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INNOVATIVEMETHOD FOR SOLVING FUZZY LINEAR PROGRAMMING PROBLEMS WITH SYMMETRIC TRAPEZOIDAL FUZZY NUMBERS

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Abstract-In this paper fully fuzzy linear programming problems with symmetric trapezoidal fuzzy numbers are discussed. A new method for solving fully fuzzy linear programming problems (FFLPP) is proposed based upon the new Ranking function. The proposed method is very easy to understand and it can apply for fully fuzzy linear programming problems occurring in real life situations as compared to the earlier methods.

Keywords–Fuzzy number, Trapezoidal fuzzy number, Ranking function, Fuzzy variable linear programming, Fully fuzzy linear programming, Fuzzy optimal solution.

I. INTRODUCTION

A linear programming problem is a special case of a mathematical programming problem. From an analytical perspective, a mathematical program tries to identify an extreme point of a function, which furthermore satisfies a set of constraints. In real world any linear programming model involves parameters whose values are assigned by experts. However, they usually cannot assign exact values to these important parameters. The decision maker has to deal with uncertainty. Representing these parameters as fuzzy data and dealing with them using the concepts of fuzzy theory seems to be the most suitable way to face the problem of uncertainty. Fuzzy logic is a form of many valued logic derived from fuzzy set theory to deal with reasoning that is fluid or approximate rather than fixed and exact. In contrast with crisp logic, where binary sets have two valued logic, fuzzy logic variables many have a truth value that ranges in degree between 0 and 1.The formulation of Fuzzy Linear Programming Problem (FLPP) was introduced by Zimmermann [13]. Many researchers proposed various types of FLPP and solve these problems using different methods. The concept of Fuzzy logic was first conceived by Loft Zadeh, a professor at the university of California at Berkley, but as a way of processing data by allowing partial set membership rather than crisp set membership or non-membership.

Allahviranloo et al. [2] proposed a new method for solving fully fuzzy linear programming problems by the use of ranking function and also solved the fuzzy integer linear programming problem by reducing it into a crisp integer linear programming problem. Amit Kumar et al. [3, 4] proposed a new method for solving fuzzy linear programs with Trapezoidal fuzzy numbers. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some level of trapezoidal fuzzy numbers. Bellman and Zadeh [5] proposed the concept of decision making in fuzzy environment. Ganesan and Veeramani [7] proposed an approach to solve a fuzzy linear programming problem involving symmetric trapezoidal fuzzy numbers without converting it into crisp linear programming problem.

Senthilkumar and Rajendran [11] proposed the solution of fuzzy linear programming problem. In 1965, Zadeh [12] introduced the concept of fuzzy set theory. Nasseri et al. [10] proposed a new method for solving linear programming problem with fuzzy right hand sides and also used the complementary slackness theorem to solve fuzzy linear programming problem with fuzzy parameters. Khabiri et al. [8] proposed a starting fuzzy solution for the fuzzy primal simplex algorithm using a fuzzy two phase method. MahdaviAmiri et al. [9] proposed a fuzzy primal simplex algorithm for solving fuzzy linear programming problems.

Fang et al. [6] presented a method for solving linear programming problems with fuzzy coefficients in constraints. Though fuzzy logic has been applied to many fields, from control theory to artificial intelligence, it still remains controversial among most statisticians, who prefer Bayesian logic, and some control engineers, who prefer traditional two-valued logic

Today, Fuzzy logic concept is used widely in many implementations like automobile engine and automatic gear control systems, air conditioners, automatic focus control, video enhancement in TV sets, washing machines, behavior-based mobile robots, sorting and handling data, Information systems, traffic control systems and so on.

Some preliminaries are presented in section 2. Section 3 describes the earlier methods. The section 4 illustrates the

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new approach with one numerical example. Obtained results are discussed in section 5. Section 6 concludes the paper.

II. PRELIMINARIES

Definition 2.1

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. this function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu_{\tilde{A}} : X \rightarrow [0,1]$. The assigned value indicate the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for $x \in X$ is called fuzzy set.

