

USING THE COMPUTER IN THE GRAPHICAL REPRESENTATION OF GEOMETRIC FIGURES AND SOLVE PROBLEMS COLLINEARITY

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Abstract- The aim of this study is to evaluate the efficiency of using computers in the teaching-learning of Geometry in middle-school, by the students from the specialization of Mathematics, “Vasile Alecsandri” University of Bacau. During the pedagogical practice stage, the students in the 3rd year, although better trained in Mathematics than the students from other departments, face various problems related to their practical skills in using the computer in teaching, as well as to their lack of teaching experience.

The research was conducted at the Middle School "Octavian Voicu" from Bacau and consisted in assisting and observing 20 lessons of Mathematics and 20 lessons of Information and Communication Technologies, involving a group of 108 middle-school students and 22 teachers of various specializations.

Computers provides techniques for acquiring knowledge in an electronic format, calculus techniques, explanatory mathematical texts, graphs, sounds and diagrams, e-learning solutions, including online testing and evaluation, as well as web-based learning tools designed for Mathematics.

The applied tests and questionnaires have shown the efficacy of using the computer in building active thought and competences in the graphical representation of geometrical figures and shapes, as well as in solving problems of collinearity in plan. In relation to these problems, we are looking for a solution to comprise the best teaching-learning strategies using the calculus technique.

Keywords: Finding-ameliorative research, co linearity in plan, computing technology, geometric representation

1. INTRODUCTION

This paper describes an experimental research regarding the effect of introducing scientific software into the learning experience upon attitudes and the learning process. The research aims at investigating the effect of using computers in building attitudes and using strategies for learning Geometry with the help of the computer. In order to research, analyse and describe a wide range of perspectives, the research team consisted of 24 students from the department of Mathematics, attending their initial teacher training.

The objective of this study is to present a practical model for using the computer by the students in the 3rd year, from the specialization of Mathematics, during their initial teacher training stage of pedagogical practice, through teaching Geometry in middle school.

The theme of using the computer in the graphical representation of geometric figures and solve problems collinearity is, attractive and useful for teachers of Mathematics, as well as for students, because it stimulates their imagination and thought, makes them taste the beauty of Mathematics, the harmony of numbers, shapes and geometrical elegance.

2. MATERIALS AND METHODS

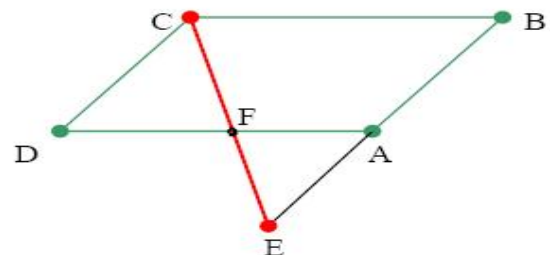
A. Demonstrating the colinearity using the reciprocal of the theorem of opposed angles

If the point B is situated on the line DE, and A and C are on one and another side of the line DE and $\angle ABD \cong \angle CBE$, the points A, B and C are colinear.

Problem 1.

ABCD a parallelogram. On the lines BA, respectively DA, the points E and F, so that $BE = AD$ and $DF = AB$. Demonstrate that the points C, E and F are colinear.

Solution



From the isosceles triangles DCF and AFE there is $m(\angle CFD) = \frac{180^\circ - m(\angle D)}{2}$ and $m(\angle EFA) = \frac{180^\circ - m(\angle D)}{2}$, where $m(\angle CFD) = m(\angle EFA)$, so the points C, E and F are colinear.

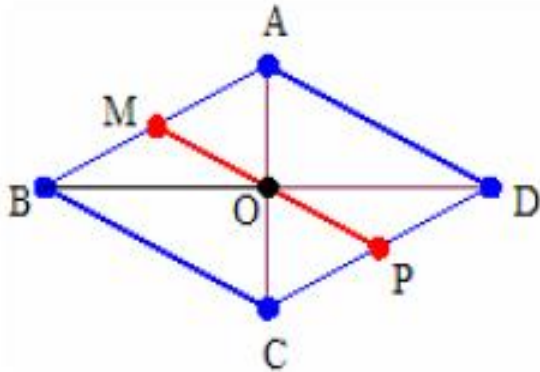
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Problem 2.

