MNK Publication

International Journal of Latest Research in Science and Technology **ISSN (Online):2278-5299**
Volume 5, Issue 2. Page No. 106-109, March-April 2016
https://www.mnkpublication.com/journal/ijlrst/index.php

IMPROVING THE NNA FOR THE TRAVELLING SALESMAN PROBLEM USING THE MVM

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Abstracts- The Travelling Salesman Problem (TSP) consists of finding the lowest cost or the shortest path tour between n cities. In the theory of computational complexity, the TSP belongs to the class of NP-complete problems and is one of the simplest but most intensively studied problems in optimization. Even though the problem is computationally difficult, a large number of heuristics and exact methods are known. In this presentation, we propose a new heuristic algorithm for the asymmetric TSP. It is well known that NNA is very sensitive to the starting city. We basically use the first iteration of the Modified Vogel Method (MVM) in order to determine the best-starting city for the Nearest Neighbor Algorithm (NNA). Note that the resulting solution tour can be considerably improved. This can yield, in some cases, to the optimal solution.

Keywords. Travelling Salesman Problem, Nearest Neighbor Algorithm, Modified Vogel Method, Vogel Approximation Method, Local Search Algorithm, Combinatorial Optimization, NP-Complete Problem

I. The Travelling Salesman Problem

The Travelling Salesman Problem (TSP) consists of finding the lowest cost or the shortest path tour between n cities. In term of graphs, it is to find the Hamiltonian cycle associated with the shortest total distance. The TSP, known in the theory of computational complexity as an NP-hard problem in combinatorial optimization, plays a very important role in operation research and theoretical computer science. The TSP is one of the simplest but the most intensively studied problem in optimization. It has several applications and is used as a benchmark for many optimization methods. Even though the problem is computationally difficult, a large number of algorithms for solving the Traveling Salesman Problem had been developed and can be classified in three main categories [7, 8].

The first one uses the exact methods based on different techniques to solve the mathematical formulation using the cutting plane method [4] or the Branch and Bound method [7]. For the cutting plane method, the Concorde solver algorithm [2] is developed and seems to behave very well. It has been currently considered the best TSP solver and has provided the optimal solutions with an efficiency of more than 96% to the TSPLIB cities [10]. For instance, with a number of cities in around the thousand order, the TSP can often be solved exactly by Concorde, with all computations carried out locally on the smartphones. It solves iteratively linear programming relaxation problem of the TSP.

The second category is the heuristic algorithms, including the NNA [11] which gains simplicity by finding an approximated solution instead of an optimal one.

The third category called metaheuristic algorithms includes the Evolutionary Algorithm (EAs) [13], Ant Colony Optimization (ACO) and the Local Search Algorithm. One of the best known local search algorithm is the Lin-Kernighan (LK) algorithm [9].

Let set $V = \{1, \dots, n\}$ the set of n cities, $G = (V,T)$ be the graph network and $C = (C_{i,j})$ be the matrix where the coefficients $C_{i,j}$ represent the cost for travelling from city i to j, then the TSP can be mathematically formulated [4] as

$$
\begin{cases}
\min\limits_{X} \; TC(X) = \sum\limits_{(i,j)\, i \neq j} C_{i,j} X_{i,j} & \\
\sum\limits_{j=1,\; j\neq i}^{n} X_{i,j} = 1\,; \quad i = 1, \dots, n & (1)\\
\sum\limits_{i=1,\; i\neq j}^{n} X_{i,j} = 1\,; \quad j = 1, \dots, n & (2)\\
\sum\limits_{i\in U, j\in \bar{U}} X_{i,j} \geq 1\,; \quad \forall\, U \subset V, \quad 2 \leq |U| \leq n-2 & (3)\\
X_{i,j} = \begin{cases} 1 & \text{if } (i,j) \in T \\ 0 & else \end{cases} \;; \quad i,j = 1, \dots, n & (4)
\end{cases}
$$

The variables are defined by constraints (4) while the first equalities (1) and (2) ensure that each city be visited only once. The third constraints (3) enforce that there is only a single tour covering all the cities. Its alternative version is the following constraint [4].

$$
\sum_{i\in U, j\in \bar{U}} X_{i,j} \leq |U| - 1\,; \quad \forall\, U \subset V, \quad 2 \leq |U| \leq n-2 \qquad (5)
$$

And many others are discussed [5, 6, 7] in order to avoid the increase of the number of these constraints induced by the formulation (3) and (5). In this paper, we present a new heuristic algorithm for the Asymmetric Travelling Salesman

Publication History

Manuscript Received	:	*20 April 2016*
Manuscript Accepted	:	*25 April 2016*
Revision Received	:	*27 April 2016*
Manuscript Published	:	*30 April 2016*

Problem (ATSP) using the first iteration of the Modified Vogel Method (MVM) [1] to improve the nearest neighbor algorithm (NNA) introduced in the next section by finding the best-starting city.

