

THE STRESS-STRAIN STATE OF THE TWO SHITREKS OF CROSS-SECTIONAL SHAPE AND THE DEPTH OF A WEIGHTY OBLIQUELY LAYERED MASSIF SYSTEM WITH SLITS IN TERMS OF ELASTIC DEFORMATION OF ROCKS

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Abstract- On the basis of anisotropic mechanical-mathematical model of inclining multilayer massif with a doubly periodic system of slits systematically studied numerically the patterns of distribution of elastic stresses and shears near two shitreks of derived shape and the depth of the finite element method in plane strain. Calculation algorithm is designed and compiled software package for the study of the elastic state of adjacent cavities of derived depth and shapes. Conducted multivariate numerical calculation and analysis of the effect on the components of stresses and shears near the cavities geometrics, physical parameters of rocks. Shitrek - is through alongside the line course of layers.

Keywords – stress-strain state; anisotropic; elastic deformation; slits; finite element

I. INTRODUCTION

In the last century, the works of foreign scientists was mainly theoretical research of stress-strain state of underground cavities in the isotropic massif. Using the symmetry of the biharmonic solutions and based on the special properties of harmonic functions O.Müller [1], K.Stocke [2] reviewed the relevant class of problems. G.V.Kolosov, N.I.Muskhelishvili [3] in the solution of plane problems of the theory of elasticity of an isotropic body has successfully used the methods of the complex variable theory.

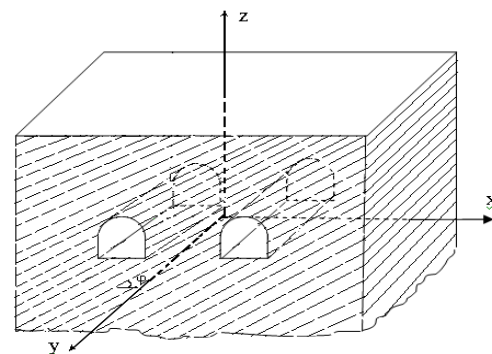
Analytic function proposed by Appel, allowed to consider the state of the one- and many related isotropic body with a circular hole. L.A.Filshtinsky considered orthotropic structures with doubly periodic system of circular holes [4], and a body with elliptical holes A.S.Kosmodamiansky, M.M.Neskorodev [5]. A.S.Kosmodamiansky investigated the stress-strain state of an anisotropic elastic body with three and endless rows of holes, and based on these decisions Zh.S.Erzhanov, K.K.Kaydarov, M.T.Tusupov [6] studied the effects of the slots on the static stress state of underground workings. Zh.S.Erzhanov, Sh.M.Aytaliev and Zh.K.Masanov [7] proposed a computational mechanics and mathematical model of the anisotropic elastic deformation of the rock mass with doubly periodic systems slots and solve the problem by bringing the elastic constants obtained transtropic body, the equivalent stiffness main massif with slots, depending on the elastic properties and the geometry of the slits. On the basis of this model studied static initial

elastic state mainly single underground cavities deep foundation of rigorous and approximate methods.

Significant contribution to the theory of finite element method and its application to solving complex problems of statics and dynamics of solid mechanics, scientists have L.Segerlind [8], B.Z.Amusin, A.B.Fadeev [9], Zh.S.Erzhanov T.D.Karimbaev [10], Sh.M.Aytaliev, Zh.K.Masanov, R.B.Baymahan, N.M.Mahmetova [11], N.T.Azhikhanov [12] and others.

II. THE TASK

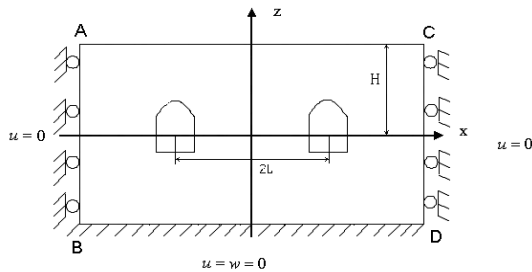
Investigated the static elastic stress and strain state of two shallow cavities laying in heavy transtropic massif depending on the degree of discontinuity conform to small sloping layers at an angle φ . Let H denote the depth of the workings of the distance between their centers $2L$.



a) three dimensional view;

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b) two dimensional view;
Fig.1. The computational domain

Anisotropic doubly periodic massif of slits systems are replaced with solid transtropic body, equivalent stiffness basic structure, by solving the problem of reduction.

