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JET BREAKUP

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Abstract- The parameter of interest in the theory of liquid jets is the jet length (and hence radius) at breakup. One such application is the use of a liquid jet to feed deuterium or deuterium-tritium fuel droplets into reaction chambers in controlled thermonuclear fusion devices.

Keywords: Jet -Jet breakup.

INTRODUCTION

Jets have been used since earliest times at odd times there we lows of governing the discharge rate from orifices supplying public water. Jets from fountains, Faucets, and fire hoze nozzles are familiar to all their behavior has been systematically studied by scientists since Renaissance time, at least.

(I) Break of Mechanically Vibrated Jet

A mechanical oscillator of frequency f and amplitude α_0 is assumed to be attached to the nozzle of the jet. The disturbances travel down the jet and amplify, eventually resulting in jet breakup. The breakup time T is related to jet breakup length z_c as

$$T = \frac{z_c}{u_z} \qquad (1-1)$$

Where, as before, u_z is the axial velocity of the jet. The solution of the linearized stability equation

$$\int_s pu_j n_j dS = -\frac{\alpha \dot{\alpha} 2\pi^2 n\gamma}{ka}(1-k^2a^2)$$

gives exponentially growing disturbances of the form

$$\alpha = \alpha_0 e^{\omega t} = \alpha_0 e^{2\pi f t} \qquad (1-2)$$

As an approximation, we will assume breakup to occur at time T when α equals the jet radius a. Thus

$$T = \frac{z_c}{u_z} = \frac{1}{2\pi f}\ln\left(\frac{a}{\alpha_0}\right) \qquad (1-3)$$

Because of the insensitivity of the logarithm function, to this order of approximation, we can replace a with t_0, the nozzle radius. Then

$$T = \frac{z_c}{u_z} = \frac{1}{2\pi f}\ln\left(\frac{t_0}{\alpha_0}\right) \qquad (1-4)$$

For the inviscid, inertial jet, solutions for u_z and z from [1] give

$$T = \frac{1}{2\pi f}\ln\left(\frac{t_0}{\alpha_0}\right) = \frac{w_0\tau^2}{2g}\left[\left(\frac{1}{\tau^4}-1\right)+\beta\left(\frac{1}{\tau}-1\right)\right] \qquad (1-5)$$

Where $\beta = 2\gamma/\rho t_0 w_0^2$ and $\tau = t/t_0$. The solution of equation $(1-5)$ gives the jet radius at breakup t_c as a function of $(f, \alpha_0, t_0, w_0$ or q). For the inviscid jet, neglecting radial kinetic energy, we have that the drop radius is

$$R_0 = 2t_c \qquad (1-6)$$

In a process where we desire Γ drops/sec, a further relationship exists on the mass flow rate, namely

$$q = \rho\pi t_0^2 w_0 = \Gamma\rho\frac{4}{3}\pi R_0^3 \qquad (1-7)$$

Or thus

$$t_0^2 w_0 = \frac{4}{3}\Gamma R_0^3 \qquad (1-8)$$

Which, when combined with equations $(1-5)$ and $(1-6)$ gives a unique function of (f, α_0, t_0) relating the nozzle and oscillator design once the drop size and drop rate are specified. Further, choosing the frequency to be in the convenient audio-frequency range, and with the physically necessary fact that $\alpha_0 < t_0$, restricts the choice of t_0 to rather narrow limits.

(II)A Dimensional Analysis Approach to Breakup Length

Obviously an exact theoretical treatment of length of the jet upon breakup is a complicated one. However, the relationship between the variables may be explored by a semi-empirical technique through the use of dimensional analysis. There are two principal rules governing such a study by dimensional analysis:

Rule 1. The dimensional formula of every measured quantity is expressible as the product of powers of the fundamental quantities upon which it depends.

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Rule 2. (Buckingham π Theorem) (Reference [2])
Physical Quantities Involved in Determining Jet Breakup Length

Property	Symbol	Units	Fundamental Units
Jet breakup length	z_c	m	L
Jet diameter	d	m	L
Characteristic jet velocity	υ	$m/\sec$	L/T
Fluid density	ρ	kg/m^3	M/L^3
Fluid absolute viscosity	μ	$kg(m)(\sec)$	M/LT
Surface tension	γ	nt/m	M/T^2

Any complete homogeneous equation expressing the relationship between n measurable quantities such as $\alpha,\beta,\gamma,$….. In the form $f(\alpha,\beta,\gamma,.......)=0$ has a solution of the form $\phi(\pi_1,\pi_2,\pi_3,......,\pi_{n-r})=0.$ where the number of $\pi-$terms is $(n-r)$ independent products of the terms $\alpha,\beta,\gamma,$……, which are dimensionless in fundamental units. Thus n is the number of physical quantities involved and r is the number of fundamental dimensions required to express them. The most common fundamental units are

