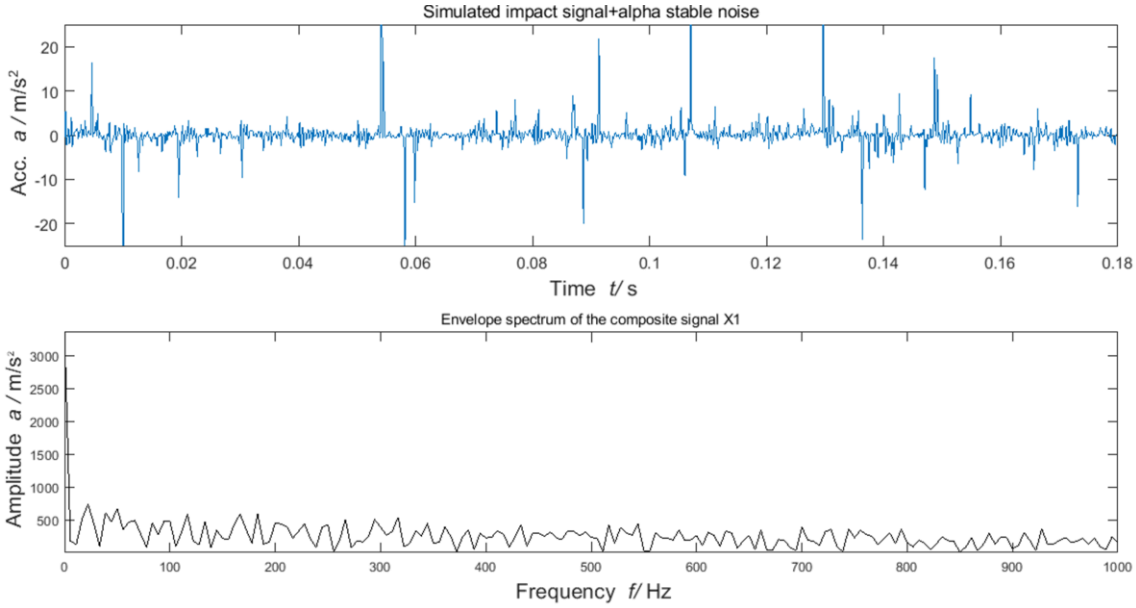

Fig.4 Composite signal X1 and its envelope spectrum

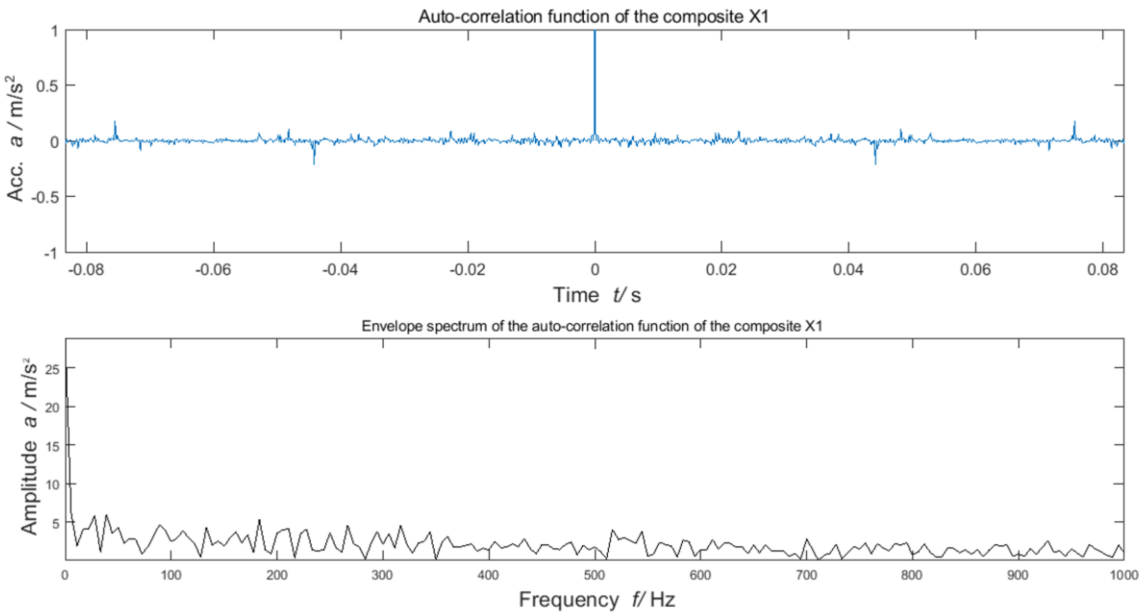

Fig.5 Auto-correlation function and its envelope spectrum of composite signal X1

In Fig.2 the simulated impact signal is not corrupted by any noise, which is the so-called SOI(signal of interest). In this signal, the natural frequency(carrier frequency) is 2000Hz; the fault frequency(modulating frequency) is 100Hz. Fig.3 is the envelope spectrum of the simulated impact signal. In Fig.3, the impact frequency is 100Hz.

When n(t) is alpha stable distributed noise, note the composite signal x(t) as composite signal X1.The simulating outcomes are as follows:

In case of the machines working in harsh industrial environment (such as mining machines), background noise has high amplitude and thus it is masking the impulsive component related to the fault (low signal–to-noise ratio).Here in Fig.4, the GSNR=5, which is close to the engineering practice. The waveform of composite signal is close to the engineering signal, which shows strong impulsive behavior. Due to the strong background noise, the periodic component cannot be directly found in the composite signal. In the envelope spectrum of the composite signal, the fault frequency (100Hz) does not stand out. Fig.5 is the auto-correlation function and its envelope spectrum. Due to the non-existence of second-order statistics in the α-stable distribution signal, the periodic component can be directly found in the waveform of the auto-correlation function, neither do the fault frequency (100Hz) stand out in the envelope spectrum of the auto-correlation function.

Therefore, the truth that the method based on the second-order statistics is invalid in dealing with α-stable distribution signal is proved.

