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# NEW OPTIMIZATION MODEL FOR CHARACTERISTIC TEMPERATURE IN HEAT EXCHANGERS WITH PARALLEL-COUNTER FLOW DESIGN FROM THE ASPECT OF COSTS

**[1]Branko Pejović, [1]Goran Tadić, [1]Vladan Mićić, [1]Milorad Tomić, [2]Sabina Begić, [3]Pero Dugić, [1]Zoran Petrović, [1]Vojislav Aleksić**
[1]University of East Sarajevo, Faculty of Technology, Zvornik, Bosnia and Herzegovina
[2]University of Tuzla, Faculty of Technology, Tuzla, Bosnia and Herzegovina
[3] Oil Refinery, Modriča, Bosnia and Herzegovina

***Abstract-*** ***Based on the complex dependence of the mean temperature difference we determine the optimal value of exit temperature when cooling the process fluid for characteristic heat exchangers with a tube bundle and a shell, in which the fluid passes once through the shell and twice through the tubes. The function of the total annual costs whose components are investment and operation costs was adopted as the function of optimization. In this respect, a detailed economic analysis of all the costs at the annual level was conducted and the correlation with the parameter which is optimized was established. Using mathematical transformations of thermodynamic relations, a suitable function of total costs was obtained and thoroughly investigated through mathematical analysis.***
***The optimal value of exit temperature of the heat transfer fluid was obtained by minimizing the complex function of total annual costs based on the basic theorem of differential calculus. The boundary function and the domain of the function of costs, considering its real domain, were also taken into consideration.***
***The method presented was verified on a characteristic example of a tube-bundle heat exchanger containing a shell of (1-2) type with parallel and counter flow. The obtained complex function of optimization was solved using a suitable numeric mathematical method, with the support of a computer program, considering the fact that the parameter which is optimized could not be expressed explicitly. Apart from the analytical method, the graphical method was also used in order to obtain a more efficient solution and to determine the extremum of function.***
***Such techno-economical approach to the problem, after determining the optimal temperature, has enabled the calculation of a series of parameters which are necessary for designing the heat exchanger observed. The paper also presents some of the possibilities for the application of the model presented.***

***Keywords -*** Heat exchangers with a tube bundle and a shell, Parallel-counter flow of the fluid; Investment and operation costs; Optimization of costs; Economic analysis; Exit temperature of the heat transfer fluid; The process fluid; Numerical analysis; Differential calculus.

## 1. INTRODUCTORY DISCUSSIONS

Apparatuses in which the process of heat exchange is carried out are called heat exchangers. The most commonly used heating fluids are vapor, hot water and hot air, whereas cooling water and cooling air are most often used as cooling fluids or heat transfer fluids. The propellants mentioned are used for the heating and cooling of process fluids, [1, 2, 3].

The exchange of heat between the fluids can occur by direct and indirect contact, Figure 1.

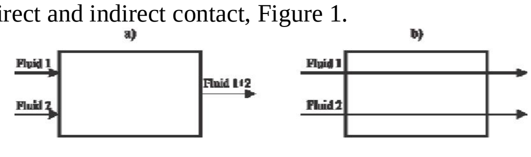


Fig. 1. The methods of heat exchange based on the contact between the fluids a) direct contact, b) indirect contact

During direct contact fluids are mixed and heat is exchanged between them simultaneously. During indirect contact fluids are usually separated by a solid wall through which heat is exchanged between the fluids, [3, 4].

In heat exchangers with indirect contact, fluids can achieve parallel flow (flow in the same direction), in the opposite direction and cross-directionally, Figure 2. During the same-directional flow fluids 1 and 2 enter the heat exchanger at the same end and flow parallelly through the heat exchanger. During the flow in opposite directions fluid 1 enters at one end and fluid 2 at the other end of the heat exchanger. During cross-directional flow, the flow of fluid 1 occurs at the right angle to the direction of the flow of fluid 2, [4, 5, 6].

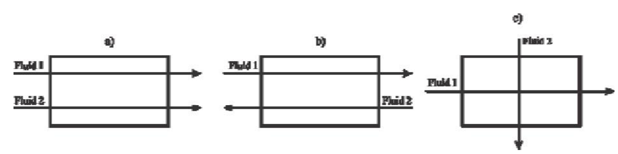


Fig. 2. The methods of heat exchange between the fluids considering the direction of their flow a) the same direction, b) opposite directions, c) cross directionally

Two types of calculations are mainly used for heat exchangers: design calculations and exploitation calculations.

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The aim of the design calculation is to determine geometrical characteristics of the exchanger for known parameters of entering and exiting flows. In exploitation calculation, geometry of the apparatus and the parameters of entering flows are known, whereas the unknown temperatures of flows at the exit of the exchangers are usually what is determined, [6, 7].

## 2. BASIC CHARACTERISTICS IMPORTANT FOR THE CALCULATION OF HEAT EXCHANGERS

Temperature difference in a heat exchanger is the driving force for heat transfer. Its value determines the total amount of transferred heat. The direction of flow of the fluid largely affects the temperature difference and the heat exchanger performances.

In practice, apart from one pass of the hot and the cold fluid through the exchanger, there are cases of heat exchange with many passes through the shell and the tubes. In that case it is necessary to introduce the correction factor of the mean temperature difference, [7, 8].

Figure 3 shows the process of heat transfer during the parallel and the counter flow. Heat transfer occurs from the hotter fluid $T_{tu}$ towards the colder fluid stream $T_{hu}$. The hotter fluid is cooled to the temperature $T_{ti}$ whereas the colder fluid is heated to temperature $T_{hi}$. The same figure shows the temperature profile along the length (L), and the area (A) of the exchanger, [8, 9, 10].

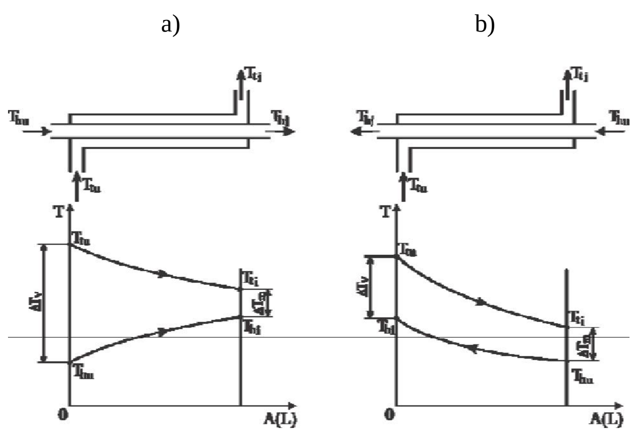


Fig. 3. Heat transfer in a tube heat exchanger
a) parallel flow, b) counter flow

If we designate the higher temperature difference with $\Delta T_v$, and the lower with $\Delta T_m$, then, considering the fact that the temperature of the fluid is changed along the length (L) and the area (A) of the heat exchanger, the log mean temperature difference is used in the calculations.[10, 11, 12]:

$$\Delta T_{sr} = \frac{\Delta T_v - \Delta T_m}{\ln \frac{\Delta T_v}{\Delta T_m}} \quad (1)$$

The previous analysis refers to (1-1) heat exchangers, i.e. when there is one pass of the fluid through the shell and one pass of the fluid through the tube, [12, 13].