The intersection of the diagonals AC and BD of the diamond $ABCD$ is the point O , and the middle of the line AB is M . Decide if M , O and the middle of the CD are colinear points.

Solution

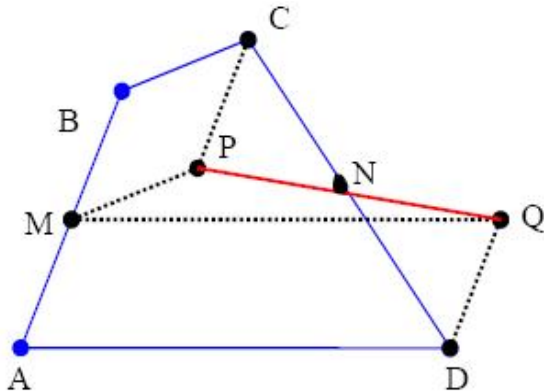


Whether point P the middle of CD . The triangles BOM and DOP are congruent, where $\angle BOM \cong \angle DOP$, and M , O and P are colinear.

Problem 3.

In the quadrangle $ABCD$, there is M and N the middle of the opposed lines AB and CD . Through M there is MP a parallel to BC and MQ parallel to AD , and in the angles C and D a parallel to AB . So there are the quadrangles $BCPM$ and $ADQM$. Demonstrate that the angles P and Q of these quadrangles are colinear to point N .

Solution



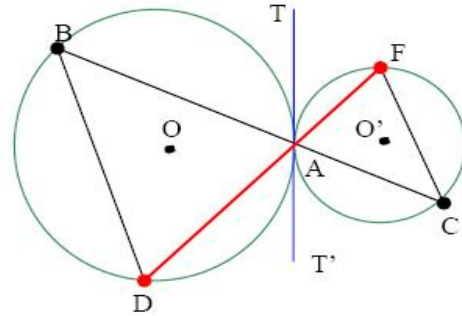
From $CP = DQ$, $\angle NCP \cong \angle NDQ$, $CN = ND$ there is the congruence of the triangles CNP and DNQ , as P and Q are on one side and another of the line CD the points P , N and Q are colinear.

Problem 4.

The circles with the centres O and O' , exterior tangents in A . A line through A intersects the circle in the centre O in B and O' in C . There is D a point belonging to the circle in centre O and F a point belonging to the circle in centre O' , so that $BD \parallel CF$ demonstrate that the points A , D and F are colinear.

Solution

Drawing a common interior tangent TAT' . We have to demonstrate that $\angle DAT' \cong \angle FAT$. There is $\angle DAT' \cong \angle ABD$, $\angle ABD \cong \angle ACF$ and $\angle FAT \cong \angle ACF$.



So that: $\angle DAT' \cong \angle FAT$, where there is the colinearity of the points D , A and F .

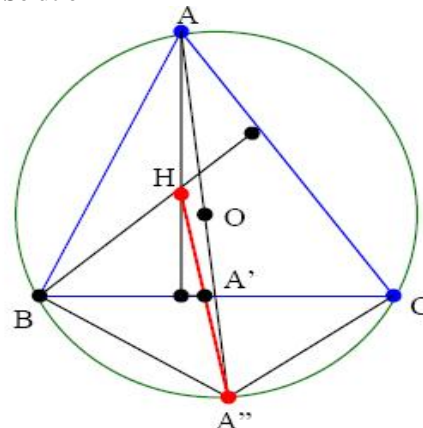
B. Demonstrating colinearity through defining a point under the condition of colinearity

The colinearity of three (or more) points can be demonstrated by redefining of the three points accomplishing the colinearity.

Problem 1.

Having O the centre of the circle subscribed in the triangle ABC , A'' the opposed point to A , A' the middle of BC and H the orthocentre of the triangle ABC . Demonstrate that H , A' , A'' are colinear.

