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FREE, VISCOUS INERTIAL JET

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***Abstract-* In some new results are established which include the modification of the solutions of some equations and the correction of some derivations**

Keywords: Viscous Inertial Jet -Jet breakup.

I. INTRODUCTION

The requirement of conservation of charge can be stated a follows suppose at the nozzle one introduces in to the fluid stream a current $\dot{Q}$, which manifests itself as surface charge. Then

$$\dot{Q}=\sigma(z)2\pi t u_z \qquad (1)$$

And the electrical stress on the surface of the jet is

$$f_E=\frac{\sigma^2}{2\varepsilon_0}\frac{\dot{Q}^2\rho}{8\varepsilon_0\pi q u_z} \qquad (2)$$

The Navier-Stokes equation for u_z then reduces to

$$\rho u_z\frac{du_z}{dz}=\rho g-\left[\frac{\dot{Q}^2\rho}{8\pi\varepsilon_0 q u_z}+\frac{\pi\rho\gamma^2}{4qu_z}\right]\frac{du_z}{dz} \qquad (3)$$

This has solution

$$u_z^2\left(1+\frac{4\pi\gamma^2}{\rho q u_z^3}-\frac{\dot{Q}^2}{4\pi\varepsilon_0 q u_z^3}\right)-w_0^2\left(1+\frac{4\pi\gamma^2}{\rho q w_0^3}-\frac{\dot{Q}^2}{4\pi\varepsilon_0 q w_0^3}\right)=2gz \qquad (4)$$

Alternatively, in terms of the jet radius t

$$\left(\frac{1}{t^4}-\frac{1}{t_0^4}\right)+\frac{2\gamma\rho\pi^2}{q^2}\left(\frac{1}{t}-\frac{1}{t_0}\right)+\frac{\dot{Q}^2\rho^2\pi^2}{4\varepsilon_0 q^4}(t^2-t_0^2)=\frac{2\pi^2\rho^2 g}{q^2}z \qquad (5)$$

Using the dimensionless quantities defined in equation

$$\tau=\frac{\mathrm{t}}{\mathrm{t}_\circ},\alpha=\frac{2\gamma}{\rho \mathrm{t}_\circ\omega_\circ^2}\qquad \xi=\left(\frac{2\mathrm{g}}{\omega_\circ^2}\right)\mathrm{z} \qquad (6)$$

With

$$\beta=\frac{\dot{Q}^2\rho^2\pi^2}{4\varepsilon_0 q^4}t_0^6 \qquad (7)$$

One has the dimensionless equation

$$\left(\frac{1}{\tau^4}-1\right)+\alpha\left(\frac{1}{\tau}-1\right)+\beta\left(\tau^2-1\right)=\xi \qquad (8)$$

The behavior of τ versus ξ is also plotted in figure (1) for $\beta=1,\alpha=0$. It is seen that surface charge has the opposite effect from surface tension, producing a more rapid variation of jet radius with length.

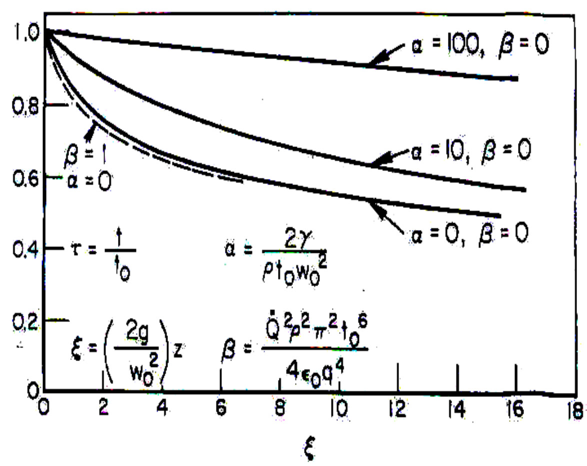

Figure (1)

It can be noted from figure (1) that, as stated earlier, jet radius is not a strong function of axial distance. Thus, as an approximation, one can locally apply the stability criteria developed for the cylindrical jet. For the inviscid jet with surface charge, the stability criterion for sausage instability was given (with a = t), from equation

$$\omega^2+\frac{3\mu k^2}{\rho}\omega-\left[\frac{\gamma}{2\rho a^3}(1-\eta^2)\eta^2-\frac{\sigma_0^2\eta^2}{2\varepsilon_0\rho a^2}\right]=0$$

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$$\omega^2 = \left[\frac{\gamma}{2\rho t^3}(1-\eta^2)\eta^2 - \frac{\sigma_0^{\ 2}}{2\varepsilon_0 \rho t^2}\eta^2 \right] \qquad (9)$$

The maximum in stability occurs at

$$\eta^2 = \frac{1}{2} - \frac{\sigma_0^{\ 2} t}{2\varepsilon_0 \gamma} \qquad (10)$$

Note that when $\sigma_0 = 0, \eta^2 = 1/2$, the approximate Raleigh result.

