



DEVELOPMENT OF MATHEMATICAL MODEL TO MONITOR MASS RATE OF CRYPTOCOCCUS DEPOSITION INFLUENCES BY PORE DISTRIBUTION ON HETEROGENEOUS FINE SAND IN ABA, ABIA STATE NIGERIA

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Abstract: *The deposition of Cryptococcus in Aba Abia state Nigeria is to determined the migration process under the influences of high degree of permeability, the study determine several formation variation through its various characteristics to establish the system, several pollution sources were examined in the formation of Aba, locating at the south east of the Niger delta, the rate of permeability were influences by other transition and regression of the geological setting in between the deltaic and other formation, these expression were considered in the developed system to generate the governing equation in the study area, subject to these relation, the expressed system generated several model at different phase, such condition produced final coupled model that represent every phase of the transport system that reflect the geological setting of Aba. The developed model will definitely serve as a tool in monitoring and assessments of Cryptococcus in the study location.*

Keywords: mass rate, Cryptococcus pore distribution, heterogeneous and fine sand

1. Introduction

The Niger Delta is situated at the southern end of Nigeria bordering the Atlantic Ocean. History told us that the proto delta developed in the northern part of the basin during the campanian transgression and ended with the Daleocene transgression whereby the formation of the modern delta began during the Ecocene and it continued into present day where generate the third circle that the modern Niger Delta was formed. Meanwhile before now, the beginning deposition of the Niger Delta are the Albian sediment which consisted of stone over lain conformably by cenoniana and younger upper cretaceous sediments, these deposit were laid from during a predominantly marine depositional circle. Furthermore, the first cycle was concluded by a phase of folding, faulting and uplifting occurring in santonian time and which definitely affect particularly in general Abakaliki anticurrium [1].The delta of deltaic environment of the Niger Delta Region strategraphic possible should explain the variable of the static water level in the Delta Region,

explaining these variables can only be known through groundwater development i.e. construction and design once a borehole has been drilled, the water level in the borehole should be measured using a dipper, this should be done immediately after drilling, and also once water levels in the borehole have recorded [8, 9 and10].

The vertical distribution of groundwater are based on the interstices occupied partially by water and partially by air, in the zone of saturation, all interstices are filled with water under hydrostatic pressure. On most of the land masses of the earth, a single zone of aeration overlies a single zone of saturation and extends upward to the ground surface. In the zone of aeration, vadose water occurs. This general zone may be further subdivided into the soil water zone, the intermediate vadose zone, and the capillary zone. The saturated zone extends from the upper surface of saturation down to underlying impermeable rock. In the absence of overlying impermeable strata, the water table, or Phreatic surface, forms the upper surface the zone of saturation. This

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is known to be surface atmospheric pressure and appears as the level at which water stand in a well permeating the aquifer [6,3].The static level of water in wells penetrating the zone of saturation is called the water table. The water table is often described as the subdued replica of the surface topography. It is generally higher under the hills and lowers under the valleys, and a contour map of the water table in any area may look the surface topography [4, 5, and 7].

Thus, the water is the surface of a water body which is constantly adjusting itself towards an equilibrium condition, with the water moving from the higher points to the lower points. If there were no recharge to or outflow from the groundwater in a basin, the water table would eventually become horizontal. But few basins have uniform recharge conditions at the surface as some areas receive more rain than others; and some portions of the basin have more permeable soil. Thus, when intermittent recharge does occur, mounds and ridges in the water table under the areas of greatest recharge; subsequent recharge creates additional mounds perhaps at other point in the basin and the flow pattern is further changed.

Meanwhile various other factors, such as variation in permeability of aquifer; impermeable strata, influence of lakes, stream and well, etc. do make the water table constantly adjusting toward equilibrium (i.e. horizontal). Because of the low flow rates in most of the aquifers, this equilibrium is rarely altered before additional disturbance occur. This is subject to the variable of the water table in the

3. Governing Equation

$$K \frac{\partial \beta}{\partial x} - \phi \frac{\partial^2 \beta}{\partial x^2} - Q \frac{\partial \beta}{\partial x} = \rho \frac{\partial \beta}{\partial t} \dots\dots\dots (1)$$

The expression above is the developed governing equation to monitor mass rate of Cryptococcus in heterogeneous deposition in aba Abia state. The developed equation are in line with the formulated system where all the parameters are

Nomenclature

- β = Mass Rate of Cryptococcus [ML⁻¹]
- ϕ = porosity [-]
- T = Time [T]
- X = Distance [M]
- ρ = Void ratio [-]
- K = permeability [LT⁻¹]

Niger Delta environment due to all these conditions causes of variable in the region [4].