Solution

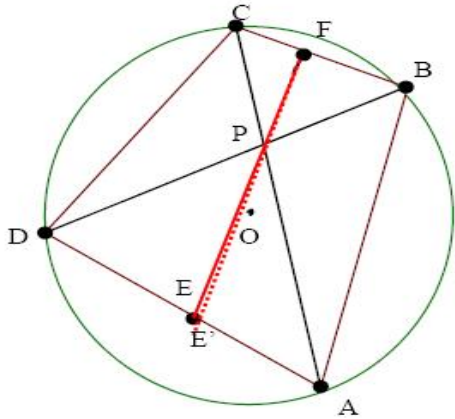


The triangle ACA'' is subscribed in a semi circle and $A''C \perp AC$. As $BH \perp AC$ there is $A''C \parallel BH$. Analogically, $A''B \parallel CH$ and so, the quadrangle $BHCA''$ is a parallelogram. One, can redefine the point A'' so that the quadrangle $BHCA''$ to be parallelogram and the conclusion is that, the diagonals of a parallelogram can be divided, and A' is the middle of BC , they belong to the line HA'' .

Problem 2.

A subscribed quadrangle has perpendicular diagonals. Demonstrate that, the perpendicular line from the intersection point of the diagonals on one of the lines, crosses the middle of the opposed one.

Solution



The subscribed quadrangle ABCD, with $AC \perp BD$ and O the meeting point of the diagonals. $PF \perp BC$ and E the middle of AD. Producing FP and E' the meeting point of the lines FP and AD.

There is $\angle DAC \equiv \angle DBC \equiv \angle CPF$. Înșă $\angle CPF \equiv \angle APE'$, as opposed angles and $\angle DAC \equiv \angle APE'$, so that the triangle E'AP is isosceles. $E'P = E'A$. Analogically, demonstrate that the triangle E'PD is isosceles, and $E'P = E'D$. From $E'P = E'A$ and $E'P = E'D$ there is $E'A = E'D$ and E' is the middle of the segment AD, where $E' = E$. So that, the line FP crosses the middle of the line AD, in other words, the points F, P and E are collinear.

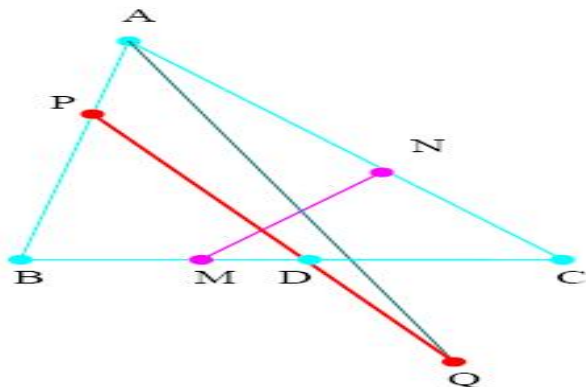
Problem 3.

In the triangle ABC the points M, N and P on the lines BC, CA, respectively AB, so that $\frac{MB}{MC} = \frac{NC}{NA} = \frac{PA}{PB}$. We name D the middle of the line BC, and through Q the simetric of A towards the middle of the line MN. Demonstrate that the points P, D, Q are collinear.

Solution

Drawing through N a parallel to BC, intersecting AB in S.

Then: $\frac{SB}{SA} = \frac{NC}{NA} = \frac{BM}{MC}$ and $SM \parallel AC$.



From $\frac{SB}{SA} = \frac{PA}{PB} = \frac{BM}{MC}$. There is $SB = PA$ and $SA = BP$.

The quadrangle MSNC is a parallelogram and SC crosses the middle of AQ, the quadrangle ASQC is a parallelogram. As $AS = CQ$ and $SA = BP$ there is BQCP a

parallelogram. We redefined D as the middle of the diagonal PQ of the quadrangle BQCP, where the points P, D and Q are collinear.

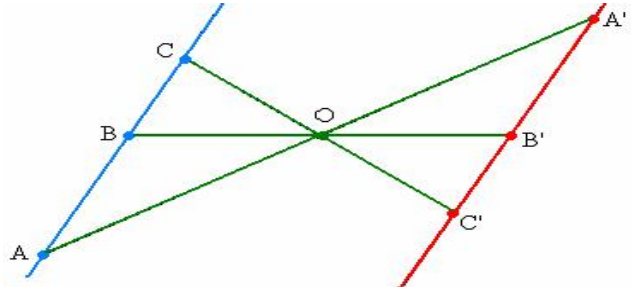
C. Demonstrating the collinearity using the result that b and c are distinct points on the same line ad and if $\angle DAB \equiv \angle DAC$, the points a, b and c are collinear.

This method is complementary to the previous ones, being used in some particular situations.

Problem 1.

