

THE EUCLIDIAN DISTANCE-BASED DETECTION METHOD APPLIED ON SCMA CODEWORDS

Sergio Vidal Beltrán¹, José Luis López Bonilla¹, Fernando Martínez Piñon²

¹Instituto Politécnico Nacional, Escuela Superior de Ingeniería Mecánica y Eléctrica- Zacatenco, México

²Instituto Politécnico Nacional. Centro de Investigación e Innovación Tecnológica- Azcapotzalco, México
 svidalb@ipn.mx, fmartinezp@ipn.mx, joseluis.lopezbonilla@gmail.com

Abstract : In this paper, we present the results of IQ parameter detection based on minimum Euclidean distance, when the data is encoded using Sparse Code Multiple Access and transmitted through a channel affected by Gaussian additive white noise. The results are presented using different values of signal to noise ratio in the channel.

Keywords – IQ parameters, SCMA, AWGN, Euclidian distance, CodeBooks

I. INTRODUCTION

Fifth generation (5G) mobile communications networks will improve the performance of mobile cellular networks by advancing speed, capacity, latency and connectivity. It is expected that 5G will achieve transmission rates in the order of Gbps, reliable access with high bandwidth, massive connectivity, very low latency (< 5ms) and reliability greater than 99.999% [1-3]. For these reasons, the radio access network must have efficient administration, control of multiple layers and a wide range of air interfaces to allow the aggregation of different networks. In order to comply with the above requirements, we seek to take advantage of the non-orthogonality of multiple access through SCMA (Sparse Code Multiple Access), which provides greater spectral efficiency than that achieved in LTE (Long Term Evolution) and UMTS (Universal Mobile Telecommunications System).

A. Sparse Code Multiple Access

SCMA does not use the QAM symbol constellations used by CDMA methods, instead it directly encodes the user bits into multidimensional codewords. Fig. 1 shows the basic diagram of an SCMA encoder with six physical resources (subcarriers) and four code words in the SCMA codebook [4]. Each user or layer assigns the binary data output of the FEC (Forward Error Correction) directly to complex code words that are assigned to physical resources according to a spreading code defined by the SCMA codebook.

SCMA is a successor to LDS (Low Density Spreading) [5], which uses low-density codebooks to reduce the complexity of symbol detection. With this method each user is assigned a codebook, and the data in it is used to map the bit stream directly to a low density vector (sparse vector) called a codeword. The multidimensional codebooks are used to complement the QAM modulation and the spreading depends on this factor. Each contains zero value in the same two dimensions of all its elements and the position of the zeros is determined randomly to prevent collisions between

users. These factors benefit the form and the coding gain [6-8].

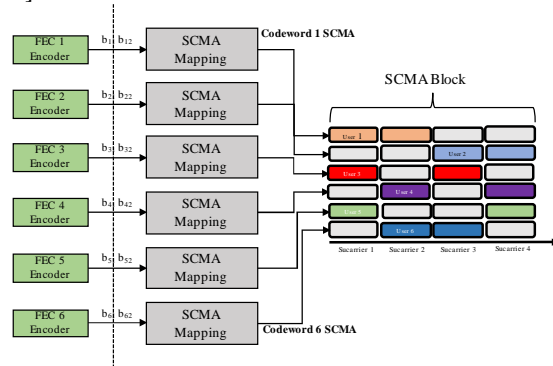


Fig. 1. SCMA Encoder.

II. DESIGN OF SCMA CODE BOOKS

SCMA, being low density codes allows to have coding gain [9, 10]. For the elaboration of this work the rotation and interleaving of mother constellations is used to generate the users' code books, considering 6 layers (users), 4 subcarriers, 4 codewords and up to 3 users connected to each radio resource, and two dimensions to design the constellations. Next, the process to generate the code books is described.