Definition 2.2

A ranking function is a function $\Re$:F(R) → R. Where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (m, n, \alpha, \beta)$ be a trapezoidal fuzzy number then $\Re(\tilde{A}) = \frac{m+n+\alpha+\beta}{4}$

Definition 2.3

A fuzzy number $\tilde{A} = (m, n, \alpha, \beta)$ is said to be a trapezoidal fuzzy number if its membership function ($\mu_{\tilde{A}}(x)$) is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{m-x}{\alpha}, m - \alpha \le x \le m, \alpha > 0 \\ 1, \quad m < x < n \\ 1 - \frac{x-n}{\beta}, n \le x \le n+\beta, \beta > 0 \\ 0, \quad otherwise \end{cases}$$

Definition 2.4

A fuzzy number $\tilde{A} = (m, n, \alpha, \alpha)$ is said to be a Symmetric Trapezoidal fuzzy number if its membership function ($\mu_{\tilde{A}}(x)$) is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x+\alpha-m}{\alpha}, m - \alpha \le x \le m, \alpha > 0 \\ 1, \quad m < x < n \\ \frac{-x+n+\alpha}{\alpha}, n \le x \le n+\alpha, \alpha > 0 \\ 0, \quad \text{otherwise} \end{cases}$$

Definition 2.5

The concept of Fuzzy decision making was first proposed by Bellman and Zadeh [5]. A linear programming problem is called fuzzy variable linear programming problem (FVLPP), if some of the parameters are crisp, and variables and right hand sides are fuzzy numbers. General form of FVLPP as follows:

Max (min) $\tilde{z} = c_j\tilde{x}_j$

Subject to

$A_{ij}\tilde{x}_j \le \tilde{B}_i$,

$\tilde{x}_j \ge 0$ Where $c_j \varepsilon R^n$, $A_{ij}\varepsilon(R)^{m\times n}$, $\tilde{B}_i\varepsilon(F(R))^m$ $\tilde{x}_j\varepsilon(F(R))^n$

Definition 2.6

Kolman and Hill was introduced a FFLP problem. A linear programming problem is called fully fuzzy linear programming problem (FFLPP), if parameters and variables are all fuzzy numbers. General form of FFLPP as follows:

Max (min) $\tilde{z} = \tilde{c}_j\tilde{x}_j$

Subject to

$\tilde{A}_{ij}\tilde{x}_j \le \tilde{B}_i$,

$\tilde{x}_j \ge 0$ Where $\tilde{c}_j\varepsilon$ $(F(R))^n$, $\tilde{A}_{ij}\varepsilon$ $(F(R))^{m\times n}$, $\tilde{B}_i\varepsilon$ $(F(R))^m$, $\tilde{x}_j\varepsilon(F(R))^n$

III. EARLIER METHOD

The Fuzzy optimal solutions of FFLP problems with inequality constraints are representing all the parameters as trapezoidal fuzzy numbers.

Maximize (Minimize) ($\tilde{C} \otimes \tilde{x}$)

Subject to

$\tilde{A} \otimes \tilde{x} \le, =, \ge, \tilde{b}$

Where $\tilde{x}$is a non-negative trapezoidal fuzzy number and $\tilde{C} = [\tilde{c}_j]_{1\times n}$, $A = [\tilde{a}_{ij}]_{m\times n}$

$\tilde{x} = [\tilde{x}_j]_{n\times 1}$ $\tilde{b} = [\tilde{b}_i]_{m\times 1}$

Example 3.1

Max (10, 14, 3, 3)$\tilde{x}_1$ + (13, 15, 5, 5)$\tilde{x}_2$ + (15, 17, 7, 7)$\tilde{x}_3$

Subject to

(13, 15, 2, 2) $\tilde{x}_1$ + (11, 13, 4, 4)$\tilde{x}_2$ + (10, 12, 6, 6)$\tilde{x}_3$ $\le$ (100, 102, 6, 6)

(14, 16, 2, 2)$\tilde{x}_1$ + (12, 14, 4, 4)$\tilde{x}_2$ +(10,12, 6, 6)$\tilde{x}_3$ $\le$ (112, 114, 8, 8)

(15, 17, 2, 2)$\tilde{x}_1$ + (13, 15, 4, 4)$\tilde{x}_2$ +(12,14, 6, 6)$\tilde{x}_3$ $\le$ (122, 124, 10, 10)

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \ge 0$

Solution:

The standard form of given FFLPP

Max (10, 14, 3, 3)$\tilde{x}_1$ + (13, 15, 5, 5)$\tilde{x}_2$ + (15, 17, 7, 7)$\tilde{x}_3$

Subject to

(13,15,2,2) $\tilde{x}_1$ + (11,13,4,4)$\tilde{x}_2$ + (10,12,6,6)$\tilde{x}_3$ + (1,1,1,1)$\tilde{x}_4$ = (100,102,6,6)