If A, B and C are distinct colinear points, A', B', C' are their simetrics towards a point O, the points A', B', C' are collinear.

Solution



If the points A, B and C are colinear there is $\angle OAB \equiv \angle OAC$.

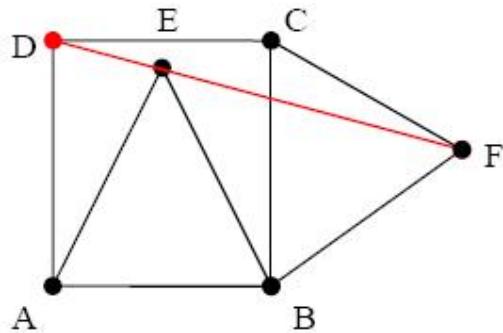
From $\triangle OAB \equiv \triangle OBA'$ there is $\angle OAB \equiv \angle OA'B'$, $\triangle OCA \equiv \triangle OCA'$ so $\angle OAC \equiv \angle OA'C'$.

There is: $\angle OAB \equiv \angle OAC$, $\angle OAB \equiv \angle OA'B'$, $\angle OAC \equiv \angle OA'C'$. Which lead to $\angle OA'B' \equiv \angle OA'C'$, so the points A', B', C' are collinear.

Problem 2.

E the interior of the square ABCD and F exterior, the triangles ABE and BCF are equilateral. Demonstrate that the points D, E and F are collinear.

Solution



In the isosceles triangle DCF (DC = CF) there is: $m(\angle DCF) = 150^\circ$, $m(\angle CDF) = m(\angle CFD) = 15^\circ$. And $m(\angle CDE) = m(\angle CDF) = 15^\circ$ the points D, E and F are collinear.

3. RESEARCH METHODOLOGY

3.1. Research hypothesis

By exploiting work in class during the pedagogical practice, we have aimed at illustrating the role of using

computers in building attitudes and developing strategies for learning Geometry with the help of the computer.

Hypothesis: the systematic use computers in teaching Geometry contributes to building certain attitudes and competences regarding the teaching-learning the methods for demonstrating concurrence in plan.

3.2. Research objectives

The objective of this study is to present a practical model for using the computer by 3rd-year students from the specialization of Mathematics, attending their initial teacher training, during their pedagogical practice, by teaching Geometry in middle school.

In order to verify the hypothesis, we have established several research objectives to direct and guide our activity: 1. Knowledge of the initial level of mathematical training regarding the problem-solving of competition in a plan; 2. Identifying the frame and reference objectives of the curriculum for mathematical education, regarding the solving of problems of competition in plan; 3. Designing and conducting a teaching process centred on using the computer in learning Geometry; 4. Applying the final evaluation of the students' level of training, regarding the solving of problems of competition in plan.

3.3. Characterization of the experimental group

The research group comprised 4 classes of 7th graders (2 experimental classes and 2 control classes), where 24 3rd-year students from the specialization of Mathematics, attending the initial teacher training study programme, have conducted, during the hours of pedagogical practice, observation, probation and final lessons.

Each student has conducted the probation lessons in the computer science laboratory, initiating the students in using computers, for geometrical and applied calculus representations in solving problems of collinearity in plan.

This fact has also led us to conduct a process of individualizing the instructive-educational act, taking into account the fact that true pedagogical artistry results from the student's ability to harmoniously combine active-formative strategies, the individual development of children and their digital competences in using the computer.

3.4. Research stages

The experimental research was conducted during the 2013-2014 school year, covering three stages:

- the stage of initial evaluation (observational) was conducted between 25 - 30 October 2013. At this stage, tests were applied in order to identify the initial level of the ability to solve competition problems.
- the ameliorative stage was conducted between November 12 – April 25, 2014. At this stage, there were organized and conducted lessons of Mathematics, frontal as well as individual, involving group and individual work.
- the stage of final evaluation (summative) was conducted between May 12 – June 10, 2014. At this stage, tests were designed, adapted and applied in order to establish the progress recorded by children, in terms of the students'

ability to use in representing geometrical figures and solve problems of competition.

3.5. Research methods and techniques

In order to verify the hypothesis and achieve the research objectives, the students have resorted to the following research methods and techniques: portfolios, questionnaires, observation, the psycho-pedagogical experiment, conversation, the methods of the analysis of activity products and the research of documents, the method of the tests, as well as techniques of mathematical-statistical presentation of the research data.