2. Theoretical background

Aba is located in south east of the Niger Delta of Nigeria, it lies within longitudes 7° 22¹ and 7° 33¹E and latitudes 5° 25¹ and 5° 34¹N, it is within the subequatorial climatic belt characterized by two major seasons: the wet and dry season generally the topography is flat plain terrain. The deposition of Cryptococcus in aba were base on the indiscriminant dumping of waste littering every part of aba, the deposited deltaic formation in natural state took advantage of it consequent deposition this type of microbes in the study location, other type of microbes may be found, but the focus of this study is the deposition Cryptococcus in heterogeneous fine sand, the deposition of the formation was as result of lacustrine influences from Imo River including influences from point bars around the environment, such predominant microbes in Phreatic zone may have accumulated in lateritic deposition were low permeability are experienced. The predominant of this specie of microbes generated lots of health hazards without the notices of the settlers in the environment, the deltaic influences through moderate deposition of permeability pressure the deposition of Cryptococcus in aba Abia state, Nigeria. Base on this condition the development of mathematical model become imperative in other to improve ways of engineering out this types pollution sources in the study location.

input as a system, these are under the consideration of all the required influential parameters that should pressure the migration of Cryptococcus in the study location.

$$K \frac{\partial^2 \beta}{\partial t} - \phi \frac{\partial^2 \beta}{\partial x} \dots\dots\dots (2)$$

$$\left. \begin{array}{l} t = 0 \\ x = 0 \\ C_{(o)} = 0 \\ \frac{\partial v}{\partial t} \Big|_{t=0, B} = 0 \end{array} \right\} \dots\dots\dots (3)$$

$$K \frac{\partial \beta}{\partial t} - Q \frac{\partial \beta^2}{\partial x^2} \dots\dots\dots (4)$$

$$\left. \begin{array}{l} t = 0 \\ x = 0 \\ q_{(o)} = 0 \\ \frac{\partial q}{\partial t} \Big|_{t=0, B} \end{array} \right\} \dots\dots\dots (5)$$

$$K \frac{\partial \beta_3}{\partial t} - \rho \frac{\partial v_3}{\partial t} \dots\dots\dots (6)$$

$$\left. \begin{array}{l} t = 0 \\ C_{(o)} = 0 \\ \frac{\partial \beta_3}{\partial t} \Big|_{t=0, B} = 0 \end{array} \right\} \dots\dots\dots (7)$$

$$Q \frac{\partial \beta_4}{\partial x} - \rho \frac{\partial \beta_4}{\partial t} \dots\dots\dots (8)$$

$$\left. \begin{array}{l} x = 0 \\ t = 0 \\ C_{(o)} = 0 \\ \frac{\partial v_4}{\partial x} \Big|_{x=0, B} = 0 \end{array} \right\} \dots\dots\dots (9)$$

$$\phi \frac{\partial^2 \beta_5}{\partial x^2} - Q \frac{\partial \beta_5}{\partial x} \dots\dots\dots (10)$$

$$\left. \begin{array}{l} x = 0 \\ \beta_{(o)} = 0 \\ \frac{\partial \beta_5}{\partial x} \Big|_{x=0, B} \end{array} \right\} \dots\dots\dots (11)$$

Applying direct integration on (2)

$$K \frac{\partial \beta_1}{\partial t} = \phi v + K_1 \dots\dots\dots (12)$$

Again, integrate equation (12) directly yield

$$Kv = \phi \beta t + Kt + K_2 \dots\dots\dots (13)$$

Subject to equation (3), we have

$$\beta_o = K_2 \dots\dots\dots (14)$$

And subjecting equation (12) to (3) we have

$$\begin{aligned} \text{At } \frac{\partial v_1}{\partial x} \Big|_{t=0} &= 0 & \beta(o) &= \beta_o \\ \text{Yield} & & & \\ 0 &= \phi\beta + K_2 & & \\ \Rightarrow V_1 &= \phi\beta_o = K_2 & \dots\dots\dots & (15) \end{aligned}$$

So that we put (13) and (14) into (13), we have

$$K\beta_1 = \phi v_{1r} - \phi\beta_o x \quad K\beta_o \dots\dots\dots (16)$$

$$K\beta_1 - \phi\beta_{1x} = K\beta_o - \phi\beta_o x \dots\dots\dots (17)$$

$$\beta_1 = \beta_o \dots\dots\dots (18)$$

Hence equation (18) entails that at any given distance x, we have constant concentration of the contaminant in the system.