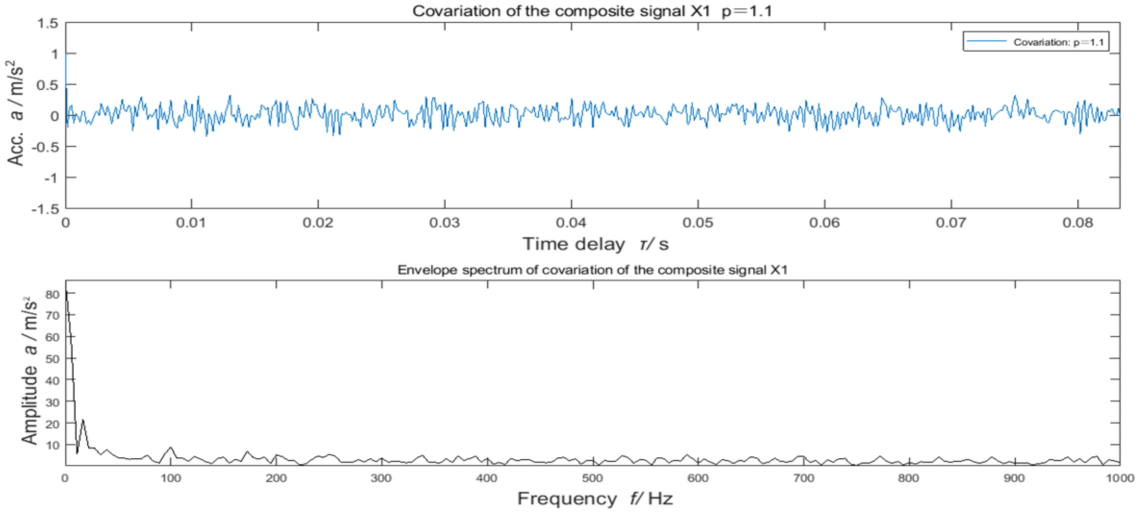

Fig.6 Covariation and its envelope spectrum of the composite signal X1

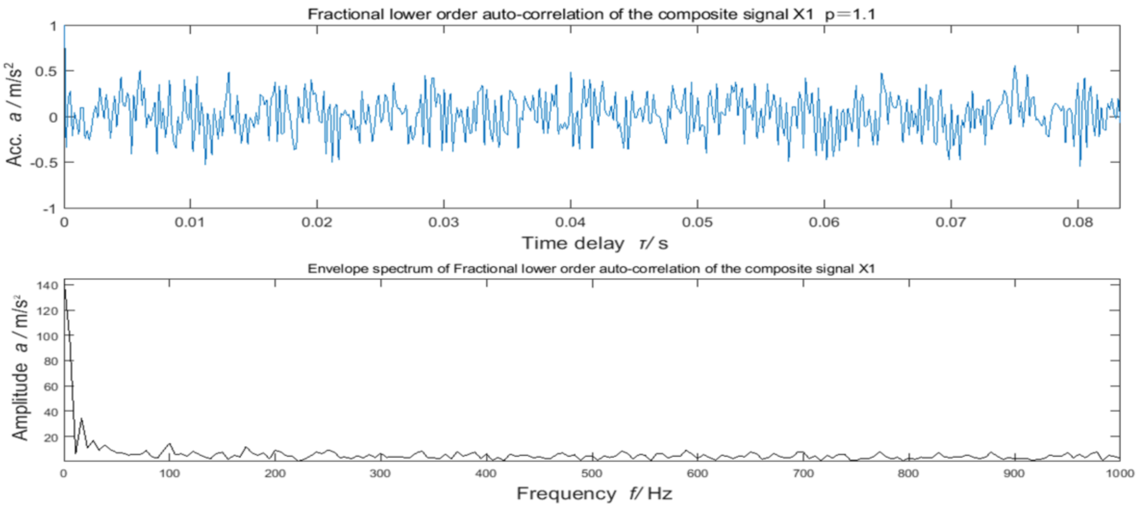

Fig.7 Fractional lower order auto-correlation