In practice, tube-bundle heat exchangers which have not only the same direction and the opposite direction streaming but a combination of the previous two, are most commonly encountered, [12, 13, 14].

When it is necessary to achieve large heat currents/flows, a combination of a series of constraints (cost, consumption of energy necessary for overcoming the drop of pressure during streaming through the apparatus and the space which must be provided in order for the exchanger to be positioned), the tendency is to use an apparatus which is more compact. That is the reason why heat exchangers with a tube bundle and a shell are most commonly used in chemical and process industry, and their different designs provide greater compactness and allow many problems imposed by real industrial processes to be solved. In this case a problem arises how to determine the mean temperature difference $\Delta T_{sr}$ at such complex real configurations of fluid stream, [8, 15, 16].

One of the designs commonly used are type 1-2 heat exchangers, in which one fluid (most commonly the hotter one) passes once through the shell and the other fluid (the colder one) which flows through the tubes passes two times, [18, 26, 27, 28].

The flow chart of the streams for this design of exchanger and the chart of temperature distribution are shown in Figure 4.

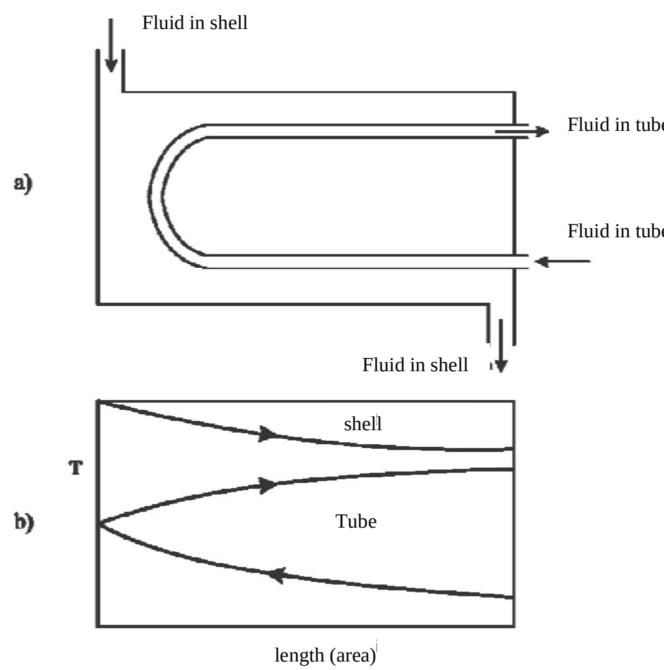


Fig.4. Fluid flows and temperature profile for (1-2) exchanger (parallel and counter flow)

Here it must be mentioned that in practice, with this type of exchanger (and those similar to it), cross partitions are installed along the flow normal to the axis of the tube which causes much better conditions for heat transfer than those obtained with the longitudinal fluid stream through the shell. Similarly, type 2-4 heat exchangers are often found in practice in which the fluid in the shell passes two times through the tubes and the fluid in the tubes passes four times through the apparatus, [27, 28, 29, 30]. Other combinations are also possible.

From Figure 4 it can be concluded that, in this case, the configuration of flows is much more complicated than with type (1-1) heat exchangers, which also makes the calculations of these apparatuses much more complex. The dependence of mean temperature difference $\Delta T_{sr}$ on the difference between the temperatures at the end of the apparatus $\Delta T_v$ and $\Delta T_m$, is also much more complex [30, 31, 32, 34].

## 3. ESTABLISHING THE RELATION FOR MEAN TEMPERATURE DIFFERENCE IN TYPE (1-2) HEAT EXCHANGER

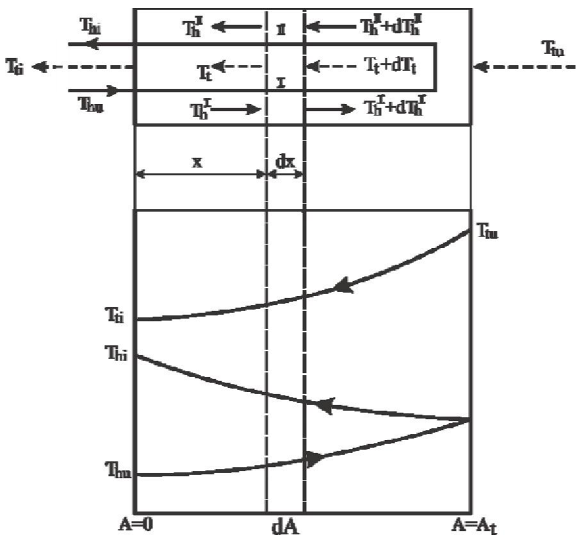


Fig.5. Determining the mean temperature difference for type (1-2) heat exchanger

It can be seen from Figure. 5 that, in the observed heat exchanger, both the parallel and the counter flow are present at the same time (the presence of cross partitions partly provides the cross-directional flow in the shell as well).

In the heat exchanger with the counter flow (1-1) heat transfer is the most efficient, therefore it can be concluded that the mean temperature difference in the apparatuses with multiple passes can only have lower value at equal flow rates and exit temperatures, [33, 34, 35].

The concept of solving the problem of designing heat exchangers with multiple passes is based on the attempt to modify the equation (1) so that it is also valid for this type of device. In this respect, the meaning of the value $\Delta T_{sr}$ will be given with the devices with multiple passes of fluid through the shell and the tubes and the equation will be defined, [36, 37, 38]. Due to the limited space, the calculation of the mean temperature difference will be given in a short form.

In the observed case, the hotter fluid flows through the shell, whereas the colder one flows through the tube with two passes, where I stands for the first pass, and II stands for the second pass of the colder fluid through the tubes, according to Figure 5.