II. The NNA Method

The NNA might be seen as a step by step method. It lets the salesman, from an initial city, to choose the nearest unvisited city as his next move.

Nearest Neighbor Algorithm (NNA)

Step 1. Choose a starting city $k = i_0$ then set $P = N - \{k\}$

Step 2. Choose r such that $\min_{j \in P} C_{k,j} = C_{k,r}$

Step 3. Update the last visited $k = r$ and set $P = N - \{k\}$
If $P = \emptyset$ then go to step 4, else go to step 2

Step 4. Link the last city to the starting city.

The NNA is a heuristic that gives an effective solution tour corresponding to the natural behaviour of a salesman with a very low number of cities. For some randomly distributed cities, the algorithm yields to the optimal path for an average of 75% of the times [5]. However, there exist many specific arranged city distributions which make the NNA algorithm give the worst tour solution [3]. Rosenkrantz and al. [11] showed that the NNA is strongly sensitive to the starting city which can impair its accuracy.

III. Application of MVM to TSP

3.1. General principle

In the application of the first iteration of the Modified Vogel Method [1], the main modification is the reduction of matrix cost in order to have at least a zero cost in each row and column. The Vogel penalties are evaluated from the new reduced matrix. Then the starting city is selected as the one associated to the row-number of the largest penalty. This is presented in the procedure presented below.

Note that we are not applying the complete MVM algorithm but just the first iteration in order to find the best-starting city. At this point the NNA is applied to find a tour which can be improved by considering the missing zero of each row. The row with the largest missing zero is then selected to be improved by considering sequentially all the lower reduced costs on that row. The solution tour with the lowest reduced cost is determined and the MVM-NNA algorithm stops.

Starting city Procedure

Step 1. Row reduction.
$\forall i$ find $u_i = \min_{j \neq i}\{C_{i,j}\}$ then
For $j = 1, \ldots, n$; $j \neq i$ set $R_{i,j} = C_{i,j} - u_i$

Step 2. Column reduction
$\forall j$ find $v_j = \min_{i \neq j}\{R_{i,j}\}$ then
For $i = 1, \ldots, n$; $i \neq j$ set $R_{i,j} := R_{i,j} - v_j$

Step 3. Penalties evaluation.
$\forall i$ find the penalty $p_i \geq 0$ as the subtraction of the two least costs of row i
$\forall j$ find the penalty $q_j \geq 0$ as the subtraction of the two least costs of column j
Find the largest penalty lp

Step 4. Starting city.
If the largest penalty is reached by p_i then the starting city is i. Else $\exists\, j$ $lp = q_j$ find i_j such that $R_{i_j,j} = 0$ then the starting city becomes i_j.

Remark 1. In this procedure the rows are reduced first then the columns. However, we can perform the reduction of the columns first.

Remark 2. If in step 3, there is a tie between a row i and a column k then the starting city is i. If there is a tie between different rows then the one with the highest penalty of the column associated to its zero is the starting city. If the tie subsists the row associated with the lowest initial cost of its zero is the starting city. If the tie subsists then it can be broken arbitrarily.

Remark 3. If in step 4 the largest penalty is null, then all penalties equal zero and any town can be the starting city.

3.2. Algorithm presentation

In this subsection the first iteration of MVM is applied in order to determine the best-starting city for NNA. From that starting city, the NNA is performed to provide a solution tour. Then it is improved to induce a new heuristic for the TSP called MVM-NNA and presented below.

MVM–NNA Algorithm for TSP

Step 1. Starting city
From the procedure find the starting city i_o then
$k = i_o$; $P = N - \{k\}$

Step 2. NNA Application
Apply the NNA algorithm from i_o to find a Tour

Step 3. Missing zero evaluation
For each row i, find the reduced cost of the assigned variable that is mz_i.