III. THE TASK EXPLAINED

The plane of the cross-sectional areas with anisotropic in plane deformation slits; efforts are at infinity (Figure 2)

$$\sigma_x^{(\infty)} = p, \sigma_z^{(\infty)} = q, \tau_{xz}^{(\infty)} = r \tag{1}$$

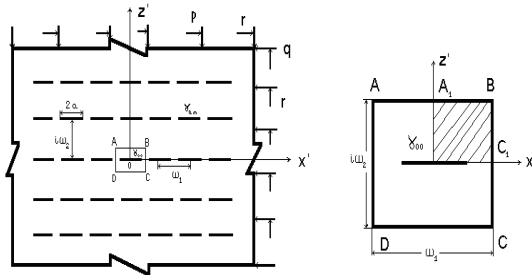


Fig.2. Surface with periodic system of slits.

Here γ_{00} - main crack; $2a, \gamma_{nm}$ - circuits and their length; n,m - indices, $\omega_1, i\omega_2$ - periods of slits in the directions of the axes x and z ; circuits are free of external loads. $E_j, \nu_j, G_2 (j=1,2)$ - elastic properties transtropic massif slots. By solving the problem of bringing to an anisotropic body with the boundary conditions (1) given elastic parameters $E_i^e, \nu_i^e, G_2^e, (i=1,2)$, transtropic solid body, equivalent stiffness anisotropic massif with slots are the following formulas:

$$\begin{aligned} E_1^e &= E_1, \nu_1^e = \nu_1, \nu_2^e = \nu_2, \\ E_2^e &= E_2^{-1} + 2\omega^{-1} < 2 \operatorname{Re} \sum_{j=1}^2 q_j \Phi_j(x + i\beta_j 0.5\omega, q) >, \\ G_2^e &= G_2^{-1} + 2\omega^{-1} < 2 \operatorname{Re} \sum_{j=1}^2 [p_j \Phi_j(x + i\beta_j 0.5\omega, r) + q_j \Phi_j(x + i\beta_j 0.5\omega, r)] > \end{aligned} \tag{2}$$

Here $< >$ - symbol averaging values, β_j - anisotropy parameters;

$$\begin{aligned} \Phi_j(z_j) &= (B_j + iC_j)z_j + \sum_{k=1}^{\infty} a_{2k-1,j} \zeta_j^{-(2k-1)} + \\ &\sum_{k=1}^{\infty} \sum_{l=1}^{\infty} a_{2k-1,j} B_{jlk} (\zeta_j^{2k-1} + \zeta_j^{-(2k-1)}) \\ B_1 &= 0.5(p + \beta_2^2 q)(\beta_2^2 - \beta_1^2)^{-1} \end{aligned} \tag{3}$$

$$\begin{aligned} B_2 &= 0.5(p + \beta_1^2 q)(\beta_2^2 - \beta_1^2)^{-1} \\ C_1 &= 0, C_2 = 0.5 r \beta_2^{-1}; \end{aligned}$$

IV. THE SOLVING PROBLEM

Hooke's law of anisotropic massif with cavities with generalized plane strain relative to the Cartesian coordinate system $Oxyz$ (see Figure 1):

$$\{\sigma\} = [\bar{D}]\{\varepsilon\}; \tag{4}$$

were $\{\sigma\} = (\sigma_x, \sigma_z, \tau_{xz})^T, \{\varepsilon\} = (\varepsilon_x, \varepsilon_z, \gamma_{xz})^T, [\bar{D}] = [d_{ij}], (i, j = 1, 2, \dots, 5)$; - deformation coefficients defined by the formulas [7].

Here $E_k^e, \nu_k^e, G_2^e (k=1,2)$ - effective elastic constants transtropic massif equivalent stiffness anisotropic massif with slits, which depends on the elastic constants of the last $E_k, \nu_k, G_2 (k=1,2)$ and the geometry of the slits $a, \omega, i\omega$.

V. THE USE OF NUMERICAL METHODS

The cross-section in plane ABCD shtrek planes of deformation using n units to m isoparametric calculation elements (Figure 1b). Constitute the basic resolution of the system of algebraic equations finite element method's $3n$ -order relative to the projections of moving points and it can be solved with the following boundary conditions [13]:

base BD calculation area ABCD non-deformable –
$$u = w = 0; \tag{5}$$

sides AB and CD under the weight of rocks moved only in the vertical direction due to a lack of influence of cavities –

$$u = 0, w = w(z). \tag{6}$$

The study estimated the area with cavities is automatically split into isoparametric elements using program FEM_3D in object-oriented environment Delphi. Each point acts the vertical force of the weight (Figure 3).