The expressions from the derived solution establish the phase of the microbes experiencing constant concentration of Cryptococcus in some region of the strata. The behaviour of

the microbes will definitely express it migration phase under the pressure of the region that developed uniform pore distribution of the strata between antecedes.

$$K \frac{\partial \beta_2}{\partial v} = -Q \frac{\partial^2 \beta}{\partial x^2} \dots\dots\dots (4)$$

We approach the system, by using the Bernoulli's method of separation of variables

$$\beta_2 = XT \dots\dots\dots (19)$$

$$\text{i.e. } K \frac{\partial \beta_2}{\partial x} = X^1 T \dots\dots\dots (20)$$

$$K \frac{\partial \beta_2}{\partial x} = X^1 T \dots\dots\dots (21)$$

Put (20) and (21) into (19), so that we have

$$KXT^1 = -QX^1T \dots\dots\dots (22)$$

$$\text{i.e. } K \frac{X^1}{X} = Q \frac{X^1}{X} = -\lambda^2 \dots\dots\dots (23)$$

$$\text{Hence } K \frac{T^1}{T} + \lambda^2 = 0 \dots\dots\dots (24)$$

$$\text{i.e. } X^1 + \frac{\lambda}{K} x = 0 \dots\dots\dots (25)$$

$$KX^1 + \lambda^2 X = 0 \quad \dots\dots\dots (26)$$

From (25), $X = A \cos \frac{\lambda}{K} X + B \sin \frac{\lambda}{\sqrt{K}} X \quad \dots\dots\dots (27)$

And (20) gives

$$T = C \ell^{\frac{-\lambda^2}{Q} t} \quad \dots\dots\dots (28)$$

And (28) gives

$$C_2 = \left(A \cos \frac{\lambda}{K} x + B \sin \frac{\lambda}{\sqrt{K}} x \right) C \ell^{\frac{-\lambda^2}{Q} x} \quad \dots\dots\dots (29)$$

Subject to equation (29) to conditions in (5), so that we have

$$\beta_o = AC \quad \dots\dots\dots (30)$$

Equation (30) becomes

$$\beta_2 = \beta_o \ell^{\frac{-\lambda^2}{K} x} \cos \frac{\lambda}{\sqrt{K}} x \quad \dots\dots\dots (31)$$

Again, at

$$\left. \begin{aligned} \frac{\partial v_2}{\partial x} &= 0, x = 0 \\ x &= 0, B \end{aligned} \right|$$

Equation (31) becomes

$$\frac{\partial \beta_2}{\partial x} = \frac{\lambda}{\sqrt{K}} \beta_o \ell^{\frac{-\lambda}{Q} x} \sin \frac{\lambda}{\sqrt{K}} x \quad \dots\dots\dots (32)$$

i.e. $0 = -\frac{\beta_o \lambda}{K} \sin \frac{\lambda}{KQ} 0$

$\beta_o \frac{\lambda}{K} \neq 0$ Considering NKP

This is the substrate utilization for microbial growth (population?)

so that $0 = v_o \frac{\lambda}{\sqrt{K}} \sin \frac{\lambda}{\sqrt{K}} B \quad \dots\dots\dots (33)$

$$\Rightarrow \frac{\lambda}{K} = \frac{n\pi}{2} n, 1, 2, 3 \quad \dots\dots\dots (34)$$

$$\Rightarrow \lambda = \frac{\lambda}{K} = \frac{n\pi\sqrt{R}}{2} \quad \dots\dots\dots (35)$$

So that equation (31) becomes

$$\Rightarrow \beta_2 = \beta_0 \ell \frac{-n^2 \pi^2 R}{2} t \text{Cos} \frac{n\pi\sqrt{K}}{2\sqrt{K}} x \dots\dots\dots (36)$$

$$\Rightarrow \beta_2 = \beta_0 \ell \frac{-n^2 \pi^2 R}{2} x \text{Cos} \frac{n\pi}{2} x \dots\dots\dots (37)$$

Several condition were experienced by the microbes on their migration process in the study location, the expression from the deposition of the formation establish conditions were microelement may influences the deposition of the microbial growth, such phase implies that the deposition of

microelement as substrate may have increase its growth of the microbes in some region of the formation. The expression here explains the behaviour of the microbes under the influences of such conditions.