Heat transfer on the elementary length dx, which has a corresponding area dA, can be defined through the following relations:

$$dq = C_t \cdot dT_t \qquad (2)$$

$$dq = C_h \cdot \left(dT_h^I - dT_h^{II}\right) \qquad (3)$$

Where heat capacities flows are the following:

$$C_t = m_t \cdot c_{pt} \qquad C_h = m_h \cdot c_{ph} \qquad (4)$$

Heat flow through the heat exchange surface will be:

$$dq = K \cdot dA \cdot \left[\left(T_t - T_h^I\right) + \left(T_t - T_h^{II}\right)\right] \qquad (5)$$

Elimination of $T_h^I$ and $T_h^{II}$ from equations (2), (3) and (5), can lead to the differential equation for change in temperature along the device. This differential equation together with using balance equation for the whole exchanger:

$$q_t = C_t \cdot \left(T_{tu} - T_{ti}\right) = C_h \cdot \left(T_{hi} - T_{hu}\right) \qquad (6)$$

can be integrated, which gives the final equation:

$$q_t = K \cdot A_t \cdot \Delta T_{sr} \qquad (7)$$

where the mean temperature difference is

$$\Delta T_{sr} = \frac{\sqrt{\left(T_{tu} - T_{ti}\right)^2 + \left(T_{hi} - T_{hu}\right)^2}}{\ln \dfrac{T_{tu} + T_{ti} - T_{hu} - T_{hi} + \sqrt{\left(T_{tu} - T_{ti}\right)^2 + \left(T_{hi} - T_{hu}\right)^2}}{T_{tu} + T_{ti} - T_{hu} - T_{hi} - \sqrt{\left(T_{tu} - T_{ti}\right)^2 + \left(T_{hi} - T_{hu}\right)^2}}} \qquad (8)$$

It must be noted that the above calculation is based upon the assumption that the total coefficient of heat flow is constant (K=const.).

In order to simplify the relation by introducing suitable shifts, the heat flow for the previous case, can be defined as:

$$q_t = K \cdot A_t \cdot \left(F \cdot \Delta T_{sr(st)}\right) \quad [W] \qquad (9)$$

Here the correction factor for log mean temperature difference is, [4, 9, 18]:

$$F = \frac{\Delta T_{sr}}{\Delta T_{sr(st)}} \qquad (10)$$

Where $\Delta T_{sr(st)}$ is determined according to the relation (1) for the counter flow.

The expression in the brackets in relation (9) represents the corrected mean logarithmic temperature difference.
Heat flow can be obtained through the equation of energy balance (Figure 3), as:

$$q_t = m_t \cdot c_{pt} \cdot \left(T_{tu} - T_{ti}\right) = m_h \cdot c_{ph} \cdot \left(T_{hi} - T_{hu}\right) [W] \qquad (11)$$

where $m_t$ and $m_h$ are mass flows of the hotter and the colder fluid in kg/s, whereas their mean specific heat capacities are $c_{pt}$ and $c_{ph}$, [1, 3, 6].

In order to conduct the analysis of performances, or the design of the heat exchanger, it is necessary to find the relationship between the total heat flow $q_t$, mean coefficient of heat transfer $K_{sr}$, heat exchange area $A_t$ and mean temperature difference between the hot and the cold fluid $\Delta T_{sr}$, in the form of [2, 4]:

$$q_t = K_{sr} \cdot \Delta T_{sr} \cdot A_t \qquad (12)$$

When it comes to tube heat exchangers, it is common to calculate the heat transfer coefficient $K_{sr}$ for the outer area of the tube as:

$$\frac{1}{K_{sr}} = \frac{1}{\alpha_o} + R_{ps} + \frac{d_s}{2\lambda_z} \cdot \ln\frac{d_s}{d_u} + R_{pu} \cdot \frac{d_s}{d_u} + \frac{1}{\alpha_c} \cdot \frac{d_s}{d_u} \qquad (13)$$

where $\alpha_c$ and $\alpha_o$ are coefficients of heat transfer in the tube and the shell, $R_{pu}$ and $R_{pi}$ – thermal resistances of fouling deposits and $\lambda_z$ heat conductivity of the wall of the tube, [5, 7, 9, 12].

## 4. FUNCTION OF COSTS IN HEAT EXCHANGERS

For practical optimization requirements, it is best to use economic criterion which compares total annual costs for different devices. This criterion shows that the best device is the one with the lowest total annual costs, [21, 22, 23, 24].
The total annual costs for a heat exchanger, expressed in currencies based on an approximate economic analysis, can be defined as, [21, 25].

$$C_{uk} = C_{inv} + C_{pog} \qquad [EUR] \qquad (14)$$

where:

$C_{inv}$ – total annual investment costs,
$C_{pog}$ – total annual operation costs.

Total annual investment costs can be expressed as

$$C_{inv} = f_k \cdot \frac{c_{rt}}{\tau_g} \qquad [EUR] \qquad (15)$$

Here is:

$c_{rt}$ [EUR] – is the price of the heat exchanger installed
$\tau_g$ [god] – the expected life time of the heat exchanger
$f_k$ – factor which takes into consideration amortization, interests, maintenance and other costs (brought down to one year)

The estimation of investment costs is based on the principle that the costs of equipment and the heat exchanger itself $C_{inv}$ rise non-linearly with the increase of dimensions.
Basic dependence which refers to the cost of the heat exchanger installed in [EUR] is, [22, 23, 24, 25]:

$$c_{rt} = c_0 \cdot \left(\frac{A_t}{A_{t0}}\right)^{\alpha} \cdot f_{mp} \cdot \frac{I}{I_0} \qquad (16)$$

where

$c_0$ [EUR] – the basic cost of heat exchanger
$A_{t0}$ [$m^2$] –the basic area of the heat exchanger
$A_t$ [$m^2$] – the area for heat exchange as a variable, which is in the interval covered by the above expression, ($A_{t,min} \div A_{t,max}$)
$\alpha$– the exponent which depends on the type of the heat exchanger
$f_{mp}$ – factor taking into consideration pressure and material
$I_0$, I – the basic and the present index of costs.

The equation (14) can be expressed in a simplified form:

$$c_{rt} = \bar{c} \cdot A_t^{\alpha} \qquad (17)$$

Where the constant is

$$\bar{c} = c_0 \cdot A_{t0}^{-\alpha} \cdot f_{mp} \cdot I \cdot I_0^{-1},$$

which is calculated for domestic economic conditions.

Relation (17) has an exponential character and holds for a particular interval $A_{t,min} \div A_{t,max}$, fig. 6.

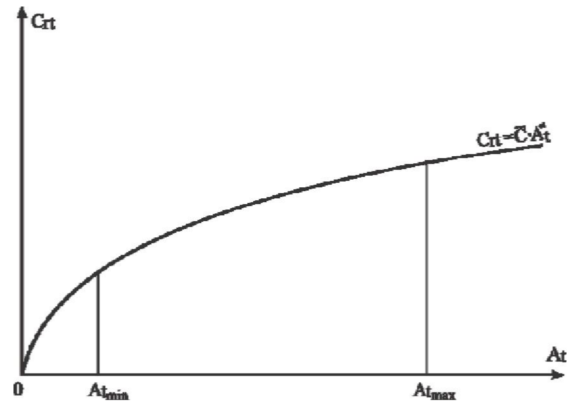

Fig. 6. Dependence between the cost of the heat exchanger and the area for heat exchange

Annual operation or production costs, for the case of cooling process fluids, can be expressed through the cost of heat transfer fluid or the costs of its pumping, transportation and preparation:

$$C_{pog} = c_{rv} \cdot \tau \cdot m_h \qquad [EUR] \qquad (18)$$

where:

$c_{rv}$ [EUR/kg] – the cost of the heat transfer fluid, the costs of its pumping, transportation and preparation

$\tau$ [h] – the number of working hours of the heat exchanger in a year

$m_h$ [kg/h] – mass flow of the heat transfer fluid

In this case, it is considered that operation (production) costs are proportional to the mass flow of the heat transfer fluid.
Generally, as it was mentioned, the total annual costs consist of investment and production components. It often happens that one of these components decreases with the increase of the parameter that is optimized R, whereas the other component rises (Fig. 7). Mathematically, it follows that the total costs in this case must have a minimum $C_{u,min}$, which corresponds to the optimal parameter $R_o$, [22, 24]. The curve of total costs can thus be designed graphically, by adding ordinates for a series of arbitrary points:

$$C_4 = C_2 + C_3 \qquad (19)$$

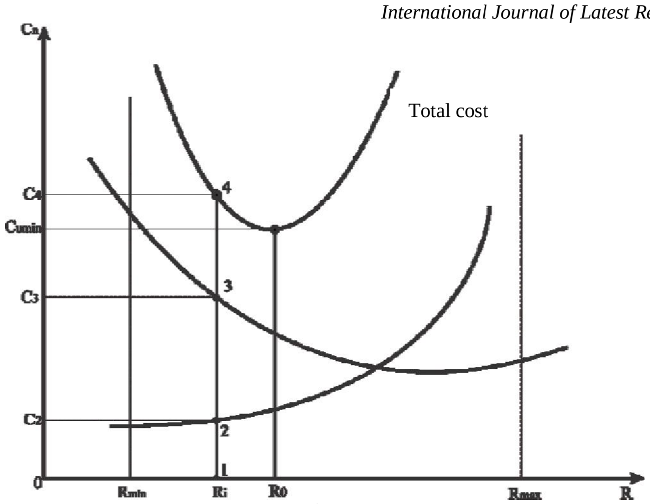


Fig. 7. Function of total annual costs

Parameters $R_{min}$ and $R_{max}$in fig. 7 represent the limitations of the parameter R.

## 5. ESTABLISHING THE FUNCTION OF TOTAL COSTS FOR THE CASE OF COOLING THE PROCESS FLUID

A common case in chemical process industry occurs in the process of cooling the process fluid using a heat transfer fluid, in a heat exchanger type (1-2), which is the topic of the paper. According to the graph in Figure 8, we will determine the optimal value of exit temperature of the heat transfer fluid for this case. It was assumed that the process fluid flows through the shell and the heat transfer fluid flows through the tube. The opposite combination of flow is also possible.

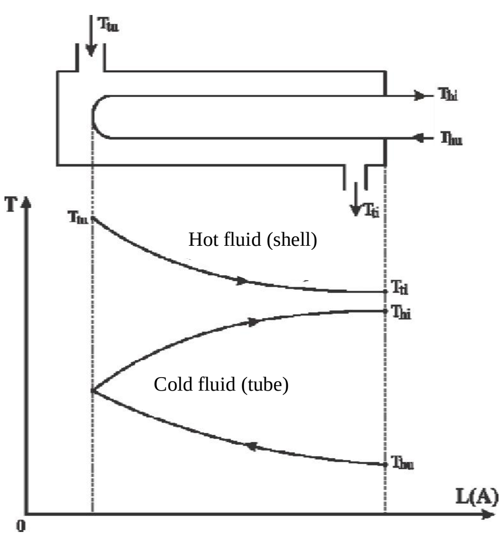


Fig. 8.Cooling of the process fluid in type (1-2) heat exchanger

We assume that the entering and the exit temperatures of the process fluid, $T_{tu}$ and $T_{ti}$ , or the mass flow $m_t$ , being primary parameters, are known. We also assume that the exit temperature of the heat transfer fluid $T_{hi}$ as well as the mass flow $m_h$ , are unknown, being secondary parameters.

There are certain limitations of the value of exit temperature of the heat transfer fluid which is optimized, considering its maximum value, [15, 27, 29].

The heat exchange area, according to (7) is

$$A_t = \frac{q_t}{K \cdot \Delta T_{sr}} \tag{20}$$

From relation (20) it follows that higher logarithmic temperature difference $\Delta T_{sr}$ at other unchanged conditions involves a smaller area $A_t$ and, therefore, a lower investment cost.

A detailed analysis of relations (1) and (9), shows that the decrease of the exit temperature of the heat transfer fluid $T_{hi}$, at other unchanged temperatures, increases the log mean temperature difference $\Delta T_{sr}$. As a result, according to (20) we obtain lower investment costs.

Substituting the relation (17) by (15), the annual investment cost will be:

$$C_{inv} = \frac{f_k}{\tau_g} \cdot \bar{c} \cdot A_t^{\alpha} \tag{21}$$

Thus the mass flow of the heat transfer fluid, according to (11) can be expressed as:

$$m_h = \frac{q_t}{c_{ph} \cdot (T_{hi} - T_{hu})} \tag{22}$$

If we consider the relation (22), operational (production) costs according to (18) will be:

$$C_{pog} = \frac{c_{rv} \cdot \tau \cdot q_t}{c_{ph} \cdot (T_{hi} - T_{hu})} \tag{23}$$

It follows from relation (23) that the decrease of exit temperature of the heat transfer fluid $T_{hi}$, will raise operation (production) costs if other conditions remain unchanged.

Therefore, in one case, with the decrease of temperature $T_{hi}$ one group of costs is reduced ($C_{inv}$), whereas the other group of costs ($C_{pog}$) increases. From this it follows, considering Figure 7, that the function of total costs must have a minimum which corresponds to the optimal exit temperature of the heat transfer fluid $T_{hi,o}$.

Substituting the expressions (21) and (23) by (14), total annual costs will be:

$$C_{uk} = \frac{c_{rv} \cdot \tau \cdot q_t}{c_{ph} \cdot (T_{hi} - T_{hu})} + \frac{f_k}{\tau_g} \cdot \bar{c} \cdot A_t^{\alpha} \tag{24}$$

Considering the relation (20), it follows that:

$$C_{uk} = \frac{c_{rv} \cdot \tau \cdot q_t}{c_{ph} \cdot (T_{hi} - T_{hu})} + \frac{f_k \cdot \bar{c}}{\tau_g} \cdot \left( \frac{q_t}{K \cdot \Delta T_{sr}} \right)^{\alpha} \tag{25}$$

Or finally:

$$C_{uk} = \frac{c_{rv} \cdot \tau \cdot q_t}{c_{ph} \cdot (T_{hi} - T_{hu})} + \frac{f_k \cdot \bar{c}}{\tau_g} \cdot \left(\frac{q_t}{K}\right)^{\alpha} \cdot \left(\frac{1}{\Delta T_{sr}}\right)^{\alpha} \quad (26)$$