Substituting this maximum into equation (9), we obtain for the maximum instability

$$\omega^2 = \left[\frac{\gamma}{2\rho t^3} - \frac{\sigma_0^{\ 2}}{2\varepsilon_0 \rho t^2} \right] \div \frac{2\gamma}{\rho t^3} \qquad (11)$$

Thus for charged jet there exists a critical value of t at which the jet becomes unstable. This is given from equation (11) as

$$\frac{\gamma}{t^3} = \frac{\sigma_0^{\ 2}}{\varepsilon_0 t^2} \qquad (12)$$

However, for the inertial jet, σ_0 is a function of t according to equation (1)

$$\sigma_0(z) = \frac{\dot{Q}}{2\pi t u_z} = \frac{\dot{Q}\rho t}{2q} \qquad (13)$$

Substituting into equation (12) one has

$$\frac{\gamma}{t_c^{\ 3}} = \frac{\dot{Q}^2 \rho^2}{4\varepsilon_0 q^2}$$

So that the critical radius for jet breakup is

$$t_c^{\ 3} = \frac{4\varepsilon_0 q^2 \gamma}{\dot{Q}^2 \rho^2} \qquad (14)$$

Substituting into equation

$$\mathrm{q} = \rho\pi \mathrm{t}^2 \mathrm{u}_z = \rho\pi \mathrm{t}_\circ^2 \omega_0 \qquad (15)$$

Gives the critical velocity, and into (5) gives the length of the jet at breakup. In terms of the dimensionless parameters

$$\tau_c^{\ 3} = \left(\frac{t_c}{t_0} \right)^3 = \frac{\alpha}{2\beta} \qquad (16)$$

Of course, if τ_c > 1, no breakup is predicted.

Free, Viscous Inertial Jet

If one examines the force balance on a differential element of the inertial jet, one has

$$\Delta\int_0^t 2\pi r dr \rho u_z^{\ 2} = \Delta\int_0^t 2\pi r dr \tau_{zz} + \rho g \pi t^2 \Delta z + 2\pi t \Delta z \tau_{0_{rz}} \quad (17)$$

Where $\tau_{0_{rz}}$ is the surface shear on the jet, or hence

$$\frac{\partial}{\partial z}\int_0^t 2\pi r dr \rho u_z^{\ 2} = \frac{\partial}{\partial z}\int_0^t 2\pi r dr \tau_{zz} + \rho g \pi t^2 + 2\pi t \tau_{0_{rz}} \quad (18)$$

For a free jet, $\tau_{0_{rz}} = 0$, and assuming $u_z = u_z(z)$ only and $\tau_{zz} = \tau_{zz}(z)$ only gives the differential equation

$$\frac{\partial u_z}{\partial z} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\frac{\tau_{zz}}{u_z} \right) + \frac{g}{u_z} \qquad (19)$$

The stresses are

$$\left. \begin{aligned} \tau_{rr} &= -p + 2\mu \frac{\partial u_r}{\partial r} \\ \tau_{zz} &= -p + 2\mu \frac{\partial u_z}{\partial z} \end{aligned} \right\} \qquad (20)$$

The continuity equation gives

$$u_r = -\frac{1}{2} r \frac{\partial u_z}{\partial z}$$

Or hence

$$\frac{\partial u_r}{\partial r} = -\frac{1}{2}\frac{\partial u_z}{\partial z} \qquad (21)$$

With

$$\tau_{rr} = -\frac{\gamma}{t} \qquad (22)$$

We then have from $(20), (21)$ and (22)

$$\tau_{zz} = -\frac{\gamma}{t} + 3\mu \frac{\partial u_z}{\partial z} \qquad (23)$$

Where t and u_z are related by equation (15) Substituting (23) into (19) Gives

$$\frac{\partial u_z}{\partial z} = \frac{3\mu}{\rho}\frac{\partial}{\partial z}\left(\frac{1}{u_z}\frac{\partial u_z}{\partial z} \right) + \frac{\gamma}{2\rho t_0 \sqrt{w_0} u_z^{3/2}}\frac{\partial u_z}{\partial z} + \frac{g}{u_z} \quad (24)$$

Which is the differential equation for $u_z(z)$, and hence t (z). One can put this into dimensionless form as follows. Equation (26)

$$w'' = \left(w + \frac{w'}{w} - \frac{\lambda}{w} \right) w' - 1 \qquad (26)$$

Dos not follow from the preceding equation (24) on condition the analysis from equation (24) it was found that the constant

