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A REPRODUCING KERNEL METHOD FOR SOLVING A CLASS OF FRACTIONAL DIFFERENTIAL EQUATIONS

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***Abstract-* This paper is devoted to the numerical treatment of a class of fractional differential equations. At first, we use Caputo fractional differential definition transform the fractional differential equations, and then solve it by reproducing kernel method. Three numerical examples are studied to demonstrate the accuracy of the present method. Results obtained by present method show that the present method is simple and accurate.**

Key Words-Reproducing Kernel; fractional differential equations; Caputo fractional differential definition; numerical solution.

1. INTRODUCTION

About fractional calculus, some early mathematicians have noticed, and have began the study on them, that already has three hundred years history. Compared with integer calculus, the application of fractional calculus is more universal , and it can solve the problems of many fields. For example, diffusion and transport theory, chain of polymer material, random walk, non-Newtonian fluid mechanic, relaxation, oscillation, control system and so on. some properties of fractional calculus is similar with integer calculus. With the deepening of the research, predecessors have given various methods for solving fractional differential equations. For different fractional differential equations we can use different methods. In this paper we will use reproducing kernel method to solve it.

In the past, reproducing kernel method has been used in many areas [1, 2, 3, 4]. In this paper, we select suitable nodes, using reproducing method to solve fractional differential equations.

Consider the following fractional differential problems,

$$\begin{cases} a(x)u'(x)+D^{\alpha}u(x)+b(x)u(x)=f(x), x\in[0,1] \\ u(0)=0 \end{cases} \quad (1)$$

Where $0<\alpha\le 1$, D^{α} denotes the fractional derivative order α, $a(x)$, $b(x)$ and $f(x)$ are sufficiently smooth known functions on [0,1].

2.Reproducing kernel method for fractional differential equations

2.1 The definition of fractional differential equations

In order to solve fractional differential equations in reproducing kernel Hilbert space, we introduce the Caputo fractional differential definition [5, 6].

Definition 2.1. The α -order Caputo derivative of $u(x)$ is defined as,

$$D^{\alpha}u(x)=\frac{1}{\tau(m-\alpha)}\int_0^x (x-t)^{m-\alpha-1}u^{(m)}(t)dt$$

(2)

Where $\alpha>0$, $x>0$, $m-1<\alpha<m$, $m\in N$.

Take (2) into (1), the equation (1) is equivalent to the equation (3)

$$\begin{cases} a(x)u'(x)+\dfrac{1}{\tau(m-\alpha)}\int_0^x (x-t)^{m-\alpha-1}u^{(m)}(t)dt+b(x)u(x), x\in[0,1] \\ u(0)=0 \end{cases} \quad (3)$$

2.2 Construct reproducing kernel Hilbert space

To solve (3) using the reproducing kernel method, we need to introduce a linear space $W_2^{\,3}[0,1]$.

$W_2^{\,3}[0,1]=\{u \mid u,u',u''$ is one - variable absolutely continuous function, $u'''\in L^2[0,1], u(0)=0,$ $u'(0)=0\}$

We give the inner product,

$$\langle u(y),v(y)\rangle = u''(0)v''(0)+\int_0^b u'''(y)v'''(y)dy \quad (4)$$

and according to [7, 8], we can prove that $W_2^{\,3}[0,1]$ is a reproducing kernel space, it's reproducing kernel $R(x,y)$ is

$$R(x,y)=\begin{cases} \dfrac{1}{120}(120+x^5+120xy-5x^4y)+30x^2y^2+10x^3y^2, x<y \\ \dfrac{1}{120}(120+y^5+10x^2y^3(3+y)-5xy(y^3-24)), y<x \end{cases} \quad (5)$$

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2.3 The approximate solution