(14,16,2,2)$\tilde{x}_1$ + (12, 14, 4, 4)$\tilde{x}_2$ +(10,12,6,6)$\tilde{x}_3$+(1,1,1,1)$\tilde{x}_5$ = (112, 114, 8, 8)

(15,17,2,2)$\tilde{x}_1$ + (13,15,4,4)$\tilde{x}_2$ + (12,14,6,6)$\tilde{x}_3$ + (1,1,1,1)$\tilde{x}_6$ = (122, 124, 10, 10)

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6 \ge 0$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6$ are non-negative trapezoidal fuzzy numbers

Max (10,14,3,3)(x_1, y_1, z_1, w_1) + (13,15,5,5)(x_2, y_2, z_2, w2) + (15,17, 7, 7)(x_3, y_3, z_3, w_3)

Subject to

(13, 15, 2, 2)(x_1, y_1, z_1, w_1) + (11, 13, 4, 4) (x_2, y_2, z_2, w2) (10, 12, 6, 6) (x_3, y_3, z_3, w_3) + (1, 1, 1, 1) (s_1, t_1, u_1, v_1) = (100, 102, 6, 6)

(14, 16, 2, 2)(x_1, y_1, z_1, w_1) + (12, 14, 4, 4) (x_2, y_2, z_2, w2) (10, 12, 6, 6) (x_3, y_3, z_3, w_3) + (1, 1, 1, 1) (s_2, t_2, u_2, v_2) = (112, 114, 8, 8)

(15, 17, 2, 2)(x_1, y_1, z_1, w_1) + (13, 15, 4, 4) (x_2, y_2, z_2, w2) (12, 14, 6, 6) (x_3, y_3, z_3, w_3) +(1, 1, 1, 1)(s_3, t_3, u_3, v_3)= (122, 124, 10, 10)

Max $\{\Re(10x_1 + 13x_2+15x_3,\ 14y_1 +15y_2+17y_3,\ 3z_1+ 5z_2+7z_3,\ 2w_1 + 4w_2 + w_3)\}$

Subject to

$(13\, x_1 + 11x_2 +10\, x_3+ s_1,\ 15\, y_1 + 13y_2 +12\, y_3+ t_1, 2\, z_1 + 4z_2 +6\, z_3+ u_1,\ 2\, w_1 + 4w_2 +6\, w_3+ v_1) = (100, 102, 6, 6)$

$(14\, x_1 + 12x_2 +10\, x_3+ s_2,\ 16\, y_1 + 14y_2 +12\, y_3+ t_2,\ 2\, z_1 + 4z_2 +6\, z_3+ u_2,\ 2\, w_1 + 4w_2 +6\, w_3+ v_2) = (112, 114, 8, 8)$

$(15\, x_1 + 13x_2 +12\, x_3+ s_3,\ 17\, y_1 + 15y_2 +14\, y_3+ t_3,\ 2\, z_1 + 4z_2 +6\, z_3+ u_3,\ 2\, w_1 + 4w_2 +6\, w_3+ v_3) = (122, 124, 10, 10)$

$(x_1, y_1, z_1, w_1), (x_2, y_2, z_2, w_2), (s_1, t_1, u_1, v_1)\ (s_2, t_2, u_2, v_2)$ are non-negative trapezoidal fuzzy numbers.

The above FFLPP is converted into the following Crisp linear programming:

Max $(\frac{1}{4}(10x_1 + 13x_2 +15\, x_3+14y_1 +15y_2+17\, y_3 +3z_1 + 5z_2+7\, z_3 +3w_1 + 5w_2 +7\, w_3))$

Subject to

$$13\, x_1 + 11x_2 +10\, x_3+ s_1 = 100$$
$$14\, x_1 + 12x_2 +10\, x_3+ s_2 = 112$$
$$15\, x_1 + 13x_2 +12\, x_3+ s_3 = 122$$
$$15\, y_1 + 13y_2 +12\, y_3+ t_1 = 102$$
$$16\, y_1 + 14y_2 +12\, y_3+ t_2 = 114$$
$$17\, y_1 + 15y_2 +14\, y_3+ t_3 = 124$$
$$2\, z_1 + 4z_2 +6\, z_3+ u_1 = 6$$
$$2\, z_1 + 4z_2 +6\, z_3+ u_2 = 8$$
$$2\, z_1 + 4z_2 +6\, z_3+ u_3 = 10$$
$$2\, w_1 + 4w_2 +6\, w_3+ v_1 = 6$$
$$2\, w_1 + 4w_2 +6\, w_3+ v_2 = 8$$
$$2\, w_1 + 4w_2 +6\, w_3+ v_3 = 10$$
$$y_1 - x_1 \geq 0$$
$$y_2 - x_2 \geq 0$$
$$y_3 - x_3 \geq 0$$