A. Gray Mapping

First, a set of Gray mapping vectors is chosen, which are the representation of each point of the constellation (mother constellation). A subset is defined S1 of Z2, where Z is a set of integers, given by (1):

$$S_1 = \{A_m(1 + j) \mid A_m = 2m - 1 - M, m = 1, \dots, M\} \quad (1)$$

$M = 4$ (code words)

A. 2.2 Constellation values for the first dimension

Publication History

Manuscript Received : 30 May 2018
 Manuscript Accepted : 8 June 2018
 Revision Received : 24 June 2018
 Manuscript Published : 30 June 2018

Values are assigned to each point of S_l . Considering $M = 4$, Gray's mapping takes the next form:

$$\begin{aligned} S_{11} 00 &\rightarrow -3(1+j) \\ S_{12} 01 &\rightarrow -(1+j) \\ S_{13} 11 &\rightarrow (1+j) \\ S_{14} 10 &\rightarrow 3(1+j) \end{aligned}$$

B. Rotation angle for the second dimension.

Rotation angle is calculated according to (2):

$$\theta_{l-1} = \frac{(l-1)\pi}{MN}, l = 1, \dots, N \quad (2)$$

Let N be the number of dimensions ($N = 2$). $S_N = U_N S_l, U_N = \text{diag}(1e^{i\theta_{l-1}}) \in \mathbb{C}^{N \times N}$, where U_N is the phase rotation matrix and 1 is an N -dimensional vector of all 1's. Fig. 2 shows the rotation of the points of the constellation for 3 dimensions:

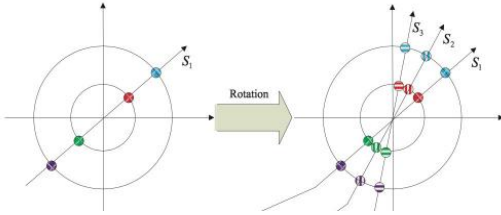


Fig. 2. Rotation vector of the mother constellation.

The Gray mapping based on the mother constellation (N -dimensional), takes the form of (3).

$$M = (S_1, \dots, S_N)^T = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1M} \\ S_{21} & S_{22} & \dots & S_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ S_{N1} & S_{N2} & \dots & S_{NM} \end{bmatrix} \quad (3)$$

For this case, all dimensions have the same power and the code words have the same average power.

C. Interleaving and reordering

The elements of the even dimensions of the mother constellation are interleaved, as shown in (4), in order to reduce the average power.

$$S'_1 = \left\{ \begin{bmatrix} -S_{1,M/2+1}, \dots, -S_{1,3M/4}, -S_{1,3M/4+1}, \dots, -S_{1,M} \\ -S_{1,M}, \dots, -S_{1,3M/4+1}, -S_{1,3M/2}, \dots, -S_{1,M/2+1} \end{bmatrix} \right\} \quad (4)$$

Therefore, the mother or base constellation is rewritten as (5):

$$M_c = (S_1, \dots, S'_1, \dots, S_N)^T \quad (5)$$

Whereas $M = 4$ and $N = 2$, the angle of rotation θ it is calculated according to (2):

$$\begin{aligned} \text{If } l=1, \\ \theta_{1-1} &= \frac{(1-1)\pi}{4 \cdot 2} = \frac{(0)\pi}{8} = 0 \\ \text{If } l=2, \\ \theta_{2-1} &= \frac{(2-1)\pi}{4 \cdot 2} = \frac{(1)\pi}{8} \end{aligned}$$

With the above, the values for the rotation angle are [11-12]:

$$\theta_0 = 0, \theta_1 = \frac{\pi}{8} \quad (6)$$

So the elements for S_l result in (7):

$$\begin{aligned} s_{1,4} &= 3(1+i) s_{1,3} = (1+i) \\ s_{2,4} &= 3(1+i)e^{i\theta_1} s_{2,3} = (1+i)e^{i\theta_1} \end{aligned} \quad (7)$$

The values of $S_{1,2}, S_{1,1}$ are generated from the negative values of $S_{1,4}$ and $S_{1,3}$. So that:

$$S'_2 = \{-s_{1,2}, s_{2,4}, -s_{2,4}, s_{2,3}\} \quad (8)$$

therefore, it is a multidimensional SCMA codebook that is defined as (9):

$$M_c = (S_1, S'_2)^T \quad (9)$$

Based on the above S_2 is defined by:

$$\begin{aligned} S_{21} 00 &\rightarrow -0.54 - 1.30j \\ S_{22} 01 &\rightarrow 1.62 + 3.91j \\ S_{23} 11 &\rightarrow -1.62 - 3.96j \\ S_{24} 10 &\rightarrow 0.54 + 1.30j \end{aligned}$$

Fig. 3 shows the mother constellation SCMA (M_c), it is observed that S_2 is formed by the rotation of S_1 which is a subset of a QAM constellation.