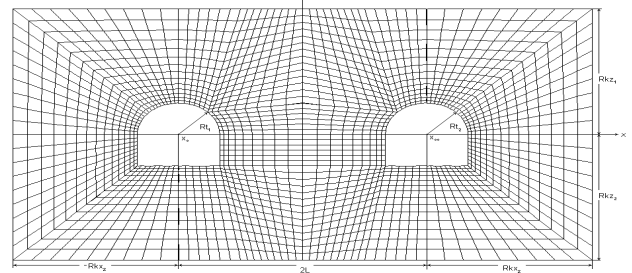


Fig.3. A layout of the estimated area for isoparametric elements

VI. SUGGESTED PROCEDURE

Solution of the fundamental system of equations with to finite element method's shear components with the boundary conditions (5), (6) rigorous methods is difficult; therefore it can be solved in an iterative method of Gauss-Seidel-relaxation factor with a given accuracy. An attractive feature of this method is as follows: firstly prepared only once and the system stiffness $[K]$ matrix used when iterating its elements and column elements of the matrix $\{U\}$; secondly, when $k+1$ - iteration for unknown

u_{m+1} , ($m=1,2,\dots,3n$), need values u_1, u_2, \dots, u_m when $k+1$ - iteration, and for u_{m+2}, \dots, u_{3n} - their values for k - iteration.

Applying a method of finite elements, we determine moving u, w and v as linear function [14]

$$\left. \begin{aligned} u &= \alpha_1 + \alpha_2 x + \alpha_3 z \\ w &= \alpha_4 + \alpha_5 x + \alpha_6 z \\ v &= \alpha_7 + \alpha_8 x + \alpha_9 z \end{aligned} \right\} \quad (7)$$

Where factors $\alpha_1, \dots, \alpha_9$ is received from [13]. At this time, find function of the form as

$$\left. \begin{aligned} u &= N_i(x, z)u_i + N_j(x, z)u_j + N_k(x, z)u_k \\ w &= N_i(x, z)w_i + N_j(x, z)w_j + N_k(x, z)w_k \\ v &= N_i(x, z)v_i + N_j(x, z)v_j + N_k(x, z)v_k \end{aligned} \right\} \quad (8)$$

Now shall determine connection between $\{f_e\}$ and $\{u_e\}$

Where $\{f_e\}$ and $\{u_e\}$ Matrix of element rigidity

$$\{f_e\} = [K]\{u_e\} \quad (9)$$

Thus, the system linear algebraic equation is formed [15]:

$$\{F\} = [K]\{U\} \quad (10)$$

VII. TESTING THE PROGRAM

To verify the correct operation of the developed algorithms and software systems solved test problem of elastic stress state circular cavity in an anisotropic massif with the horizontal plane of isotropy ($\varphi=0$) in the plane strain and hydrostatic stress distribution in a pristine environment. Because of the symmetry of the problem with a quarter of the area of the cavity is divided into 342 isoparametric elements with the help of 380 points. The basic system of equations is solved about 1140 with 1000 iterations. Unlike values of shears at characteristic points of contour obtained by iterative and strict known methods, is no more than 1-2% (Table 1).

Table 1. Comparison of the shear in the contour points obtained by iteration and rigorous methods.

θ , deg	$-\sigma_\theta^{cont} / \gamma H$			
	$-\sigma_\theta^{anal} / \gamma H$ Precise method (test)	$-\sigma_\theta^{FEM} / \gamma H$ FEM	$ \sigma_\theta^{anal} / \gamma H - \sigma_\theta^{FEM} / \gamma H $	$\frac{ \sigma_\theta^{anal} - \sigma_\theta^{FEM} }{ \sigma_\theta^{anal} }$
0	3.079	3.040	0.039	0.01
30	1.510	1.493	0.017	0.01
60	1.706	1.694	0.012	0.007
90	2.692	2.631	0.061	0.022

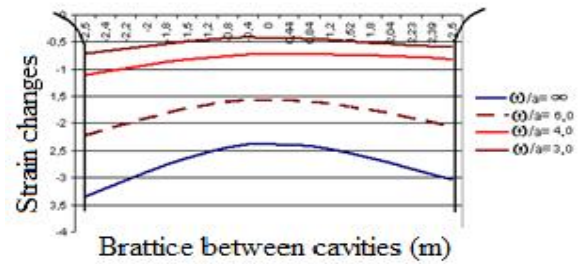
VIII. APPLICATION EXAMPLES

When calculating the components of shears and stresses near contiguous shreks of varying depth ($H = 5m, 10m, 20m$) and the shape of the profile in slots transtropic massif with a

discontinuous layer coupling ($\omega/a = 2.5, 3, 4, 6, \infty$) and inclined plane isotropy ($\varphi = 0, 30^\circ, 45^\circ, 60^\circ, 90^\circ$) the study area was divided into 2064 elements with 2189 points.