In order to solve (3), let's give a linear reversible operator L, then, (3) can be represented as follows:

$$\begin{cases}(Lu)(x) = f(x), x \in [0,1] \\ u(0) = 1\end{cases} \tag{6}$$

we can get $\psi_i(x)$,

$$\psi_i(x) = \begin{cases}(\frac{1}{24}x(4x(3+x)y + n(24 - x^3 + 12xy + 4x^2y)))|_{y=x_i}, x < y \\ (\frac{1}{24}(4y(-3xy^2 + y^3 + 3x^2(1+y)) + n(y^4 + 6x^2y(2+y) - 4x(-6+y^3))))|_{y=x_i}, y < x\end{cases} \tag{7}$$

where $i = 1,2,3...$

Then practise Gram-Schmidt orthogonalization $\{\psi_i(x)\}_{i=1}^{\infty}$, according to [9], [10] , we get

$$\begin{cases}\overline{\psi}_1(x) = \beta_{11}\psi_1(x) \\ \overline{\psi}_2(x) = \beta_{21}\psi_1(x) + \beta_{22}\psi_2(x) \\ \overline{\psi}_3(x) = \beta_{31}\psi_1(x) + \beta_{32}\psi_2(x) + \beta_{33}\psi_3(x) \\ \vdots \\ \overline{\psi}_i(x) = \beta_{i1}\psi_1(x) + \beta_{i2}\psi_2(x) + \beta_{i3}\psi_3(x) + \cdots + \beta_{ii}\psi_i(x)\end{cases} \tag{9}$$

Where β_{ik} are coefficients of Gram-Schmidt orthogonalization.

If $\{x_i\}_{i=1}^{\infty}$ is distinct point dense in [0,b], and L^{-1} is existent, we get

$$u(x) = \sum_{i=1}^{\infty}\sum_{k=1}^{i}\beta_{ik}F(x_k)\overline{\psi}_i(x) \tag{10}$$

is the solution of (1). The proof of it refers to[11], [12] we can easily prove this conclusion.

2.4 Error analysis

We make truncation error of (10), obviously,

$$u(x) = \sum_{i=1}^{\infty}\sum_{k=1}^{i}\beta_{ik}\overline{F}(x_k)\overline{\psi}_i(x) \tag{11}$$

is the numerical solution of (1), on the basis of [13, 14], we can easily to get that $\|u_m(x) - u(x)\| \to 0$, and $u_m^{(k)}(x) \to u^{(k)}(x), k = 0,1,2$.

3 .Numerical experiment

In this section, three numerical examples are studied to demonstrate the accuracy of the present method for fractional differential equations.

Example 1[13] Consider the following fractional differential equation.

$$\begin{cases}u'(x) + D^{\frac{1}{4}}u(x) + u(x) = \frac{5}{2}x^{\frac{3}{2}} + x^{\frac{5}{2}} + \frac{15\sqrt{\pi}}{8\tau(\frac{13}{4})}x^{\frac{9}{4}}, x \in [0,1] \\ u(0) = 0\end{cases}$$

The exact solution is $u_T(x) = x^2\sqrt{x}$. The numerical results are shown in Figure 1 and Table 1.

Example 2[14] Consider the following fractional differential equation.

$$\begin{cases}u'(x) + D^{0.8}u(x) + (1+x)u(x) = \frac{1.4}{\tau(3.8)}x^{1.8} + 2.5x^2 + \frac{5}{\tau(3.8)}(1+x)x^{2.8}, x \in [0,1] \\ u(0) = 0\end{cases}$$

The exact solution is $u_T(x) = \frac{5}{\tau(3.8)}x^{2.8}$. The numerical results are shown in Figure 2 and Table2.

Example 3[15] Consider the following fractional differential equation.

$$\begin{cases}u'(x) + D^{\frac{1}{2}}u(x) = 1 + \frac{2}{\sqrt{\pi}}x^{\frac{1}{2}}, x \in [0,1] \\ u(0) = 0\end{cases}$$

The exact solution is $u_T(x) = x$. The numerical results are shown in Figure 3 and Table3.