Step 4. Improving NNA
The row i_m with the largest missing zero i.e. $\max_i\{mz_i\} = mz_{im}$ is to be improved. We consider the starting city as i_m then assign sequentially all the reduced costs which are lower than mz_{im} on the row i_m. When all the associated reduced cost are evaluated, the algorithm ends.

III. Convergence Results

The convergence results of the MVM-NNA are established by the following results.

Theorem 1. If the largest missing zero is null then the associated solution tour is optimal.

Proof. We know by definition that the reduced cost is positive. Therefore if the largest missing zero is null then the reduced cost is null and has reached its minimal value. Consequently, the associated solution tour is optimal.

Theorem 2. If the largest missing zero associated to row k is unique and all the other strictly positive missing zeros are reduced during the improvement induced by using row k then the associated solution tour is optimal.

Proof. All the possibilities to decrease the total cost by row k are visited. Since the decrease of the missing zero of row k affects the other entire missing zeros then these rows are dependent on row k. Therefore all the possible situations are considered and the associated cost is optimal.

Remark 4. If there is only one strictly positive missing zero then the theorem applies.

Remark 5. The only way to improve the solution provided by the MVM-NNA algorithm is to reduce the non-null missing zero.

Remark 6. If the MVM-NNA algorithm is applied to find the initial solution tour for a Local Search algorithm, then local search has to be performed near the cities associated to non-null missing zero. The optimal solution tour will very likely be in their neighborhood.

Remark 7. The MVM-NNA algorithm can be used to determine more performed populations in the application of Evolutionary algorithm (EAs). It suffices to consider the different situations associated the improvement of the algorithm by using the improvement of the rows associated to the non-null missing zero. The resulting population will very likely contains the optimal solution tour.

Theorem 3. If all the strictly positive missing zeros are improved then the resulting solution tour is optimal.

Proof. If all the strictly positive missing zeros are improved then all the possibilities to reduce the total reduced cost are considered. Therefore, all situations are enumerated and the resulting solution tour is optimal.

IV. Example of illustration

Let us consider an AsymmetricTraveling Salesman Problem (ASTP) with 8 cities defined by the following matrix

$$\begin{bmatrix} - & 76 & 43 & 38 & 51 & 42 & 19 & 80 \\ 42 & - & 49 & 26 & 78 & 52 & 39 & 87 \\ 48 & 28 & - & 40 & 63 & 44 & 68 & 61 \\ 72 & 31 & 29 & - & 42 & 49 & 50 & 38 \\ 30 & 52 & 38 & 47 & - & 64 & 72 & 82 \\ 66 & 51 & 83 & 51 & 22 & - & 37 & 71 \\ 77 & 62 & 93 & 54 & 69 & 38 & - & 26 \\ 42 & 58 & 66 & 76 & 41 & 52 & 83 & - \end{bmatrix}$$

To determine the starting city using MVM, we reduce the matrix to

$$\begin{bmatrix} - & 57 & 24 & 19 & 32 & 12 & \mathbf{0} & 61 \\ 16 & - & 23 & \mathbf{0} & 52 & 15 & 13 & 61 \\ 20 & \mathbf{0} & - & 12 & 35 & 5 & 13 & 33 \\ 43 & 2 & \mathbf{0} & - & 13 & 9 & 21 & 9 \\ \mathbf{0} & 22 & 8 & 17 & - & 23 & 42 & 52 \\ 44 & 29 & 61 & 29 & \mathbf{0} & - & 15 & 49 \\ 51 & 36 & 67 & 28 & 43 & 1 & - & \mathbf{0} \\ 1 & 17 & 25 & 35 & \mathbf{0} & 0 & 42 & - \end{bmatrix}$$

The Vogel penalties of the rows and the columns are

$p_1 = 12$; $p_2 = 13$; $p_3 = 5$; $p_4 = 2$; $p_5 = 8$; $p_6 = 15$; $p_7 = 1$; $p_8 = 0$

$q_1 = 1$; $q_2 = 2$; $q_3 = 8$; $q_4 = 12$; $q_5 = 0$; $q_6 = 1$; $q_7 = 13$; $q_8 = 9$

The largest penalty is therefore $p_6 = 15$ the starting city is then 6 which yields the tour