Reciprocal value of mean logarithmic temperature difference according to (8), will be

$$\frac{1}{\Delta T_{sr}} = \frac{\ln \dfrac{T_{tu} + T_{ti} - T_{hu} - T_{hi} + \sqrt{(T_{tu} - T_{ti})^2 + (T_{hi} - T_{hu})^2}}{T_{tu} + T_{ti} - T_{hu} - T_{hi} - \sqrt{(T_{tu} - T_{ti})^2 + (T_{hi} - T_{hu})^2}}}{\sqrt{(T_{tu} - T_{ti})^2 + (T_{hi} - T_{hu})^2}} \quad (27)$$

In order to have a simpler analysis and to solve the problem faster, relations (26) and (27) can be simplified by introducing substitutions:

$$C_{uk} = y \qquad T_{hi} = x \qquad T_{hu} = B$$

$$T_{tu} + T_{ti} - T_{hu} = C \qquad T_{tu} - T_{ti} = A \quad (28)$$

$$\frac{c_{rv} \cdot \tau \cdot q_t}{c_{ph}} = D \qquad \frac{f_k \cdot \bar{c}}{\tau_g} \cdot \left(\frac{q_t}{K}\right)^{\alpha} = E$$

Substituting relation (27) by (26), considering the substitutions (28), the function of total costs can be expressed as

$$y = \frac{D}{x - B} + E \cdot \left( \frac{\ln \dfrac{C - x + \sqrt{A^2 + (x - B)^2}}{C - x - \sqrt{A^2 + (x - B)^2}}}{\sqrt{A^2 + (x - B)^2}} \right)^{\alpha} \quad (29)$$

The analysis of the function (29) shows that the same is defined for $x \neq B$ or according to (28), $T_{hi} \neq T_{hu}$
In order to conduct a simpler mathematical analysis, function (29) can be expressed through two functions, $y_1$ and $y_2$, as:

$$y = D \cdot y_1 + E \cdot y_2 \quad (30)$$

where:

$$y_1 = \frac{1}{x - B} \quad (31)$$

$$y_2 = \left( \frac{\ln \dfrac{C - x + \sqrt{A^2 + (x - B)^2}}{C - x - \sqrt{A^2 + (x - B)^2}}}{\sqrt{A^2 + (x - B)^2}} \right)^{\alpha} \quad (32)$$

In this matter, considering definition of function $y_1$it must be $x \neq B$ which was not called into questions.

Square root sizes in equation (29) are positive for all values of the parameters A and B and the variable x.

Also by mathematical analysis can be shown that the logarithm in the same equation can't be negative for the real values of the parameters A, B and C.

## 6. OPTIMIZATION OF THE FUNCTION OF TOTAL COSTS

In case we presume that the total coefficient of heat flow is K=const., the function of total costs (26) or (29), can be differentiated according to the exit temperature of the heat transfer fluid $T_{hi}$=x, which gives its optimal value, [10, 19, 20]:

$$\frac{dC_{uk}}{dx} = 0 \qquad y' = \frac{dy}{dx} = 0 \quad (33)$$

Thus, for the case of minimum it must be $y'' = \frac{d^2 y}{dx^2} > 0$

Differentiation of (30) results in:

$$y' = D \cdot y_1' + E \cdot y_2' \quad (34)$$

The first derivative of function (31) is

$$y_1' = -\frac{1}{(x - B)^2} \quad (35)$$

Derivative of the complex function (32), can be expressed as, [19, 20]:

$$y_2' = \alpha \cdot \left( \frac{\ln \dfrac{C - x + \sqrt{A^2 + (x - B)^2}}{C - x - \sqrt{A^2 + (x - B)^2}}}{\sqrt{A^2 + (x - B)^2}} \right)^{\alpha - 1} \cdot \left(\frac{u}{v}\right)' \quad (36)$$

Where new functions introduced are:

$$u = \ln \frac{C - x + \sqrt{A^2 + (x - B)^2}}{C - x - \sqrt{A^2 + (x - B)^2}} =$$

$$= \ln\left(C - x + \sqrt{A^2 + (x - B)^2}\right) - \ln\left(C - x - \sqrt{A^2 + (x - B)^2}\right)$$
(37)

$$v = \sqrt{A^2 + (x - B)^2} \quad (38)$$

Using the quotient rule for derivatives, it follows that [19, 20]:

$$\left(\frac{u}{v}\right)' = \frac{u'\cdot v - v'\cdot u}{v^2} \tag{39}$$

Derivative of the complex function (37), after certain mathematical operations and transformations, is the following:

$$u' = \frac{\left(C - x + \sqrt{A^2 + (x-B)^2}\right)'}{C - x + \sqrt{A^2 + (x-B)^2}} - \frac{\left(C - x - \sqrt{A^2 + (x-B)^2}\right)'}{C - x - \sqrt{A^2 + (x-B)^2}} \tag{40}$$

or, finally

$$u' = \frac{2\cdot\left[A^2 + (x-B)^2\right] + 2\cdot(C-x)\cdot(x-B)}{\left[(C-x)^2 - A^2 - (x-B)^2\right]\cdot\sqrt{A^2 + (x-B)^2}} \tag{41}$$

From (38) it follows:

$$v' = \frac{x-B}{\sqrt{A^2 + (x-B)^2}} \qquad v^2 = A^2 + (x-B)^2 \tag{42}$$

Substituting (37), (38), (41) and (42) by(39) we have:

$$\left(\frac{u}{v}\right)' = \frac{1}{A^2 + (x-B)^2}\cdot \left[\frac{2\cdot\left[A^2 + (x-B)^2\right] + 2\cdot(C-x)\cdot(x-B)}{\left[(C-x)^2 - A^2 - (x-B)^2\right]\cdot\sqrt{A^2 + (x-B)^2}}\cdot\sqrt{A^2 + (x-B)^2}\right] - \frac{1}{A^2 + (x-B)^2}\cdot\left[\frac{x-B}{\sqrt{A^2 + (x-B)^2}}\cdot \ln\frac{C - x + \sqrt{A^2 + (x-B)^2}}{C - x - \sqrt{A^2 + (x-B)^2}}\right] \tag{43}$$

After simple mathematical operations from (43) we obtain quotient derivative in the form of:

$$\left(\frac{u}{v}\right)' = \frac{2\cdot\left[A^2 + (x-B)^2\right] + 2\cdot(C-x)\cdot(x-B)}{\left[(C-x)^2 - A^2 - (x-B)^2\right]\cdot\left[A^2 + (x-B)^2\right]} - \frac{x-B}{\sqrt{\left[A^2 + (x-B)^2\right]^3}}\cdot \ln\frac{C - x + \sqrt{A^2 + (x-B)^2}}{C - x - \sqrt{A^2 + (x-B)^2}} \tag{44}$$