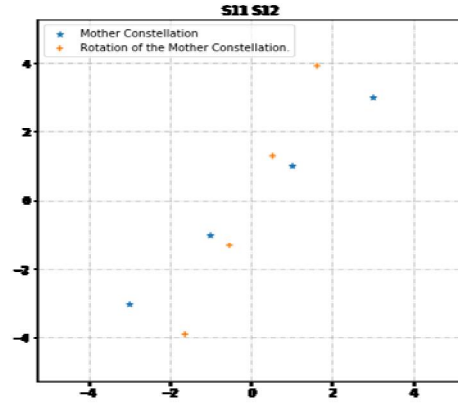


Fig. 3. Mother constellation and rotation of the constellation

D. Code books for different users

The code books are obtained from the mother constellation (M_c), for each user. The code words are rotated, taking care that the structure and the Euclidean distance remain unchanged.

Considering that $u \in \mathbb{Z}$ and that d_f represents the users that can use a given radio resource, the angle of rotation φ_u is defined by (10):

$$\varphi_u = (u-1) \frac{2\pi}{Md_f} + \epsilon_u \frac{2\pi}{M}, \forall u = 1, \dots, d_f \quad (10)$$

The optimal values of φ_u are:

$$\varphi_1 = 1, \varphi_2 = \exp\left(\frac{i\pi}{6}\right), \varphi_3 = \exp\left(\frac{i\pi}{3}\right) \quad (11)$$

The values of φ_u they are assigned to the non-zero positions of the graph factor following Latin Squares rules [9]; considering $M = 4$ y $N = 2$, the graph factor is given by (12):

$$F = \begin{bmatrix} \varphi_1 & 0 & \varphi_2 & 0 & \varphi_3 & 0 \\ \varphi_2 & 0 & 0 & \varphi_3 & 0 & \varphi_1 \\ 0 & \varphi_1 & \varphi_1 & 0 & 0 & \varphi_3 \\ 0 & \varphi_3 & 0 & \varphi_1 & \varphi_1 & 0 \end{bmatrix} \quad (12)$$

Considering that f_j be the j -th column of F without zero elements, an operator is defined for the j -th user as observed in (13)

$$\Delta_j = \text{diag}(f_j) = f_j, |j = 1, 2, \dots, 6 \quad (13)$$

$$x_j = V_j \Delta_j M_c, |j = 1, 2, \dots, J \quad (14)$$

In this example it is considered that $J = 6$ layers and $K = 4$ resources, the codebook for each user contains $M = 4$ symbols representing the alphabet:

00 01 11 10

Each symbol in the codebook is a column vector that contains 4 rows; in each row there is a complex number. Two of the rows will be filled and two will contain the complex value $0 + 0j$, that is, they are empty. A code book X_j for a user it is generated by concatenating the code words corresponding to different symbols transmitted by that user. The columns represent the code words while the rows represent the resources (subcarriers). For the first user the code book is represented by (15).

$$X_1 = \begin{matrix} S/C_2 \\ S/C_2 \\ S/C_3 \\ S/C_4 \end{matrix} \begin{bmatrix} 0.00 + 0.00j & 0.00 + 0.00j & 0.00 + 0.00j & 0.00 + 0.00j \\ 0.20 - 0.75j & 0.11 - 0.42j & -0.11 + 0.42j & -0.20 + 0.75j \\ 0.00 + 0.00j & 0.00 + 0.00j & 0.00 + 0.00j & 0.00 + 0.00j \\ -0.10 - 0.24j & 0.50 + 1.21j & -0.50 - 1.21j & 0.10 + 0.24j \end{bmatrix} \quad (15)$$

carriers 2 and 4 are occupied, while subcarriers 1 and 3 contain zeros (they are empty). The user data is transmitted using two subcarriers; the digital value 00 is represented in its complex values by $0.20 - 0.75j$ in subcarrier 2 and by $-0.10 - 0.24j$ in the subcarrier 4. Fig. 4 shows the representation of the code book in the complex plane (left figure), and the representation of the digital values in its complex representation (code words).