By simplex method, the Fuzzy optimal solutions are $\tilde{x}_1= (0, 0, 3, 3)$, $\tilde{x}_2 = (0, 0, 0, 0)$ $\tilde{x}_3 = (10, 8.5, 0, 0)$ and Max $\tilde{z}$ = 312

IV. PROPOSED METHOD

A new method is proposed to find the Fuzzy optimal solution of fully fuzzy liner programming problems with symmetric trapezoidal fuzzy numbers. The steps of the proposed method are as follows.

Algorithm

Step 1: Formulate the chosen problem into the following fully fuzzy linear programming problem:

Max (min) $\tilde{z} = \tilde{c}_j\tilde{x}_j$

Subject to $\tilde{A}_{ij}\tilde{x}_j \leq \tilde{B}_i$,

$\tilde{x}_j \geq 0$

Step 2: Using the Ranking function $\Re(\tilde{c}_j, \tilde{A}_{ij}) = \frac{m+n+\alpha-\beta}{2}$, the FFLPP transform into FVLPP.

Step 3: Solve the FVLPP by using simplex method / Big-M method. Let the solution be $\hat{x}_j$. Hence the solution of FFLPP is $\widetilde{x_j}$.

Step 4: Convert all the inequality constraints into equations by adding slack / surplus variable and the cost of this variable zero.

Step 5: Compute the value of $\tilde{Z}$ == C_BY_j-C_j, j≠B j=1, 2,..., n. If all $\tilde{Z} \geq 0\ \forall j$ for maximization problem and $\tilde{Z} < 0\ \forall j$ for minimization problem then the current solution is optimal, otherwise go to step 6.

Step 6: Determine the basic variable $\tilde{x}_k$ which will be replaced by the non-basic variable where k = arg min$\{\Re(\tilde{B}_i)\}$ i=1, 2, ….m, in maximization problem and k = arg max$\{\Re(\tilde{B}_i)\}$ i=1, 2,…,m, in minimization problem.

Step 7: Perform the pivot operation, return to step 5.

Example 4.1

Max $(10, 14, 3, 3)\tilde{x}_1 +(13, 15, 5, 5)\tilde{x}_2 +(15,17, 7, 7)\tilde{x}_3$

Subject to

$(13, 15, 2, 2)\ \tilde{x}_1 + (11, 13, 4, 4)\tilde{x}_2 + (10, 12, 6, 6)\tilde{x}_3 \leq (100, 102, 6, 6)$

$(14, 16, 2, 2)\tilde{x}_1 + (12, 14, 4, 4)\tilde{x}_2 + (10, 12, 6, 6)\tilde{x}_3 \leq (112, 114, 8, 8)$

$(15, 17, 2, 2)\tilde{x}_1 + (13, 15, 4, 4)\tilde{x}_2 + (12, 14, 6, 6)\tilde{x}_3 \leq (122, 124, 10, 10)$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3 \geq 0$

Solution:

Standard form

Max $(10, 14, 3, 3)\tilde{x}_1 + (13, 15, 5, 5)\tilde{x}_2 + (15, 17, 7, 7)\tilde{x}_3$

Subject to

$(13,15,2,2)\ \tilde{x}_1 + (11,13,4,4)\tilde{x}_2 + (10,12,6,6)\tilde{x}_3 + (1,1,1,1)\tilde{x}_4 = (100, 102, 6, 6)$

$(14,16,2,2)\tilde{x}_1 + (12,14,4,4)\tilde{x}_2 + (10,12,6,6)\tilde{x}_3 + (1,1,1,1)\tilde{x}_5 = (112, 114, 8, 8)$

$(15,17,2,2)\tilde{x}_1 + (13,15,4,4)\tilde{x}_2 + (12,14,6,6)\tilde{x}_3 + (1,1,1,1)\tilde{x}_6 = (122, 124, 10, 10)$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5, \tilde{x}_6 \geq 0$

Its equivalent FVLPP is

Max $\tilde{Z}= 12\hat{x}_1 + 14\hat{x}_2 + 16\hat{x}_3 + 0\hat{x}_4 + 0\hat{x}_5 + 0\hat{x}_6$