Substituting (44) by (36) we finally obtain:

$$y_2' = \alpha\cdot\left(\frac{\ln\dfrac{C - x + \sqrt{A^2 + (x-B)^2}}{C - x - \sqrt{A^2 + (x-B)^2}}}{\sqrt{A^2 + (x-B)^2}}\right)^{\alpha-1}\cdot \left[\frac{2\cdot\left[A^2 + (x-B)^2\right] + 2\cdot(C-x)\cdot(x-B)}{\left[(C-x)^2 - A^2 - (x-B)^2\right]\cdot\left[A^2 + (x-B)^2\right]} - \frac{x-B}{\sqrt{\left[A^2 + (x-B)^2\right]^3}}\cdot \ln\frac{C - x + \sqrt{A^2 + (x-B)^2}}{C - x - \sqrt{A^2 + (x-B)^2}}\right] \tag{45}$$

Substituting (35) and (45) by (34), the first derivative of the optimization function was completely determined as:

$$y' = -\frac{D}{(x-B)^2} + E\cdot\alpha\cdot\left(\frac{\ln\dfrac{C - x + \sqrt{A^2 + (x-B)^2}}{C - x - \sqrt{A^2 + (x-B)^2}}}{\sqrt{A^2 + (x-B)^2}}\right)^{\alpha-1}\cdot \left[\frac{2\cdot\left[A^2 + (x-B)^2\right] + 2\cdot(C-x)\cdot(x-B)}{\left[(C-x)^2 - A^2 - (x-B)^2\right]\cdot\left[A^2 + (x-B)^2\right]} - \frac{x-B}{\sqrt{\left[A^2 + (x-B)^2\right]^3}}\cdot \ln\frac{C - x + \sqrt{A^2 + (x-B)^2}}{C - x - \sqrt{A^2 + (x-B)^2}}\right] \tag{46}$$

If we, considering (33), take the function of first order (46) to be equal zero (y′=0), the obtained equation can be expressed in suitable form for numerical analysis, that is, to be solved using unknown variable x:

$$\frac{2\cdot\left[A^2 + (x-B)^2\right] + 2\cdot(C-x)\cdot(x-B)}{\left[(C-x)^2 - A^2 - (x-B)^2\right]\cdot\left[A^2 + (x-B)^2\right]} - \frac{x-B}{\sqrt{\left[A^2 + (x-B)^2\right]^3}}\cdot \ln\frac{C - x + \sqrt{A^2 + (x-B)^2}}{C - x - \sqrt{A^2 + (x-B)^2}} = \frac{D}{(x-B)^2}\cdot\frac{1}{E\cdot\alpha}\cdot\left(\frac{\ln\dfrac{C - x + \sqrt{A^2 + (x-B)^2}}{C - x - \sqrt{A^2 + (x-B)^2}}}{\sqrt{A^2 + (x-B)^2}}\right)^{1-\alpha} \tag{47}$$

The resulting equation (47) cannot be expressed explicitly, it represents a transcendental equation.

If we take the left side of the equation (47) and denote it with φ(x) and the right side with ψ(x) and represent both functions in the same coordinate system, Figure 9, the solution sought $x_o$, if it exists at all, is then obtained by a graphical method, in the cross-section of curves φ(x) and ψ(x), point P. The solution obtained, according what was said earlier, represents the optimal temperature of the heat transfer fluid at the exit, i.e. $x_o=T_{hi,o}$. It must be noted that only the solution which is in the real temperature domain is taken into consideration.

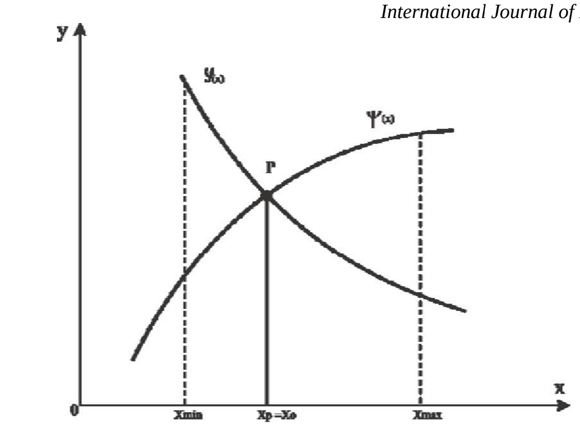


Fig. 9. Principled solution of transcendental equation of the problem using graphical method

According to Figure 9, the solution $x_o$ is obtained by iterative method using a suitable computer program. Prior to that, functions φ(x) and ψ(x) are graphically represented also using a computer program.

If we want to determine the character of the local extremum of the function y=y(x), the second derivative of the same function $y'' = \frac{d^2y}{dx^2}$ can be determined according to (46) as well. Considering the fact that this derivative is an extremely complex function, it is more practical to take the starting function y=y(x) according to (29) and represent it graphically for a real temperature region and decide from the graph which extremum is in question.

## 7. ANALYSIS OF THE FUNCTION OF COSTS FROM THE ASPECT OF TEMPERATURE LIMITATIONS

The analysis will be performed for the region in which the function of costs y(x) according to (29), is continuant and defined, i.e. for a wider region of real values of exit temperatures for the heat transfer fluid: $T_{hu} \div T_{ti}$, fig. 5. Moreover, the limitation of maximum exit temperature of the heat transfer fluid will be taken into consideration, which is for water $T_{hi,max}$=45÷50 $^0$C. The main reason for this is a more intensive formation of lime deposits at these temperatures, [28, 29].

Generally, according to Figure 10, when the function of costs y=y(x) has a minimum in point M, there are two possibilities.

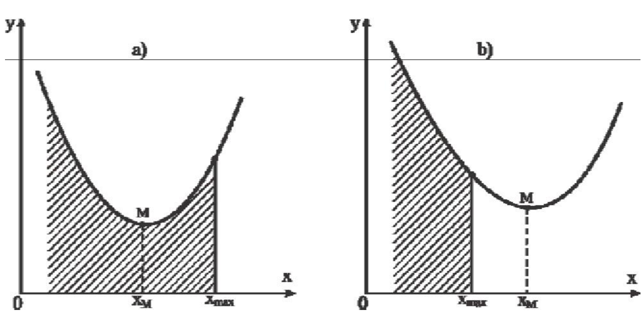


Fig. 10. Possibilities of the minimum of the function of costs considering the exit temperature of the heat transfer fluid

In the first case (Figure 10a) the minimum is in the region $x_M \le x_{max} = T_{hi,max}$. For this case, solution $x_M$ is the optimal one.

In the second case (Figure 10b) the minimum is in the region $x_M > x_{max} = T_{hi,max}$. This solution cannot be adopted as optimal considering that the maximum allowed exit temperature of the heat transfer fluid $T_{hi,max}$ is exceeded. For this case, the optimal solution is $T_{hi,max}$ since it gives the lowest possible costs.

When the heat transfer fluid is water, which often happens in practice, it can be adopted that $T_{hi,max}$=50 $^0$C, [30, 31].