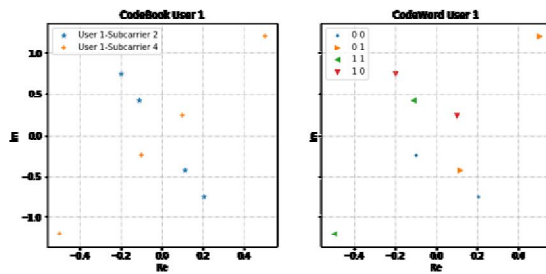


Fig. 4. Code books and code words for user 1

In this work, only the codebook for the first user is presented; to generate all the books of the other users, the process of rotation of the matrix is followed.

III. SIGNAL TO NOISE RATIO

Assuming a channel with bandwidth B [13], with a reception power P_r , and a spectral density of noise power $N_0/2$ the signal-to-noise ratio is given by (16).

$$\gamma = \frac{P_r}{N_0 B} \quad (16)$$

Considering that E_b is the bit energy and E_s is the energy per symbol of a signal, so $\gamma_b = E_b/N_0$ and $\gamma_s = E_s/N_0$ are the signal to noise ratio of the bit and the symbol, respectively [13]. So for an M -ary signaling scheme with $k = \log_2(M)$

bits per symbol, the energy of the signal for each modulated symbol is given by (17):

$$E_s = k E_b \quad (17)$$

Then the signal to noise ratio per symbol turns out to be:

$$\gamma_s = \frac{E_s}{N_0} = k \frac{E_b}{N_0} = k \gamma_b \quad (18)$$

A. AWGN Channel model and Euclidian distance

Random noise is added to the signal; noise magnitude depends on the signal to noise ratio that it is specified [14]. The signal to noise ratio is expressed in dB, in Fig. 5, the diagram of a signal to which noise is added is shown.

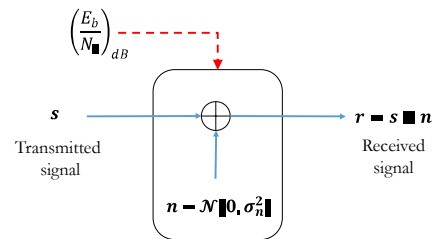


Fig. 5. AWGN channel

Considering Fig. 5, given a given signal-to-noise ratio, it is desired to incorporate Gaussian additive white noise. Considering the signal to noise ratio (γ), refers to $\gamma_b = E_b/N_0$ when using binary modulations (BPSK), for multilevel modulations (QPSK, MQAM) is considered $\gamma_s = E_s/N_0$.

The amount of noise added by the AWGN channel is controlled by the signal-to-noise ratio γ [14]. Considering the complex IQ plane for all digital modulations, the variance of the required noise (noise power) to generate random noise of the Gaussian type is given by (19).

$$\sigma^2 = \frac{N_0}{2} \quad (19)$$

To generate a noise vector n with a normal distribution with zero mean and standard deviation given by (19), we have the following [14].

$$n = \begin{cases} (\sigma) (\text{aleat}(s)) \rightarrow \text{if } s \text{ is real} \\ (\sigma) (\text{aleat}(s) + j \text{aleat}(s)) \rightarrow \text{if } s \text{ is complex} \end{cases} \quad (20)$$

where

n = it is the noise vector
 s = transmitted signal
 aleat = random values

Finally, the received signal that passes through an AWGN channel is:

$$r = s + n \quad (21)$$

B. IQ detector based on minimum Euclidean distance

For coherent detection, the transmitter and the receiver agree on the same reference constellation to modulate and demodulate user information. The first step in IQ detection is to calculate the Euclidean distance between two given vectors (the reference array and the symbols received with noise). Each symbol in the received vector must be compared with each symbol of the reference array, and then calculate the minimum Euclidean distance.

Considering that $x = (x_1 + x_2, \dots, x_p)$, $y = (y_1 + y_2, \dots, y_p)$, be two vectors in the p -plane. The Euclidean distance is given by (22).

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \dots + (x_p - y_p)^2} \tag{22}$$

IV. Simulation and Results

Considering the following user data: [1 0] [0 0] [1 0] [1 1] [1 0] [0 0] [0 1][0 1][1 0][1 1], its representation in code words using (15), is given by:

- [0 0] → [0.202-0.754j]
- [0 1] → [0.113-0.422j]
- [0 0] → [0.202-0.754j]
- [1 0] → [-0.202+0.754j]
- [0 0] → [0.202-0.754j]
- [1 0] → [-0.202+0.754j]
- [1 1] → [-0.113+0.422j]
- [0 1] → [0.113-0.422j]
- [1 1] → [-0.113+0.422j]
- [0 1] → [0.113-0.422j]

Considering a SNR = -3dB, the transmitted signals are affected by the AWGN channel (Fig.5), so the values that are recovered in the receiver are:

- 0.483-0.589j
- 0.128-0.153j
- 0.154-0.532j
- 0.308+1.093j
- 0.070-0.58j
- 0.063+1.053j
- 0.096+0.779j
- 0.012-0.586j
- 0.006-0.071j
- 0.087-0.113j

The received data is shown in the IQ plane in Fig. 6.