Table 1The numerical results of Example1

x	$u_T(x)$	$u_{20}(x)$	$\lvert u_T(x) - u_{20}(x)\rvert$	$\lvert u_T'(x) - u_{20}'(x)\rvert$
0.1	0.0031	0.0029	1.766E−04	9.573E−05
0.2	0.0178	0.0178	5.501E−05	3.438E−04
0.3	0.0492	0.0493	4.646E−05	6.610E−04
0.4	0.1011	0.1013	1.376E−04	9.348E−04
0.5	0.1767	0.1770	2.237E−04	1.192E−03
0.6	0.2788	0.2791	3.081E−04	1.449E−03
0.7	0.4099	0.4103	3.936E−04	1.715E−03
0.8	0.5724	0.5729	4.822E−04	1.996E−03
0.9	0.7684	0.7690	5.759E−04	2.299E−03
1	1	1.0006	6.764E−04	2.630E−03

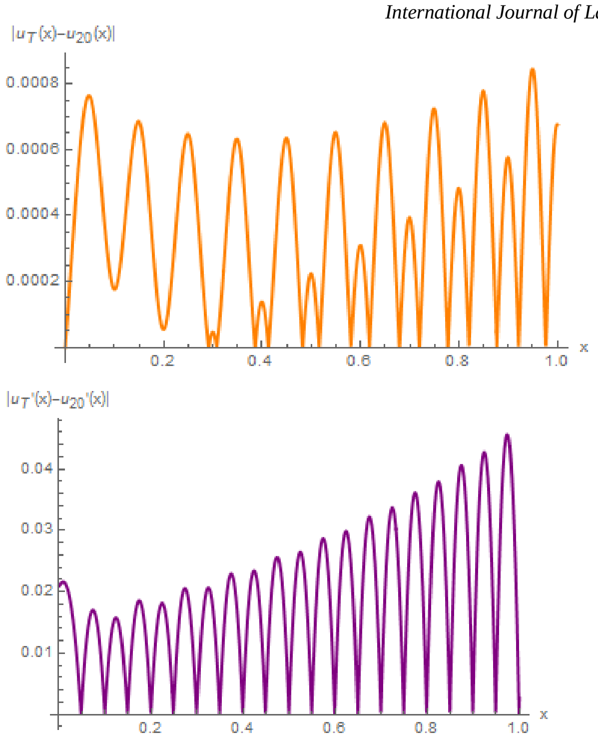
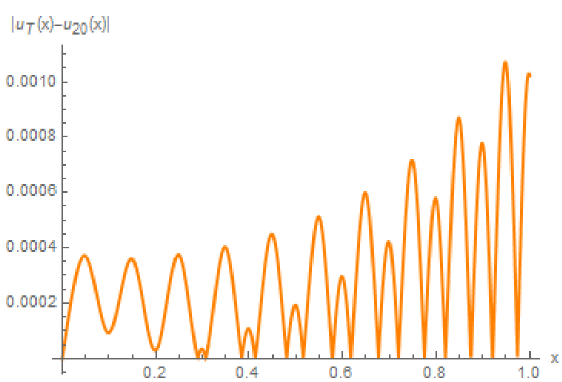


Figure 1: The reproducing kernel method for Example 1, the first picture is $|u_T(x)-u_{20}(x)|$, the second picture is $|u_T'(x)-u_{20}'(x)|$

Table 2 The numerical results of Example2

x	$u_T(x)$	$u_{20}(x)$	$\lvert u_T(x)-u_{20}(x)\rvert$	$\lvert u_T'(x)-u_{20}'(x)\rvert$
0.1	0.0016	0.0015	9.303E−05	5.436E−04
0.2	0.0117	0.0117	3.045E−05	9.119E−04
0.3	0.0365	0.0366	3.352E−05	1.266E−03
0.4	0.0818	0.0819	1.057E−04	1.688E−03
0.5	0.1529	0.1531	1.906E−04	2.221E−03
0.6	0.2548	0.2551	2.932E−04	2.887E−03
0.7	0.3923	0.3927	4.191E−04	3.747E−03
0.8	0.5702	0.5708	5.758E−04	4.854E−03
0.9	0.7930	0.7938	7.728E−04	6.285E−03
1	1.0651	1.0661	1.022E−03	8.140E−03