and its envelope spectrum of the composite signal X1

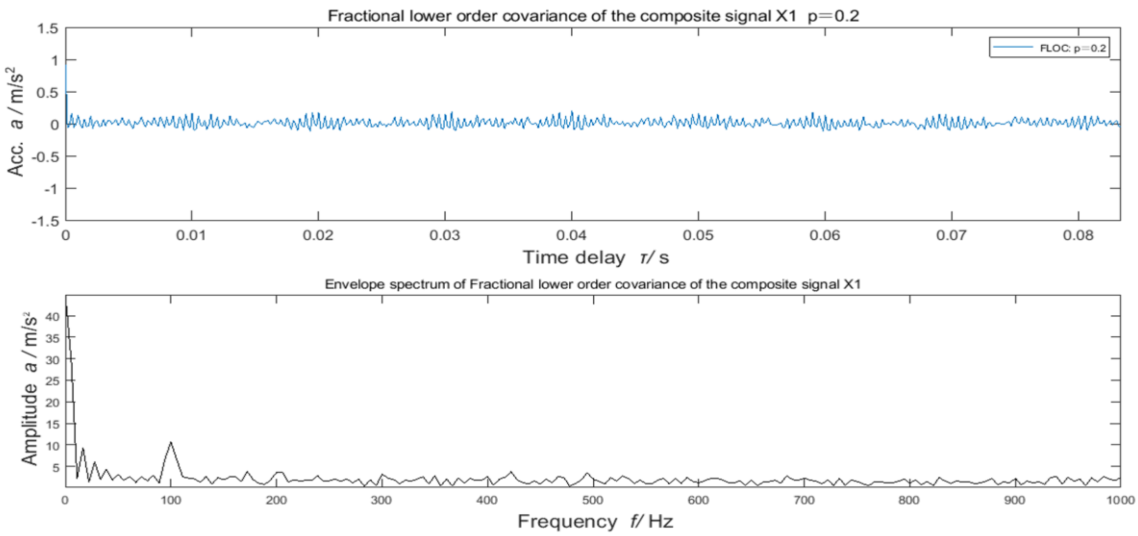

Fig.8 Fractional lower order covariance and its envelope spectrum of the composite signal X1