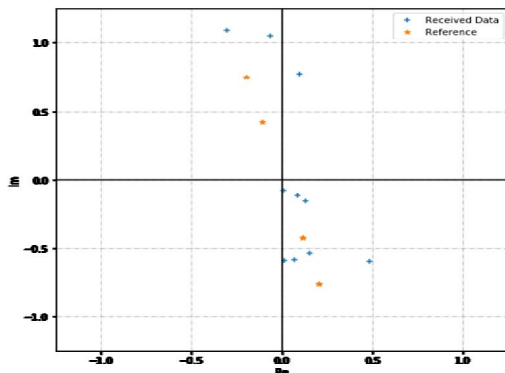


Fig. 6. Received data and its reference points

From Fig. (6), we can observe the influence of the AWGN channel, the received data are far from the points of the reference constellation, then the Euclidean distance between each received data and the reference constellation is calculated. The minimum distance is selected to associate the received data with the closest point of the reference. Fig. 7 shows the association of the received data to its closest reference point using the minimum Euclidean distance.

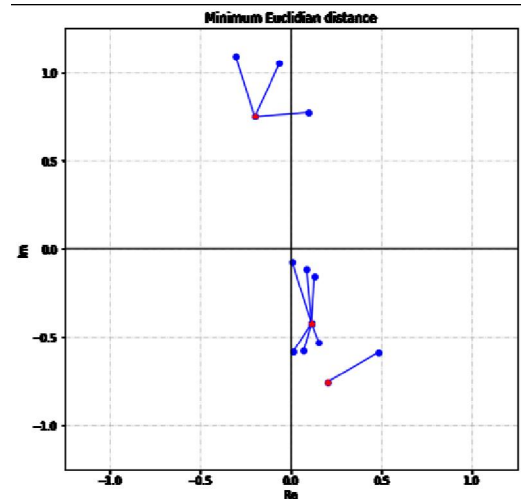


Fig. 7. association of the received data to its closest reference point

Based on the minimum Euclidean distance, the received data is decoded as:

- 0.483-0.589j → [0 0]
- 0.128-0.153j → [0 1]
- 0.154-0.532j** → **[0 1]**
- 0.308+1.093j → [1 0]
- 0.070-0.58j** → **[0 1]**
- 0.063+1.053j → [1 0]
- 0.096+0.779j** → **[1 0]**
- 0.012-0.586j → [0 1]
- 0.006-0.071j** → **[0 1]**
- 0.087-0.113j → [0 1]

For the third transmitted data [0 0], of (15) it is known that the code word corresponds to [0.202-0.754j], however, the received value was [0.154-0.532j], which is closest to the point [0.113-0.422j], which causes that when calculating the minimum Euclidean distance, the IQ detector erroneously decides that the data received is [0 1] instead of [0 0], it is observed that for this example 4 of the 10 transmitted data are misinterpreted, this is due because the SNR proposed is -3dB, for higher SNRs, errors in detection decrease considerably; in

Fig. 8 the representation in the IQ map of 500 data transmitted and SNR = 0dB is shown.

In Fig 9, for each point the calculation of the Euclidean distance is made to associate the received values to its nearest reference point.