4. RESEARCH RESULTS AND DISCUSSION

The pre-test and post-test questionnaires have included a series of questions related to the use of the computers in teaching-learning Geometry in middle school. To what extent do you believe that the use of the computer is useful in learning Geometry in the current educational context? The answers may be provided on a scale from 1 to 5, where: 1= to a very small extent/ not at all and 5 = to a very high extent.

Table 1 Questionnaire on the use of the computer in learning Geometry

	1	2	3	4	5
1. To what extent do the computer help you in learning Geometry?					
2. To what extent does your level of Mathematical knowledge help you in learning Geometry?					
3. What role does the attitude regarding the use of the computer play in learning Geometry?					
4. What role does the experience in using play in learning Geometry?					
5. Which is the level of engagement in exploration as a learning strategy?					
6. How relevant is graphical representation (visual strategies) as a support in solving problems.					
7. Mention the things you liked about using computers in learning Geometry?					
8. Are you going to continue using computers software in learning?					

The students' answers to the eight questions included in the pre-test and post-test questionnaires were: Computer confidence: 62% - 67%; Confidence in mathematical training: 62% - 65%; The attitude towards the use of the computer: 55% - 64%; The relevance of the experience in using the computers: 65% - 78%; Commitment to exploration as a learning strategy: 61% - 65%; The relevance of graphical representation as a support in problem solving: 64% - 70%.

We may observe a slight percentage increase regarding the computer confidence, confidence in Mathematics and the

attitude towards the computer throughout the semester. There is also a slight percentage increase regarding the two strategy scales.

In relation to question 7, the things they liked most about using the computer, the students made frequent references to the following aspects: - graphics and visualization is helpful also for understanding; - the speed and usefulness of building geometrical shapes and performing calculi; - The accuracy in understanding problems as a result of graphic and calculus exploration; - The ability to verify written calculi and confirm answers.

Many students have reacted to using the computer in solving problems as if these had been a game. Approximately half of the questioned students say they would continue to use computers. The tutors have also reported certain difficulties related to the use of computer science laboratories and the specific software. There were no relevant differences based on gender. The relevance of knowing the English language was generally recognized.

Table 2 Giving marks based on performance descriptors

Mark Item No.	Very well	Well	Satisfactory
Item 1	P the middle of CD . The triangles BOM and DOP are congruent, where $\angle BOM \cong \angle DOP$, and M, O and P are collinear.	Point P the middle of CD and triangles BOM and DOP are congruent	Builds triangles BOM and DOP
Item 2	In the isosceles triangle DCF ($DC = CF$) there is: $m(\angle DCF) = 150^\circ$, $m(\angle CDF) = m(\angle CFD) =$ and $m(\angle CDE) = m(\angle CDF) =$ the points D, E and F are collinear.	$m(\angle DCF) = 150^\circ$, $m(\angle CDF) = m(\angle CFD)$ and $m(\angle CDE) = m(\angle CDF)$	In the isosceles triangle DCF ($DC = CF$)
Item 3	As $\angle AEC \cong \angle AFC$ (90°) there is the quadrangle $AEFC$ subscribed, where $\angle CEF \cong \angle CAD$ and $m(\angle ATF) + m(\angle CAD) =$ There is $m(\angle BEH) = 90^\circ - m(\angle ABE)$. As $\angle ABC \cong \angle ADC$ there is $\angle CEF \cong \angle BEH$, and H and F are situated on one and another side of the line BC , the points H, E and F are colinear.	As $\angle AEC \cong \angle AFC$ (90°) there is the quadrangle $AEFC$ subscribed, where $\angle CEF \cong \angle CAD$ and $m(\angle ATF) + m(\angle CAD) =$	As $\angle AEC \cong \angle AFC$ (90°) there is the quadrangle $AEFC$ subscribed
Item 4	Multiplied the result is $\frac{ME}{NE} \cdot \frac{NE}{NA} \cdot \frac{FA}{FD} \cdot \frac{EA}{EA} \cdot \frac{EA}{EA} = 1$. From the similarity of the triangles $AC'E$ and $C'BA_1$, $AB'D$ and $A_1B'C$ there is $\frac{CE}{EA} = \frac{A_1B}{A_1C}$. Multiplied: $\frac{CE}{EA} \cdot \frac{EA}{EA} \cdot \frac{EA}{EA} = 1$. Where: is $\frac{ME}{NE} \cdot \frac{NE}{NA} \cdot \frac{FA}{FD} = 1$, and according to the reciprocal of the theoreme of Menelaos, the point M, N and P are	Using the theorem of Menelaus for the triangle A_1DE divided by $MB'C'$, NAB' and PAC' , there is: $\frac{ME}{NE} \cdot \frac{NE}{NA} \cdot \frac{FA}{FD} = 1$ respectively	Having D and E points where the parallel through A divides the lines A_1B' , respectively A_1C' .