In order to successfully extract the fault frequency of the composite signal through envelope spectrum analysis, covariation, fractional lower order auto-correlation and fractional lower order covariance are used respectively. The corresponding outcomes are shown in Fig. 6, Fig. 7, Fig. 8. In Fig.6, Fig.7, Fig.8, the envelope spectrum show obvious peak at the frequency of 100Hz corresponding to the fault signature frequency.

The envelope spectrum analysis based on fractional lower order statistics can extract the fault signature frequency, while the envelope spectrum based on the second-order statistics can not. In the meantime, making comparisons among Fig.6, Fig.7, Fig.8, it can be easily found that the fractional lower order covariance envelope spectrum performs best in processing the α-stable distributed noise.

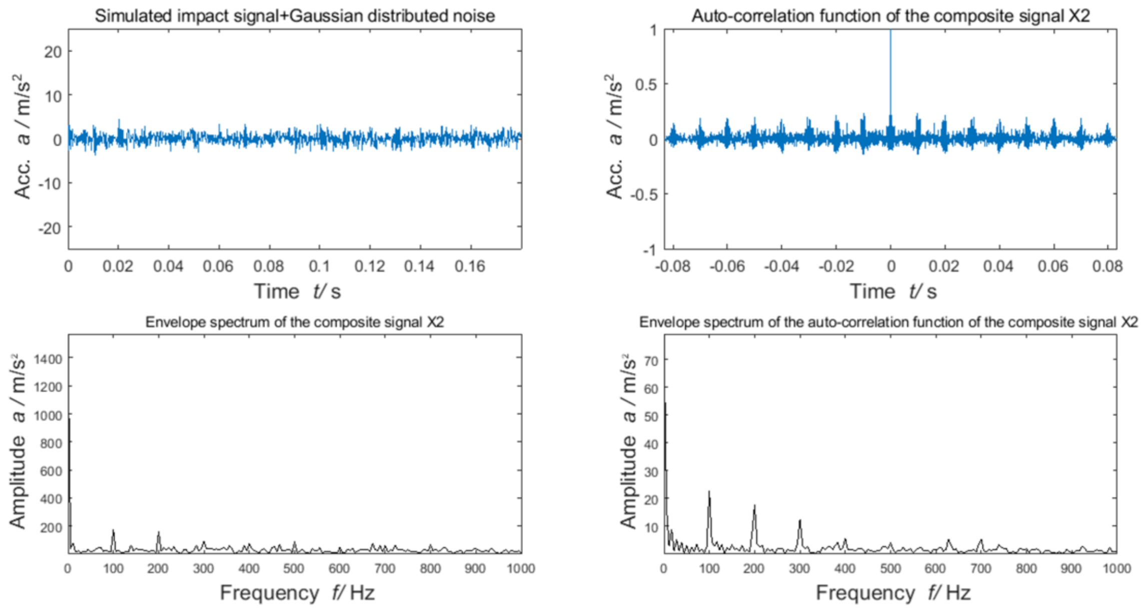

Fig.9 Conventional envelope spectrum analysis of the composite signal X2

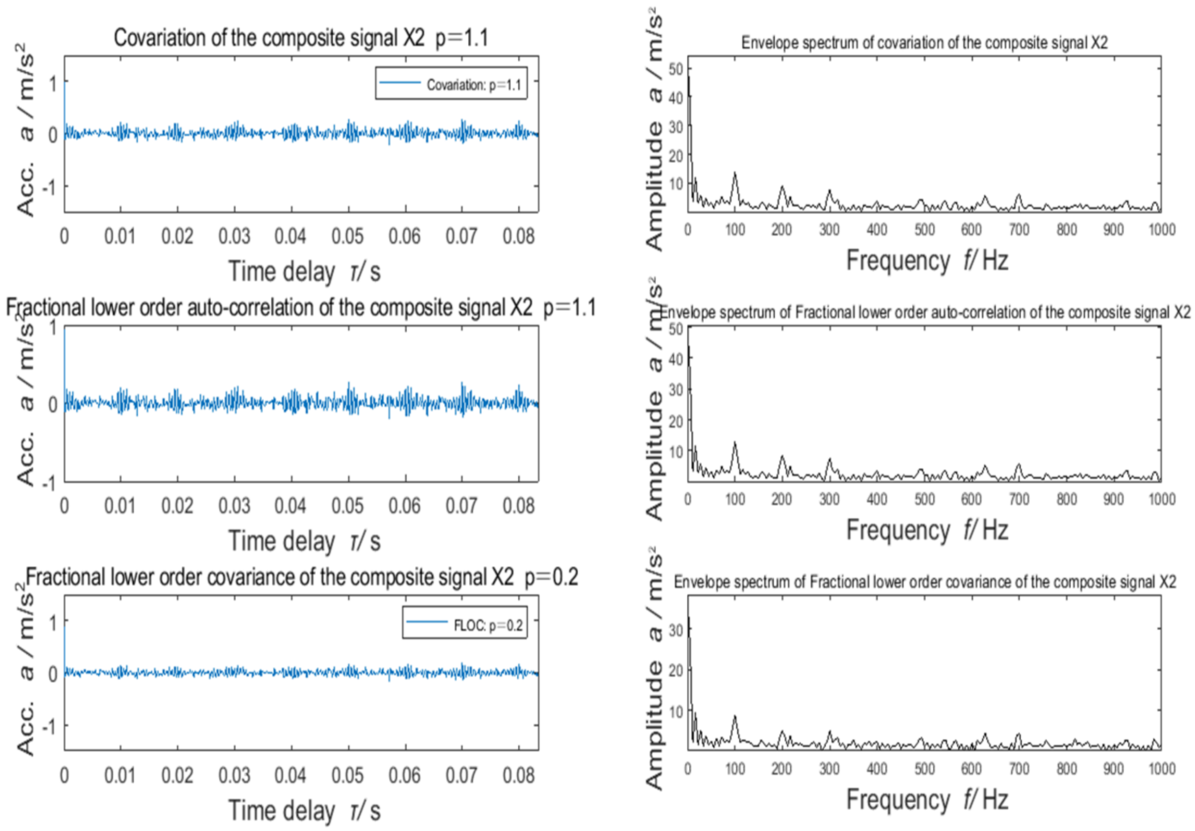

Fig.10 Fractional lower order statistics envelope spectrum analysis of the composite signal X2

When n(t) is Gaussian distributed noise, note the composite signal x(t) as composite signal X2.The simulating outcomes are as follows:

In Fig.9, the fault signature frequency (100Hz) can be successfully extracted through conventional envelope spectrum analysis, which conforms the fact that the conventional envelope spectrum analysis is valid in dealing with signal in Gaussian distributed noise.Fig.10 shows that the fractional lower order statistics envelope spectrum analysis is dealing with signal in Gaussian distributed noise.Fig.10 shows that the fractional lower order statistics envelope spectrum analysis is also appropriate for the case where the background noise is Gaussian distributed .

V. CONCLUSIONS

Due to the non-existence of the second-order statistics of the α-stable distributed signal, the auto-correlation function envelope spectrum failed in extracting the impact frequency of the fault rolling element. However, the envelope spectrum of the fractional lower order statistics can extract the impact frequency of the fault rolling element effectively. In this paper, it is verified that the fractional lower order covariance envelope spectrum analysis is a powerful tool in the fault diagnosis of the rolling element bearing in both alpha stable distribution and Gaussian distributed background noise.

ACKNOWLEDGMENT

This research is a part of the research that is sponsored by the scientific research fund of Tianjin University of Technology and Education (No. KYQD202003).

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