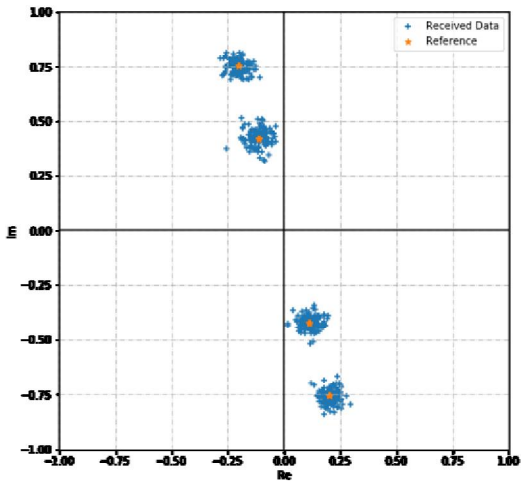


Fig. 8. Received data and its reference points

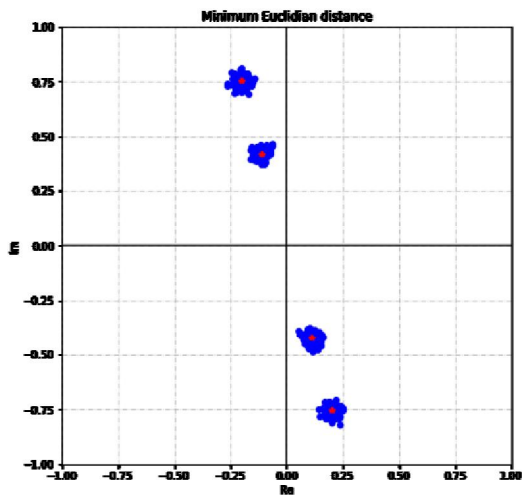


Fig. 9. association of the received data to its closest reference point

From Fig. 9, it can be seen that the received data are very close to the reference constellation whereby the error rate decreases considerably.

V. CONCLUSIONS

This work presents a detailed description of the method of phase rotation and interleaving of mother constellations to construct the code words in an SCMA system, additionally the Euclidian distance-based detection method is presented as an easy-to-implement alternative to demodulate SCMA data even in the presence of low signal-to-noise ratio values.

ACKNOWLEDGMENT

The authors thank the Instituto Politécnico Nacional for the support received in carrying out this work.

REFERENCES

[1] METIS. Mobile and wireless communications Enablers for the Twenty-twenty Information Society. Novel radio link concepts and state of the art analysis. 2013

[2] Gupta A. y Jha R. K. "A survey of 5g network: Architecture and emerging technologies". IEEE Access, 3:1206–1232, 2015.

[3] ITU. Framework and overall objectives of the future development of IMT for 2020 and beyond. 2015

[4] Dai, L., Wang, B., Yuan, Y., Han, S., y Wang, Z. "Non-orthogonal multiple access for 5G: solutions, challenges, opportunities, and future research trends". IEEE Communications Magazine, 53(9):74–81, Sept 2015.

[5] Van de Beek, J. y Popovic, B. M. "Multiple access with low-density signatures". In GLOBECOM 2009 - 2009 IEEE Global Telecommunications Conference, pages 1–6, Nov 2009.

[6] Bayesteh, A., Nikopour, H., Taherzadeh, M., Baligh, H., y Ma, J. "Low complexity techniques for scma detection". 2015 IEEE Globecom Workshops (GC Wkshps), pages 1–6, Dec 2015.

[7] Nikopour, H. y Baligh, H. "Sparse code multiple access". 2013 IEEE 24th Annual International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC), pages 332–336, Sept 2013.

[8] Yu, L., Lei, X., Fan, P., y Chen, D. "An optimized design of SCMA codebook based on star-QAM signaling constellations". In 2015 International Conference on Wireless Communications Signal Processing (WCSP), pages 1–5, Oct 2015.

[9] Minnick, R. C., Elspas, B., Short, R. A. "Symmetric Latin Squares". IEEE Transactions on Electronic Computers. 1963

[10] Yang, H., Fang, X., Liu, Y., Li, X., Luo, Y., y Chen, D. "Impact of overloading on link-level performance for sparse code multiple access". 2016 25th Wireless and Optical Communication Conference (WOCC), pages 1–4, May 2016.

[11] Bao, J., Ma, Z., Ding, Z., Karagiannidis, G. K. y Zhu, Z. "On the design of multiuser codebooks for uplink SCMA systems". IEEE Communications Letters, 20(10):1920–1923, Oct 2016.

[12] Bao, J., Ma, Z., Mahamadu, M. A., Zhu, Z., y Chen, D. "Spherical codes for SCMA codebook". 2016 IEEE 83rd Vehicular Technology Conference (VTC Spring), pages 1–5, May 2016.

[13] Proakis J.G. Digital Communications, fifth edition, New York, McGraw–Hill, 2008

[14] Andrea Goldsmith, Wireless Communications, Cambridge University Press, first edition, August 8, 2005