## 8. EXAMPLE

For a multi-tube heat exchanger type 1-2, according to Figure 11 and air as the process fluid the known parameters are entering and exit temperatures and mass flow:

$t_{tu}$=100$^0$C $t_{ti}$=36$^0$C $m_t$=1000 kg/h

Air is cooled using water at the exit temperature $t_{hu}$=19$^0$C

Cooling water passes through the tubes and air through the shell. The total coefficient of heat flow is constant K=150 W/m$^2$K.The life time and the number of working hours of the heat exchanger are

$\tau_g$=7 years τ=7000 h

The cost of water, its transportation and pumping is: $c_{rv}$=0,0015 EUR/kg.

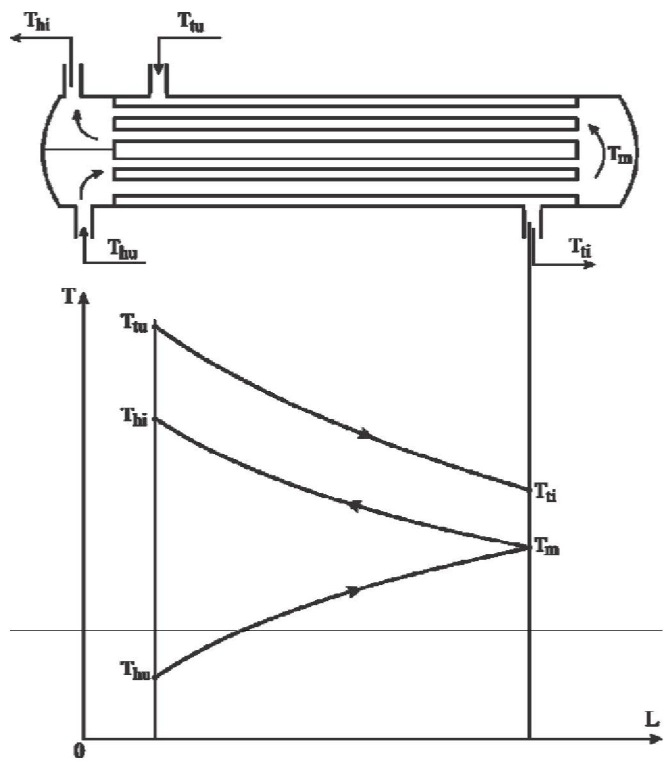


Fig. 11. Tube-bundle heat exchanger with two passes through the tubes and one pass through the shell

The problem is solved in the following order:

a) The envisaged profile of temperature distribution, it is given in Figure 11.
b) The constants, considering given temperatures, are the following, according to (28):

$B=T_{hu}=292,15$ K $A=T_{tu}-T_{ti}=64$ K

$C=T_{tu}+T_{ti}-T_{hu}=390,15$ K

c) The constants considering techno-economic indicators are the following, according to (28):

$$D=\frac{c_{rv}\cdot\tau\cdot q_t}{c_{ph}}$$

$$D=\frac{0{,}0015\cdot 7000\cdot 3600\cdot 17938}{4180}=162214\ \text{[EURK]}$$

$$E=\frac{f_k\cdot\bar{c}}{\tau_g}\cdot\left(\frac{q_t}{K}\right)^{\alpha}=\frac{1{,}1\cdot 525}{7}\left(\frac{17938}{150}\right)^{0{,}65}=1849\ [\text{EUR K}^{0.65}]$$

Now for this case it is assumed that $K = 150\text{W/m}^2\text{K}$ [26, 28].

Here we have adopted a factor which takes into consideration interests, amortization, maintenance and other fixed costs $f_k=1{,}1$.

Heat flow is

$$q_t=m_t\cdot c_{pt}\cdot(T_{tu}-T_{ti})=\frac{1000\cdot 1009\cdot(373{,}15-309{,}15)}{3600}$$

$q_t=17938$ [W]

Specific heat capacity of air was adopted for mean temperature.

In order to determine consonants $\bar{c}$ and α approximately according to (17) and Figure 6, we can determine, for the expected range of areas $A_{t,min}\div A_{t,max}$, the price of several heat exchangers, according to manufacturer's catalogues. If we adopt in the given range of areas for example 4 heat exchangers, points 1, 2, 3 and 4 according to Figure 12 are completely determined. Heat exchangers with known areas $A_{t1}$, $A_{t2}$, $A_{t3}$ and $A_{t4}$ have a known price $c_{rt1}$, $c_{rt2}$, $c_{rt3}$ and $c_{rt4}$. Using these data, constants $\bar{c}$ and α in relation $c_{rt}=\bar{c}\cdot A_t^{\alpha}$ can be determined by the method of least squares, for example, [19, 20].

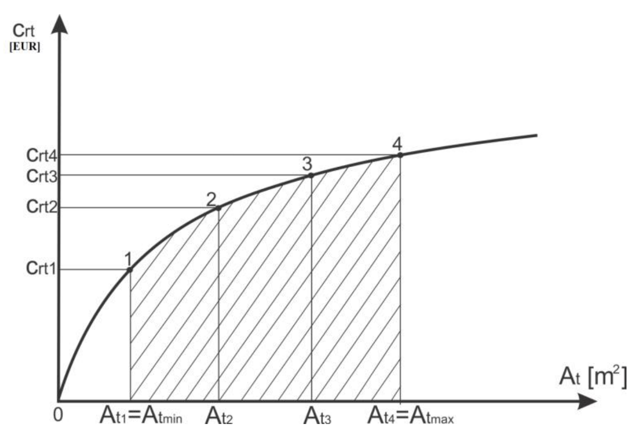


Fig.12. Determining constants $\bar{c}$ and α in relation (17)

Due to the scale of the procedure and limited space, we will only give the final result of the previous calculation, relation (17):

$$c_{rt}=\bar{c}\cdot A_t^{\alpha}$$

Where the result is $\bar{c}=525$ α=0,65.

d) Optimal exit temperature of cooling water is determined using a suitable program according to relation (47), where functions ψ(x) and φ(x) are drawn according to Figure 9. Iterative method gives the optimal exit temperature

$x_o=T_{hi,o}=$ 320.06 K $t_{hi,o}=46.91\ ^0$C

The resulting solution is not too far away from maximum allowed exit temperature $t_{hi,max}=50\ ^0$C, therefore it can be adopted for further calculation.

Graphic representation of function (29) in the coordinate system xOy, for real exit temperatures, leads to the conclusion that the function of costs for the calculated exit temperature has a minimum.

After this, all the other values which are necessary for designing heat exchangers, are determined according to the classic procedure, [30, 31, 33, 36].

## 9. CONCLUSION

Using the thermo-hydraulic calculation, the main dimensions of heat exchangers are determined based on the given operation conditions. This problem can be solved constructively in many ways, all of which fulfill the given conditions. Among these various possible combinations of solution, as it was shown in the paper, the optimal solution of the parameter which is optimized was determined for the criterion of minimal total annual costs.