Subject to

$14\hat{x}_1+ 12\hat{x}_2 +11\hat{x}_3 + \hat{x}_4 = (100, 102, 6, 6)$

$15\hat{x}_1+ 13\hat{x}_2 +11\hat{x}_3 + \hat{x}_5 = (112, 114, 8, 8)$

$16\hat{x}_1+ 14\hat{x}_2 +13\hat{x}_3 + \hat{x}_6 = (122, 124, 10, 10)$

$\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4, \hat{x}_5, \hat{x}_6 \geq 0$

Table: 4.1.1

Basis	$\hat{x}_1$	$\hat{x}_2$	$\hat{x}_3$	$\hat{x}_4$	$\hat{x}_5$	$\hat{x}_6$	RHS	$\Re(\tilde{B}_i)$
$\hat{x}_4$	14	12	11	1	0	0	(100, 102, 6, 6)	101
$\hat{x}_5$	15	13	11	0	1	0	(112, 114, 8,8)	113
$\hat{x}_6$	16	14	13	0	0	1	(122, 124, 10, 10)	123
$\tilde{Z}$	-12	-14	-16	0	0	0	(0,0,0, 0)	

Here -16 is the most positive in $\tilde{Z}$. So $\hat{x}_3$ is an entering variable and min $\{\Re((100, 102, 6, 6)\ \Re(112, 114, 8,8), \Re(122, 124, 10,10)\}$ is 101. So $\hat{x}_4$ is a leaving variable, pivotal element is 11.

Table:4.1.2

Basis	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	$\tilde{x}_6$	RHS
$\tilde{x}_3$	$\frac{14}{11}$	$\frac{12}{11}$	1	$\frac{1}{11}$	0	0	$(\frac{100}{11}, \frac{102}{11}, \frac{6}{11}, \frac{6}{11})$
$\tilde{x}_5$	1	1	0	-1	1	0	(12, 12, 12, 12)
$\tilde{x}_6$	$-\frac{6}{11}$	$-\frac{2}{11}$	0	$-\frac{13}{11}$	0	0	$(\frac{42}{11}, \frac{58}{11}, \frac{32}{11}, \frac{32}{11})$
$\tilde{Z}$	$\frac{92}{11}$	$\frac{38}{11}$	0	$\frac{16}{11}$	0	0	$(\frac{1600}{11}, \frac{1632}{11}, \frac{96}{11}, \frac{96}{11})$

Since $\tilde{Z} \geq 0$, the fuzzy optimal solution of the FVLPP is $\tilde{x}_1$ = (0, 0, 0, 0), $\tilde{x}_2$ = (0, 0, 0, 0), $\tilde{x}_3 = (\frac{100}{11}, \frac{102}{11}, \frac{6}{11}, \frac{6}{11})$

Therefore, the fuzzy optimal solutions of FFLPP is $\widetilde{x_1^*}$ = (0, 0, 0, 0),, $\widetilde{x_2^*}$ = (0, 0, 0, 0), $\widetilde{x_3^*} = (\frac{100}{11}, \frac{102}{11}, \frac{6}{11}, \frac{6}{11})$ and the fuzzy optimal value is $\tilde{z} = (\frac{1600}{11}, \frac{1632}{11}, \frac{96}{11}, \frac{96}{11}) \cong 312$.

V. RESULTS AND DISCUSSIONS

The results of the fully fuzzy linear programming problems have chosen from the above example. Number of Iterations was obtained by using various sizes of FLP problems for the existing and the proposed methods. The comparative results are clearly shown in the following table 5.1.

Table: 5.1

Sl. no	No. of constraints	No. of variables	Earlier method		Proposed method	
			No. of Iterations	Fuzzy optimal value ($\tilde{z}$)	No. of Iterations	Fuzzy optimal value ($\tilde{z}$)
1	3	3	19	312	2	312
2	2	2	11	147	3	147
3	2	2	12	176	3	176
4	2	2	13	1172	2	1172
5	2	2	13	115	3	115
6	3	3	20	396	3	396
7	3	3	21	212	3	212
8	3	3	19	198	3	198
9	3	3	11	356	3	356
10	4	4	25	204	2	204

It is obvious from the results; the fuzzy optimal values of the existing method and the proposed methods are same.

VI. CONCLUSIONS

In this paper, a new method is proposed for solving the fuzzy optimal solution of FFLP problem.The FFLP problem is converted into FVLP problem using new Ranking function. Ranking function is reasonable and effective for calculating the trapezoidal weights of criteria. But in the prop osed method, time consumption is less compared to the earlier method. Proposed method requires less number of iterations. Therefore it is easy to solve fully fuzzy linear programming problems.

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