Now, we consider equation (7), we have the same similar condition with respect to the behaviour

$$Q \frac{\partial \beta_3}{\partial x} = -\rho \frac{\partial \beta}{\partial t} \dots\dots\dots (6)$$

$$v_3 = XT^1 \dots\dots\dots (38)$$

$$\frac{\partial \beta_3}{\partial x} = X^1 T \dots\dots\dots (39)$$

$$\text{i.e. } K \frac{\partial \beta_3}{\partial t} = XT^1 \dots\dots\dots (40)$$

Put (20) and (21) into (19), so that we have

$$KX^1 T = -XT^1 \dots\dots\dots (41)$$

$$\text{i.e. } K \frac{x^1}{x} = -\rho \frac{T^1}{T} \dots\dots\dots (42)$$

$$K \frac{T^1}{T} + \lambda^2 = 0 \dots\dots\dots (43)$$

$$X^1 + -\frac{\lambda}{K} t = 0 \dots\dots\dots (44)$$

$$\text{And } KT^1 + \lambda^2 t = 0 \dots\dots\dots (45)$$

$$\text{From (44), } x = A \text{Cos} \frac{\lambda}{K} x + B \text{Sin} \frac{\lambda}{\sqrt{K}} t \dots\dots\dots (46)$$

and (39) give

$$T = C \ell \frac{-\lambda^2}{\rho} t \quad \dots\dots\dots (47)$$

By substituting (46) and (47) into (38), we get

$$\beta_3 = \left(A \cos \frac{\lambda}{Q} tx + B \sin \frac{\lambda}{\sqrt{Q}} t \right) C \ell \frac{-\lambda^2}{\rho} t \quad \dots\dots\dots (48)$$

Subject equation (48) to conditions in (7), so that we have

$$\beta_0 = AC \quad \dots\dots\dots (49)$$

Equation (49) becomes

$$\beta_3 = \beta_0 \ell \frac{-\lambda^2}{\rho} t \cos \frac{\lambda}{Q} t \quad \dots\dots\dots (50)$$

Again, at $\frac{\partial \beta_3}{\partial x} \Big|_{t=0, B} = 0 \quad t = 0$

Equation (50) becomes

$$\frac{\partial \beta_3}{\partial x} = \frac{\lambda}{Q} C \ell \frac{-\lambda}{\rho} t \sin \frac{\lambda}{Q} t \quad \dots\dots\dots (51)$$

i.e. $0 = \beta_0 \frac{\lambda}{Q} \sin \frac{\lambda}{Q} 0$

$\beta_0 \frac{\lambda}{Q} \neq 0$ Considering NKP again

Due to the rate of growth, which is known to be the substrate utilization of the microbes we have

$$0 = -\beta_0 \frac{\lambda}{\sqrt{Q}} \sin \frac{\lambda}{\sqrt{Q}} B \quad \dots\dots\dots (52)$$

$$\Rightarrow \frac{\lambda}{Q} = \frac{n\pi}{2} \quad n, 1, 2, 3 \quad \dots\dots\dots (53)$$

$$\Rightarrow \lambda = \frac{n\pi\sqrt{Q}}{2} \dots\dots\dots (54)$$

So that equation (50) becomes

$$\beta_3 = Qo\ell \frac{-n^2\pi^2R}{2\rho} x \text{Cos} \frac{n\pi}{2}t \dots\dots\dots (55)$$

The behaviour of microbes if not for deposition of substrate utilizations may be influences by the degree of predominant formation characteristics in the strata. Such condition were applicable in the express model at [55], the behaviour of the Now, we consider equation (8), we have

transport system consider the condition of the microbes in this formation influences and was monitored in such phase as it expressed in the study location through it geological setting.

$$K \frac{\partial\beta_4}{\partial x} - \rho \frac{\partial\beta_4}{\partial x} \dots\dots\dots (8)$$

Using Bernoulli's method, we have

$$\beta_4 = XT \dots\dots\dots (56)$$

$$\frac{\partial\beta_4}{\partial x} = X^1T \dots\dots\dots (57)$$

$$\frac{\partial\beta_4}{\partial t} = X T^1 \dots\dots\dots (58)$$

Put (57) and (58) into (56), so that we have

$$KX^1T = -X T^1\rho X^1T \dots\dots\dots (59)$$

$$\text{i.e. } K \frac{X^1}{X} = \rho \frac{T^1}{T} \dots\dots\dots (60)$$

$$K \frac{X^1}{X} = \varphi \dots\dots\dots (61)$$

$$\rho v \frac{T^1}{T} = \varphi \dots\dots\dots (62)$$

$$X = A \ell \frac{\varphi}{\rho} x \dots\dots\dots (63)$$

Put (62) and (63) into (56), gives

$$\beta_4 = A \ell \frac{\varphi}{\rho} \bullet B \ell \frac{-\varphi}{\rho}t \dots\dots\dots (64)$$

$$\beta_4 = AB \ell^{(t-x)} \frac{\varphi}{\rho} \dots\dots\dots (65)$$

Subject equation (66) to (8)