λ dos not lead to anon dimensional equation we found that the a approximate λ should be

$$\lambda=\frac{1}{2}\left(\frac{\gamma}{\rho t_0\sqrt{w_0}}\right)\left(\frac{\rho}{3\mu g}\right)^{\frac{3}{2}} \qquad (25)$$

Correction of some derivations

To obtain the required form. Below we give the calculate of this equation. We start from equation (24)

Let $w=\left(\frac{\rho}{3\mu g}\right)^{\frac{1}{3}}u_z$, $k=\left(\frac{\rho}{3\mu g}\right)^{\frac{1}{3}}$

$,w=ku_z$ $,u_z=\frac{w}{k}$, $\xi=\left(\frac{\rho}{3\mu g}\right)^{\frac{2}{3}}gz$

$,\ \xi=k^2gz$ $,z=k^{-2}g^{-1}\xi$

$$\frac{\partial u_z}{\partial z}=\frac{\partial u_z}{\partial \xi}\frac{\partial \xi}{\partial z}=k^2g\frac{\partial u_z}{\partial \xi}=\frac{\partial w}{\partial \xi}\frac{\partial u_z}{\partial w}\frac{\partial \xi}{\partial z}=\frac{\partial w}{\partial \xi}\frac{1}{k}k^2g=kg\frac{\partial w}{\partial \xi}$$

$$\frac{\partial u_z}{\partial z}=kg\,\frac{\partial w}{\partial \xi} \quad ,\ \frac{\partial}{\partial z}=\frac{\partial}{\partial \xi}\frac{\partial \xi}{\partial z}$$

$$\frac{\partial u_z}{\partial z}=\frac{3\mu}{\rho}\frac{\partial}{\partial z}\left(\frac{1}{u_z}\frac{\partial u_z}{\partial z}\right)+\frac{\gamma}{2\rho t_0\sqrt{w_0}u_z^{3/2}}\frac{\partial u_z}{\partial z}+\frac{g}{u_z}$$

$$kg\frac{\partial w}{\partial \xi}=\frac{3\mu g}{\rho}\frac{1}{g}\frac{\partial \xi}{\partial z}\frac{\partial}{\partial \xi}\left(\frac{k}{w}kg\frac{\partial w}{\partial \xi}\right)+\frac{\gamma k^{3/2}}{2\rho t_0\sqrt{w_0}w^{3/2}}kg\frac{\partial w}{\partial \xi}+kg\frac{1}{w}$$

$$kg\frac{\partial w}{\partial \xi}=\frac{3\mu g}{\rho}\frac{1}{g}k^2g\frac{\partial}{\partial \xi}\left(\frac{k}{w}kg\frac{\partial w}{\partial \xi}\right)+\frac{\gamma k^{3/2}}{2\rho t_0\sqrt{w_0}w^{3/2}}kg\frac{\partial w}{\partial \xi}+kg\frac{1}{w}$$

$$kgw'=kg\,\frac{\partial}{\partial \xi}\left(\frac{w'}{w}\right)+\frac{\gamma k^{3/2}kgw'}{2\rho t_0\sqrt{w_0}w^{3/2}}+\frac{kg}{w}$$

$$w'=\frac{\partial}{\partial \xi}\left(\frac{w'}{w}\right)+\frac{\gamma k^{3/2}w'}{2\rho t_0\sqrt{w_0}w^{3/2}}+\frac{1}{w}$$

$$w'=\left(\frac{-1}{w^2}(w')^2+\frac{w''}{w}\right)+\frac{1}{w}+\lambda\frac{w'}{w^{3/2}}$$

$$ww'=-\frac{(w')^2}{w}+w''+1+\lambda\frac{w'}{w^{1/2}}$$

$$w''=ww'+\frac{(w')^2}{w}-1-\lambda\frac{w'}{w^{1/2}}$$

$$w''=\left(w+\frac{w'}{w}-\frac{\lambda}{w^{1/2}}\right)w'-1$$

In a similar manner the equation which includes the effects of surface charge in this formulation has been obtained in the form

$$w''=\left(w+\frac{w'}{w}-\frac{\lambda}{w}+\frac{\Omega}{w^2}\right)w'-1 \qquad (27)$$

While the correct form as before, should be

$$w''=\left(w+\frac{w'}{w}-\frac{\lambda}{w^{1/2}}+\frac{\Omega}{w^2}\right)w'-1$$

CONCLUSION

In this paper the derivation of equation (27) from equation (24) was in error in the original text [1]. The correct version is then equation (27)

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