	collinear.		
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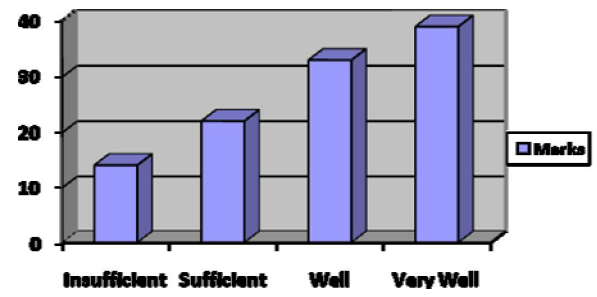
Table 3 the results obtained to the final test

Marks	Frequency
Very well	39
Well	33
Sufficient	22
Insufficient	14

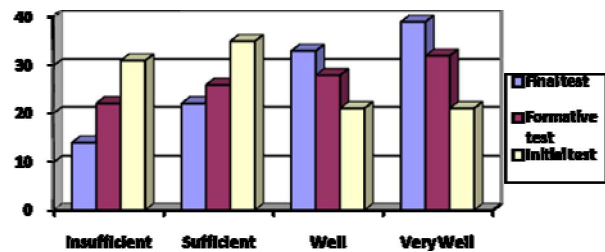
We shall further present only the final evaluation test.

1. The intersection of the diagonals AC and BD of the diamond $ABCD$ is the point O , and the middle of the line AB is M . Decide if M, O and the middle of the CD are collinear points.
2. Point E the interior of the square $ABCD$ and F exterior, the triangles ABE and BCF are equilateral. Demonstrate that the points D, E and F are collinear.
3. Considering the triangle ABC subscribed in the circle with the centre O . There is AD a diameter, F the reflection of C on AD , AE the height from A and H the reflection of E on AB . Demonstrate the colinearity of the points H, E and F .
4. The triangle ABC and B', C' two points on the lines CA and AB , and A_1 the middle of the line BC . The parallel through A to BC intersects the line $B'C'$ in M, A_1C' and A_1B' intersects CA , respectively AB , in N and P . Demonstrate that the points M, N and P are collinear.

Graph 1 the results of the experimental sample to the final test



Graph 2 Comparative analyses of the evaluation test results



Following the analysis and interpretation of the data collected during the initial evaluation, there were applied differentiated learning tasks, providing support particularly to children with knowledge gaps or poor mathematical skills. Proper methods and procedures were applied to approach these issues (exercises on correct graphical representation of geometrical figures, exercises on the collinearity of points in a plane and solving collinearity problems).

Concerned with ensuring an active participation of children in the activities they have conducted, the university students have tried to give an accurate motivation for tasks, stimulate and maintain their interest by means of procedures that engaged them emotionally, use the computer and other attractive materials in the most appropriate moments of the lesson.

5. CONCLUSIONS

The children's attitude towards Mathematics and the use of the computer, the role of technology in the process of learning Mathematics have also been analysed in connection with the paradigm of scientific, pedagogical, psychological and technological training for the teaching-learning-evaluation of Mathematical knowledge, achieved during faculty studies through courses, pedagogical and technological training but, especially, through pedagogical practice. The questionnaire confirms the formative and emotional potential of using technology.

Many teachers (75%) appreciate the relevance of the efficient use of the computer in enhancing the motivational potential, in relation to learning Mathematics. They also believe that the graduates from the specialization of Mathematics should build the skills needed to exploit the available technological resources. The students from pedagogical practice and the children from the lessons of Mathematics are encouraged to use the AEL lesson packs, widely available, as well as special software such as GeoGebra, Allgebra, Matlab, Maple, and Mathematica.

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