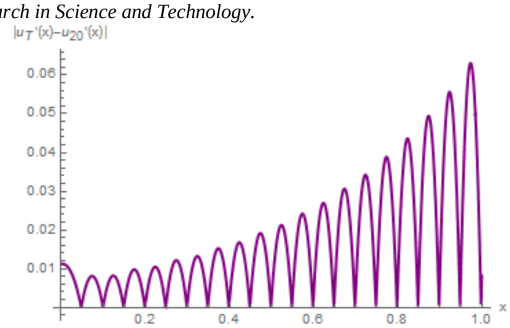


Figure 2: The reproducing kernel method for Example 2, the first picture is $|u_T(x)-u_{20}(x)|$, the second picture is $|u_T'(x)-u_{20}'(x)|$

Table 3 The numerical results of Example3

x	$u_T(x)$	$u_{20}(x)$	$\lvert u_T(x)-u_{20}(x)\rvert$	$\lvert u_T'(x)-u_{20}'(x)\rvert$
0.1	0.1	0.0996	3.135E−04	5.016E−04
0.2	0.2	0.1997	2.768E−04	2.773E−04
0.3	0.3	0.2997	2.538E−04	1.933E−04
0.4	0.4	0.3997	2.369E−04	1.481E−04
0.5	0.5	0.4997	2.236E−04	1.196E−04
0.6	0.6	0.5997	2.127E−04	9.995E−05
0.7	0.7	0.6997	2.035E−04	8.554E−05
0.8	0.8	0.7998	1.955E−04	7.454E−05
0.9	0.9	0.8998	1.885E−04	6.578E−05
1	1	0.9998	1.823E−04	5.887E−05

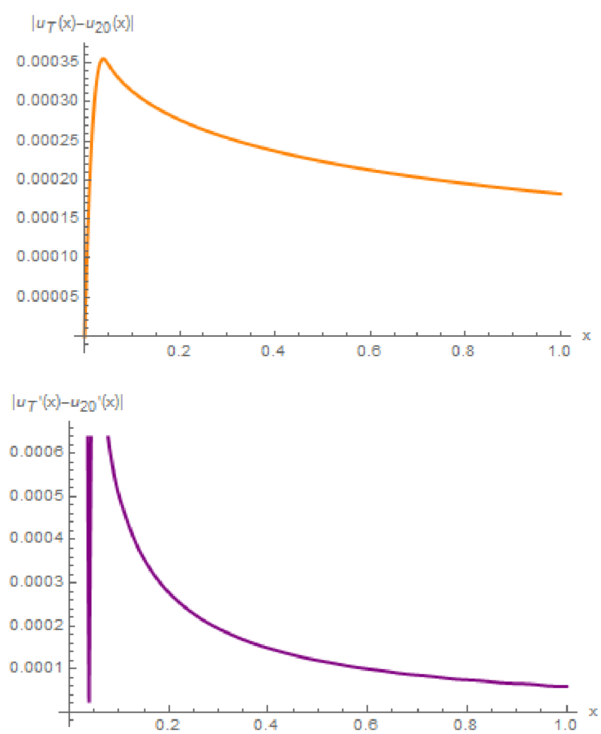


Figure 3: The reproducing kernel method for Example 3, the first picture is $\left|u_T(x)-u_{20}(x)\right|$, the second picture is $\left|u_T'(x)-u_{20}'(x)\right|$

4 CONCLUSIONS AND REMARKS

In this paper, we devote to the numerical treatment of a class of fractional differential equations. The numerical results demonstrate that the method is quite accurate and efficient for fractional differential equations. This makes it easy to solve fractional differential equations. All computations are performed by the Mathematica10.2 software package.

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