Techno-economic calculations, which are included during thermo-hydraulic design, obviously make the procedure more complicated to a large extent. The reason for this is mainly of mathematical nature, considering the fact that, in almost every case, we obtain complex mathematical functions whose derivatives are even more complex functions. The solutions cannot be expressed explicitly, which makes the problem even more complex. One has to be particularly cautious in cases where many local minimums of the optimization function occur, which sometimes is the case with such problems.

It is not sufficient to approach the problem of optimization of heat exchangers purely thermo-dynamically, and economic conditions must also be taken into account. Due to such approach, it can be stated that optimization of heat exchangers is a very complicated tasks for engineers. It is particularly convenient to apply optimization of heat exchangers in those cases when working conditions are not precisely defined, as it was the case in the paper. In this case the parameters which have secondary importance are optimized.

Based on the model shown it is possible to optimize other groups of heat exchangers with multiple passes of the fluid through the shell and the tubes. It is also possible to optimize other parameters in heat exchangers, the speed of the fluid through the tube and the shell, for example.
It is also possible to solve the case when the total coefficient of heat transfer is not known in advance, or when its value is unreliable. Here, iterative method must be applied considering the fact that not all characteristic temperatures are known.

Optimization of type 1-1 heat exchangers (the fluid passes once through the shell and the tubes) is considerably simpler since the expression for mean temperature difference here is much simpler. In theory and in practice, there are elaborate models for this case, which cannot be said for type 1-2 heat exchangers which are the subject of this paper.

The derived model can be directly applied with type 1-2 condensers or evaporators. We would obtain somewhat simpler relations since the temperatures of the cold and the hot fluid are constant in this case.

Solving the problem of optimization of heat exchangers in which the fluid passes more than once through the shell and the tubes requires a multidisciplinary approach, it is difficult to solve such problem without the experts specialized in various fields (thermodynamics, heat spreading, fluid mechanics, economy, numeric mathematics, mathematical programming, mathematical analysis, etc.). In the same way, it must be supported by modern computer technique and suitable numeric programs, both the existing and those that are yet to be devised. This is certainly one of the reasons why optimization of heat exchangers is rarely conducted in practice. This is not always justified, especially since economic aspect must not be neglected.

## REFERENCES


[1] W. E. Boyce and R. C. Diprima Elementary Differential Equations and Boundary Value Problems. New York, John Waley, 1986

[2] SS. Kutateladze, Fundamentals of Heat Transfer, New York, Academic Press, 1983.

[3] Y. A. Cengel and R. H. Turner, Fundamentals of Thermal-Fluid Sciences, Mc Graw-Hill, New York, 2005.

[4] Robert J. Ribando, Heat Transfer Tools, Mc Graw, New York, 2002.

[5] Schumpeter, J. A. The Theory of Economic Development, Oxford Unuversity Press,1981

[6] Schmalenbach E : Kostenrechmung und Preispolitik, Koln, 1963.

[7] D. Q. Kern and A. D. Kraus, Extended Surface Heat Transfer, Mc Graw-Hill, New York, 1982.

[8] H. Gröber, S. Erk, and U. Grigull, Fundamentals of Heat Transfer, Mc Graw-Hill, New York, 1981.

[9] Y. Jaluria and K. E. Torrance, Compatational Heat Transfer, Hemisphere, New York, 1986.

[10] T. M. Shih, Numerical Heat Transfer, Hemisphere, 1984.

[11] M. Jakob, Heat Transfer, John Wiley, New York, 1979.

[12] D. K. Edwards, V. E. Denny, Transfer Processes. 2nd ed. Hemisphere, Nashington, 1979.

[13] S. W. Chi., Heat Theory and Practice, Hemispher, Washington, 1976.

[14] Baehr H. D., Termdynamik, Springer-Verlag, Berlin, 1993.

[15] Büki G., Energetika, MK, Budapest, 1997.

[16] Reynolds W. S. Thermodynamics, Mc Graw-Hill, Book Co, New York, 1978.

[17] Kozić Đ., Vasiljević B., Bekavne V.: Handbook of Thermodynamics, MF, Beograd 2007.

[18] Michael J. M. Howard N. S. Fundamentals of Engineering Thermdynamics, Wiley, New York, 1994.

[19] Bronštejn I.N.; Semendljajev K. A.: Sprovočnik po matematike, GIFML, Moskva, 1992.

[20] Kurepa S. Mathematical analysis, Termička knjiga, Zagreb, 1999.

[21] Perović D. Theory of Costs, Svetlost, Sarajevo, 1994.

[22] Markovski S. Fundamentals of Theory of Costs, Informator, Zagreb, 1981.

[23] Milojević M. Business System, Naučna knjiga, Beograd, 1979.

[24] Smith J.G.: Business Strategy, Basic Blackwell, 1985.

[25] Šoškić B. Theory of Values, Savremena administracija, Beograd, 1971.

[26] N. Afgan and E. U. Schlunder. *Heat Exchanger: Design and Theory Sourcebook.* Washington, DC: McGraw-Hill/Scripta, 1974.

[27] R.A. Bownman, A. C. Mueller, and W. M. Nagle. "Mean Temperature Difference in Design." *Trans.* ASME 62, 1940. p. 283. Reprined with permission of ASME International.

[28] A. P. Fraas, *Heat Exchanger Design*. 2d ed. New York: John Wiley&Sons, 1989.

[29] K. A. Gardner. "Variable Heat Transfer Rate Correct Multipass Exchangers, Shell Side Film Controlling". *Transactions of the* ASME 67 (1945).

[30] W. M. Kays and A. L. London. *Compact Heat Exchangers*. 3rd ed. New York: McGraw-Hill, 1984. Reprinted by permission of William M. Kays.

[31] W. M. Kays and H. C. Perkins. In *Handbook of Heat Transfer*, ed. W. M. Roshenow and J. P. Hartnett. New York: McGraw-Hill, 1972. Chap. 7.

[32] A. C. Mueler. *"Heat Exchangers*". In *Handbook of Heat Transfer*, ed. W. M. Roshenow and J. P. Hartnett. New York: McGraw-Hill, 1972. Chap.18.

[33] M. N. Özißik., *Heat Transfer-A Basic Apprach*. New York: McGraw-Hill, 1985.

[34] E. U. Schlunder: *Heat Exchanger Design Handbook*. Washington, DC: Hemisphere, 1982.

[35] Standards of Tubular Exchangers Manufacturers Association. New York: Tublar Exchangers Manufacturers Association, latest ed.

[36] R. A. Stevens, J. Fernandes, and J. R. Woolf. "Mean Temperature Difference in One, Two, and Three Pass Crossflow Heat Exchangers."*Transactions of the* ASME 79 (1957).

[37] J. Taborek, G. F. Hewitt, and N. Afgan. *Heat Exchangers Theory and Practice*. New York: Hemisphere, 1983.

[38] G. Walker. *Industrial Heat Exchangers*. Washington, DC Hemisphere, 1982.