$$\beta_4 (o) = \beta o \dots\dots\dots (66)$$

So that equation (67) becomes

$$\beta_4 = Ko \ell^{(t-x)} \frac{\varphi}{\rho v}$$

$$\dots\dots\dots (67)$$

Exponential condition were found in the course of considering the lithology of the formation, the structural stratification of the formation are assume to pressure the migration condition of the microbes, therefore the concentration were confirm to Considering equation (10), we have

deposit in exponential phase at some certain region of the formation ,the expression at [66] expressed the behaviour of the microbes in that phase

$$\phi \frac{\partial \beta_5}{\partial x} - Q \frac{\partial \beta_5^2}{\partial x^2} \dots\dots\dots (10)$$

$$\beta_5 = X^{11}T \dots\dots\dots (68)$$

$$\frac{\partial \beta_5}{\partial x} + X T^1 \dots\dots\dots (69)$$

$$\frac{\partial \beta_5}{\partial t} + X T^1 \dots\dots\dots (70)$$

Put (69) and (70), so that we have

$$\phi X^1 T - TX T^1 \dots\dots\dots (71)$$

$$\phi \frac{X^{11}}{X} T - Q \frac{T^1}{T} \dots\dots\dots (72)$$

$$\phi \frac{X}{X} = \varphi \dots\dots\dots (73)$$

$$Q \frac{T^1}{T} = \varphi \dots\dots\dots (74)$$

$$X^1 = A \ell \frac{\varphi}{Q} t \dots\dots\dots (75)$$

Put (74) and (75) into (68), gives

$$\beta_5 = A \ell \frac{\varphi}{Q} \bullet B \ell \frac{-\varphi}{Q} x \dots\dots\dots (76)$$

$$\beta_5 = AB \ell^{(x-t)} \frac{\varphi}{Q} \dots\dots\dots (77)$$

Subject (76) to (10)

$$\beta_5 (o) = \beta_o \dots\dots\dots (78)$$

So that equation (78) becomes

$$\beta_5 = v_o \ell^{(x-x)} \frac{\varphi}{Q} \dots\dots\dots (79)$$

Similar condition were experienced in some region of the formation, but with some variations, the expression in [79] establish similar behavioral experienced through the derived solution in the formation, the variation of its concentration may express fluctuation in the exponential phase of the transport system.

Now, assuming that at the steady flow, there is no NKP for substrate utilization, our concentration here is zero, so that equation (79) becomes

$$\beta_5 = 0 \dots\dots\dots (80)$$

Therefore, $C_1 + C_2 + C_3 + C_4 + C_5 \dots\dots\dots (81)$

We now substitute (18), (37), (55), (67) into (81) so that we have the model of the form

$$\beta = \beta_o + \beta_o \ell \frac{-n^2 \pi^2 Q}{2Q} x \text{Cos} \frac{n\pi Q}{2} t \bullet \beta_o \ell \frac{-n^2 \pi^2 Q}{2T} t \text{Cos} \frac{n\pi}{2} t + Q_o \ell^{(t-x)} \frac{\varphi}{Q} \dots\dots\dots (82)$$

$$\Rightarrow \beta = q_o + 1 + \ell \frac{n^2 \pi^2 Q}{2Q} x \text{Cos} \frac{n\pi}{2} \bullet C_o \ell \frac{-n^2 \pi^2 Q}{2T} t \text{Cos} \frac{n\pi}{2} t + C_o \ell^{(t-x)} \frac{\varphi}{T} \dots\dots\dots (83)$$

The condition of microbes are under non linear phase due to it heterogeneous deposition of the formation, the reflection of the structural stratification no doubt made the behaviour of the concentration to experiences vacillation in some region of the strata, therefore the application of these type of mathematical method should be in line with the system as different model under different phase considered in the study area, subject to this relation, the transport condition will definitely influences the rate of migration in the system. These conditions were through integrating these parameters into the derived solution

through the state variable to produces the final model for the study.

4. Conclusion

The behaviour Cryptococcus in the study location implies that high accumulation must have deposited in lateritic soil were low permeability are experienced, such condition pressure the migration of the microbes to deposit in heterogeneous formation influenced by lacustrine. Furthermore, the variation has explain the characteristics of

formation variation and the output from these influence, the study has also provide a platform for practicing Engineers to understand the relationship between fluctuation concentration of the microbes deposition. The static deposition of the microbes were expressed in some region of the shallow depth due it location at the upland deltaic environment, Finally the study will enhance the design of productive of boreholes in the study area including quality groundwater; it will also prevent abortive well construction and water pollution, because the study will definitely stand as baseline to all professional in the study areas.

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