$(6 \to 5 \to 1 \to 7 \to 8 \to 2 \to 4 \to 3 \to 6) \qquad TC = 254$

This solution tour is presented in bold on the following matrix

$$\begin{bmatrix} - & 57 & 24 & 19 & 32 & 12 & \mathbf{0} & 61 \\ 16 & - & 23 & \mathbf{0} & 52 & 15 & 13 & 61 \\ 20 & 0 & - & 12 & 35 & \mathbf{5} & 13 & 33 \\ 43 & 2 & \mathbf{0} & - & 13 & 9 & 21 & 9 \\ \mathbf{0} & 22 & 8 & 17 & - & 23 & 42 & 52 \\ 44 & 29 & 61 & 29 & \mathbf{0} & - & 15 & 49 \\ 51 & 36 & 67 & 28 & 43 & 1 & - & \mathbf{0} \\ 1 & \mathbf{17} & 25 & 35 & 0 & 0 & 42 & - \end{bmatrix} \quad \begin{matrix} mz_1 = 0 \\ mz_2 = 0 \\ mz_3 = 5 \\ mz_4 = 0 \\ mz_5 = 0 \\ mz_5 = 0 \\ mz_7 = 0 \\ mz_8 = 17 \end{matrix}$$

By considering the missing zero in the column at the right, note that the sum mz of the missing zeros, i.e. $mz = \sum_{i=1}^{8} mz_i = 22$ is the total reduced cost.

Note that if we were applying a Local Search algorithm then local search has to be performed near the cities 8 and 3. They were the only cities with non-null missing zeros $mz_8 = 17$ and $mz_3 = 5$.

Similarly, if we were applying an Evolutionary algorithm (EAs) then we can consider all the population capable of improving the reduced cost. They are three possibilities for row 8 and one for row 3.

In the continuation of applying MVM-NNA, we also notice that the largest missing zero is provided by row 8. To improve the solution, we can apply an NNA iteration by considering the new starting city 8 and then decreasing the reduced cost 17 to the next lowest 1. This connects the city 8 to 1 and induces the solution tour

$(8 \to 1 \to 7 \to 6 \to 5 \to 3 \to 2 \to 4 \to 8) \qquad TC = 251$

This solution tour is presented in bold on the following matrix

$$\begin{bmatrix} - & 57 & 24 & 19 & 32 & 12 & \mathbf{0} & 61 \\ 16 & - & 23 & \mathbf{0} & 52 & 15 & 13 & 61 \\ 20 & \mathbf{0} & - & 12 & 35 & 5 & 13 & 33 \\ 43 & 2 & 0 & - & 13 & 9 & 21 & 9 \\ 0 & 22 & \mathbf{8} & 17 & - & 23 & 42 & 52 \\ 44 & 29 & 61 & 29 & \mathbf{0} & - & 15 & 49 \\ 51 & 36 & 67 & 28 & 43 & \mathbf{1} & - & 0 \\ \mathbf{1} & 17 & 25 & 35 & 0 & 0 & 42 & - \end{bmatrix} \quad \begin{matrix} mz_1 = 0 \\ mz_2 = 0 \\ mz_3 = 0 \\ mz_4 = 9 \\ mz_5 = 8 \\ mz_6 = 0 \\ mz_7 = 1 \\ mz_8 = 1 \end{matrix}$$

The total reduced cost is $mz = 19$ and can no longer be improved by decreasing the reduced assigned cost of row 8. Since the only other initially non-null missing zero of the third row becomes equal to zero, we are in the situation of Theorem 3 and the solution tour associated to the total cost $TC^* = 251$ is optimal.

This example was introduced in [12] and solved by using the Branch and Bound method. Seven assignment problems, with a complexity of n^3, were solved while just 3 NA iterations, with a complexity of n^2, were necessary for the MVM-NNA algorithm.

V. CONCLUSION

In this paper, we proposed a new heuristic approach called MVM-NNA to improve the NNA for solving the Asymmetric Traveling Salesman Problem. Note that we do not apply a complete MVM method but just the first iteration of that method in order to determine the best-starting city for the NNA algorithm. The MVM-NNA method provides the best-starting city for the NNA and therefore avoids the worst case scenario. In some cases, the algorithm detects the optimality of the tour and also indicates a way for improvement.

Using MVM-NNA may also improve the local search algorithms [5, 9] by identifying the neighborhood of the network where the search should be processed. The approach can also be used to constitute more performed sample of populations for the Evolutionary Algorithm (EAs) [13].

The MVM-NNA method can also be adapted for the Symmetric Traveling Salesman Problem (STSP). Because of the matrix symmetry, the algorithm can be considerably simplified. Furthermore, the MVM-NNA opens a new approach which links the TSP to the technique used to solve some transportation models.

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