M	L	T	θ	H
Mass	Length	Time	Temperature	Thermal energy

To illustrate the use of dimensional analysis, consider the jet breakup length to depend on the variables tabulated in above table. Thus the number of physical quantities involved is $n=6$. These depend on only three fundamental units: $M,L,T,$ so that $r=3$. We may therefore expect $(n-r)=3$ dimensionless groupings $(\pi-terms)$. The problem is assumed to be of the form

$$f(z_c,d,\upsilon,\rho,\mu,\gamma)=0 \qquad (2-1)$$

And

$$\phi(\pi_1,\pi_2,\pi_3)=0 \qquad (2-2)$$

Where

$$\pi=z_c^a d^b \upsilon^c \rho^d \mu^e \gamma^f \qquad (2-3)$$

Or hence

$$\pi = L^a L^b \left(\frac{L}{T}\right)^c \left(\frac{M}{L^3}\right)^d \left(\frac{M}{LT}\right)^e \left(\frac{M}{T^2}\right)^f$$

$$=L^{a+b+c-3d-e} M^{d+e+f} T^{-c-e-2f} \qquad (2-4)$$

Since π is a dimensionless quantity, the exponents must be zero. Hence

$$\begin{aligned} a+b+c-3d-e &= 0\\ d+e+f &= 0 \qquad (2-5)\\ c+e+2f &= 0 \end{aligned}$$

We have three equations in six unknowns, so that three unknowns may be
Chosen arbitrarily, provided that they are independent of the others. The independency is established if the determinant of the coefficients of the remaining terms does not vanish.

First solution

Since we desire z_c to appear as a function of the other variables, it is logical to choose $a=1$. Since the simplest dimensionless grouping would be (z_c/d), as a (guess) choose $b=-1$. Also as a (guess), choose $f=0$. Then equation (2 – 5) becomes

$$\begin{aligned} -c+3d+e &= 0\\ d+e &= 0 \qquad (2-6)\\ c+e &= 0 \end{aligned}$$

Which has solution $c=d=e=0$. To check for validity of the assumptions on the exponents, we must have the determinant of the coefficients in equation (2 - 6) not vanish. This determinant is

$$\begin{vmatrix} -1 & 3 & 1\\ 0 & 1 & 1\\ 1 & 0 & 1 \end{vmatrix} = 1 \neq 0$$

Hence our assumptions are valid. The first dimensionless grouping. Is then established from equation (2 - 3) as

$$\pi_1 = z_c^1 d^{-1} \upsilon^0 \rho^0 \mu^0 \gamma^0 = \left(\frac{z_c}{d}\right) \qquad (2-7)$$

Second solution

If viscosity is to appear at all, then we must try $e=1$. Since π_1 contains z_c already, it is logical to take $a=0$ for the second solution. As a third choice (guess) at $f=0$ again. Then equation (2 - 5) becomes

$$\begin{aligned} b+c-3d &= 1\\ d &= -1\\ c &= -1 \end{aligned}$$

Which has solution $b=c=d=-1$. Again, independency is established, and the second dimensionless grouping π_2 is then given through equation (2 - 3) as

$$\pi_2 = z_c^0 d^{-1} \upsilon^{-1} \rho^{-1} \mu^1 \gamma^0 = \left(\frac{\mu}{d\upsilon\rho}\right) \qquad (2-8)$$

The reciprocal of this term is the well-known Reynolds Number, so we may equally well take

$$\pi_2 = \left(\frac{d\upsilon\rho}{\mu}\right) = \mathrm{Re} \qquad (2-9)$$

Third solution

If surface tension is to appear at all, then we must try $f=1$. Since π_1 already contains z_c, it is again logical to take $a=0$. Since π_2 already contains μ, as a (guess), choose $e=0$. Then equation (2 - 5) becomes

$$b + c - 3d = 0$$
$$d = -1$$
$$c = -2$$

Which has solution $b=d=-1$ and $c=-2$. Then

$$\pi_3 = z_c^0 d^{-1}\upsilon^{-2}\rho^{-1}\mu^0\gamma^1 = \left(\frac{\gamma}{d\upsilon^2\rho}\right) \qquad (2-10)$$

The reciprocal of this term is the Weber Number, so we may equally well take

$$\pi_3 = \left(\frac{d\upsilon^2\rho}{\gamma}\right) = We \qquad (2-11)$$

Thus from equation (2 - 2),

$$\phi\left[\left(\frac{z_c}{d}\right), \mathrm{Re}, We\right] = 0 \qquad (2-12)$$

The simplest functional relationship one might assume is

$$\left(\frac{z_c}{d}\right) = k\ \mathrm{Re}^x\ We^y \qquad (2-13)$$

Where k, x, and y are constants to be determined experimentally. However, Weber formulated a slightly different form for low-velocity jets (Reference [3])

$$\frac{z_c}{d\sqrt{We}} = \ln\left(\frac{d}{2\alpha_0}\right)\left[1 + \frac{3\sqrt{We}}{\mathrm{Re}}\right] \qquad (2-14)$$

Where, as before, α_0 is the amplitude of the initial disturbance.

The general solution of (2 - 5)

Let $a=k$, $b=\lambda$, $c=h$

$$e + 3d + 0 = k + \lambda + h$$
$$e + d + f = 0$$
$$e + 0 + 2f = -h$$

$$\begin{vmatrix} 1 & 3 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -1 \neq 0$$

CONCLUSION

In the dimensional analysis. A general solution of equation (2 - 5) was obtained showing all the possible solutions of the dimensional analysis method

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