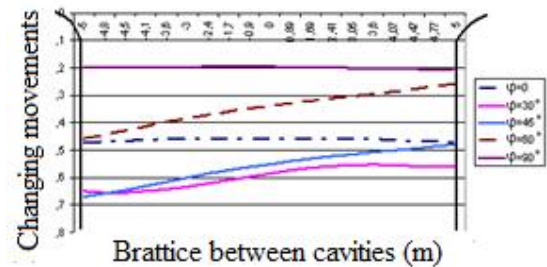
The results of calculations are presented in the form of graphics and diagrams; They analyzed in detail regarding the impact of input parameters on the elastic state of underground structures.

Figure 4 shows the variation of the vertical elastic stress values $-\sigma_z / \gamma H$ on the brattice between cavity vaulted profile depending on the ω/a . Figure 5 depending on the value of the vertical shear of w (mm) on the brattice diagonal and the angle of incidence of cavities plane isotropy.



$L=2.5M; H=10M; \psi=90^\circ; \varphi=45^\circ;$

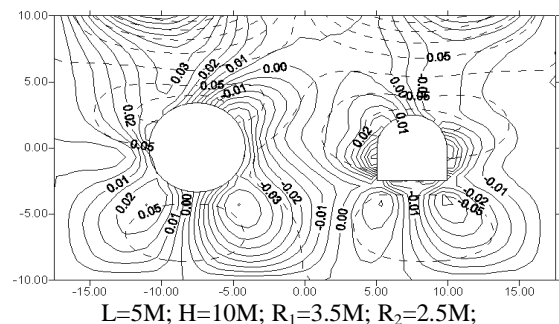
Fig.4. – Changing the value of the stress $-\sigma_z / \gamma H$ on the brattice in depending from ω/a .



$L=5M; H=5M; \psi=45^\circ; \omega/a=6;$

Fig.5. – Depending on the values of the vertical shears w on the brattice of the angle of incidence of the plane of isotropy rocks.

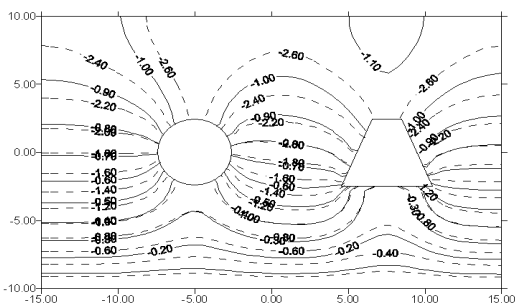
From these figures it can be seen that the parameter slits ω/a , angle of incidence of the plane of isotropy φ significant impact on the stress and strain state of cavities: with decreasing values ω/a strain varies considerably, angle φ equates to an asymmetric distribution of vertical shear w . Other things being equal ω/a significantly affects the shears near the cavities of various forms; with a decrease in the value of its recent increase (Figures 6,7).



$L=5M; H=10M; R_1=3.5M; R_2=2.5M;$

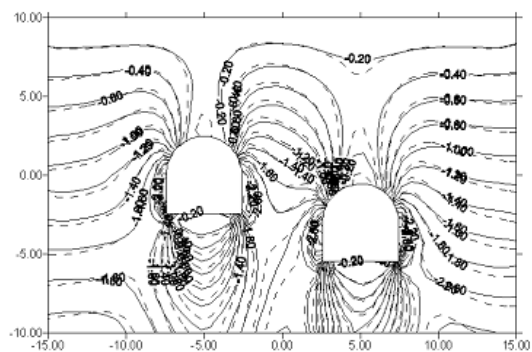
$\psi=90^\circ$; $\varphi=30^\circ$; — $\omega/a=6.0$; - - $\omega/a=2.5$

Fig.6. Isolines shears u (mm) around the cavity in different shapes.



L=5M; H=10M; $\psi=45^\circ$; $\varphi=0$; — $\omega/a=\infty$; - - $\omega/a=3$ Fig.7.
Isolines of vertical shear w (mm) around the cavity in different shapes.

If you find cavities at different levels of the stress distribution is very difficult; they vary with increasing ω/a (Figure 8).



$H_1=10M; H_2=10M; \psi=30^\circ; \varphi=90^\circ; -\omega/a=6$; -- $\omega/a=2.5$
Fig.8. Isolines change vertical shear w_x plane at different location

IX. CONCLUSIONS

At an angle of inclination of the plane of isotropy $\varphi=0,90^\circ$ (and the plane of the slits) slots massif with cavities, ceteris paribus both stress and shear are distributed symmetrically around the vertical axis Oz and increase with the depth of emplacement of structures; reduces stress, increasing shear with reduction ω/a ; when $\varphi \neq 0,90^\circ$ both the stress and the shear are asymmetric about a vertical axis Oz . When the length of the brattice $5D$ and more, where D -cavities of the largest diameter, interference structures is negligible.

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