MNK Publication

International Journal of Latest Research in Science and Technology **ISSN (Online):2278-5299**
Volume 3, Issue 3: Page No. 15-24. May-June 2014
https://www.mnkpublication.com/journal/ijlrst/index.php

AN ACCURATE AND COMPUTATIONALLY EFFICIENT APPROXIMATION TO PSYCHROMETRIC CALCULATIONS

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Abstract – ***The present paper deals with a computationally efficient procedure to calculate the psychrometric properties of moist air. Taking into account the simplified approximations by ASHRAE and the accurate correlations for the dry air (Lemmon et al., 1999) and saturated steam (Wagner & Kretzschmar, 2008) properties, a simple model is built to predict the thermodynamic behaviour of any moist air working system. All properties are calculated using polynomial correlations of higher grade or of lower grade, according to their accuracy: thus, the computational efficiency is achieved, while the values of each property is satisfactorily accurate (average errors are lower than 0.05%). Apart from improved correlations for extensive properties (volume, enthalpy and entropy) and intensive properties (partial pressure, dew point temperature) of moist air, a new approximation is presented to calculate the wet-bulb temperature without solving any iteration based complex equation. The results of the presented method are compared to the ASHRAE approximation and the accurate correlations by Lemmon and Kretzschmar, in terms of error, average error and standard deviation of error in a range of dry – bulb temperature between 0°C and 80oC***

Keywords **–** Psychrometrics, Dry air properties, Saturated air properties, Specific enthalpy, Specific entropy, Wet-bulb temperature

I. INTRODUCTION

The psychometric calculations are necessary for each air based energy systems: HVAC plants, evaporative coolers, cooling towers etc. The psychrometric charts are widely used to calculate quickly the thermodynamic properties of moist air, while the psychrometric correlations are used to design complex cooling devices, as evaporative coolers and cooling towers. Given that the accuracy of the calculations is directly connected to the quality of design of these complex devices, the accuracy of the computational procedures is rather critical, especially when these devices are modelled using large computational grids, which require high computational power and significant accuracy.

The most known, common, and widely used psychrometric calculation has been developed and presented by ASHRAE [1]. This method is based on an exponential approximation of saturation pressure of water and on a first grade polynomial correlation of specific enthalpy. The computationally efficiency of this method is quite satisfactory, although an iterative calculation of wet-bulb is needed. Apart from correlations, ASHARE also provides tables for dry air, saturated air and saturated steam properties; the tables data do not generally agree with the corresponded values calculated using the correlation, however the errors are not significant (see Table I).

Arnold Wexler and Richard Hyland presented [2] a comprehensive study about the properties of dry air, moist air and water. In this study, extremely complex correlations (based on Virial formulas) have developed, in order to study the properties of moist air within a wide range of pressure and temperature conditions.

TABLE I: SATURATION PRESSURE OVER LIQUID WATER, ACCORDING TO ASHRAE

Temperature [°C]	Temperature [K]	Saturation pressure over liquid water (eq. 6 [1]) [Pa]	Saturation pressure over liquid water (Table 3 [1]) [Pa]	error
0	273.15	611.213	611.15	0.002%
10	283.15	1227.995	1228.2	0.017%
20	293.15	2338.8	2339.2	0.017%
30	303.15	4246.03	4246.7	0.016%
40	313.15	7383.46	7384.4	0.013%
50	323.15	12349.86	12351.3	0.012%
60	333.15	19943.76	19945.8	0.010%

The accuracy of this approximation is undoubtedly high, however the computational efficiency is very low (as an significant number of derivations has to be carried out) and the whole methodology is not based on the newest temperature standards, as it is older than ITS-1990 [3].

The most recent and well-known works about the properties of dry air and water steam have been presented by Eric Lemmon et al. [4] and Wolfgang Wagner & Hans – Joachim Kretzschmar [5], respectively. The first study deals with the properties of dry air under different conditions, providing extensive correlations for each property; the second study focuses on the water steam properties and is based on the industrial formulation IAPWS IF1997.

Publication History

Manuscript Received	:	*22 May 2014*
Manuscript Accepted	:	*31 May 2014*
Revision Received	:	*10 June 2014*
Manuscript Published	:	*30 June 2014*

As the psychrometric calculations are applied to devices working under atmospheric pressure (so the dry air behaves as an almost ideal gas), the tabled data for dry air properties under 1*atm* can be used without any significant error, while the water steam properties should always be calculated as a function of temperature and pressure.

II. WATER VAPOUR CALCULATIONS

According ASHRAE [1], the saturation pressure over liquid water, as the temperature varies between 0^oC and 200^oC, is calculated as a function of temperature:

$$p^{sat}(T) = \exp\begin{bmatrix} \frac{-5.8002206\,10^{3}}{T} + 1.3914993 \\ -4.8640239\,10^{-2}\cdot T + 4.1764768\,10^{-5}\cdot T^2 \\ -1.4452093\,10^{-8}\cdot T^3 \\ +6.5459673\ln(T) \end{bmatrix} \quad \{1\}$$

A similar expression is proposed by Perry's Chemical Engineer's Handbook [6]:

$$p^{sat}(T) = \exp\begin{pmatrix} 73.649 - \frac{72582}{T} \\ -7.3037\cdot\ln(T) + 4.1653\,10^{-6}\cdot T^2 \end{pmatrix} \quad \{2\}$$

The IAPWS based correlation [5] proposes the following complex correlation:

$$p^{sat}(T) = 10^6 \cdot P\left(T + \frac{a}{T+b}\right) \quad \{3\}$$

where $P(\theta) = \left(\frac{2\cdot\gamma(\theta)}{-\beta(\theta) + \sqrt{\beta^2(\theta) - 4\cdot\alpha(\theta)\cdot\gamma(\theta)}}\right)^4$, $\alpha(\theta) = \theta^2 + c\cdot\theta - d$, $\beta(\theta) = e\cdot\theta^2 + f\cdot\theta - g$ and $\gamma(\theta) = h\cdot\theta^2 + i\cdot\theta + j$. The polynomial coefficients a-j are given on Table II.

TABLE II: POLYNOMIAL COEFFICIENTS OF EQ. {3}

a	-0.23855557567849	***f***	1202.82470247
b	650.17534844798	***g***	3232555.0322333
c	1167.05212767	***h***	14.91510861353
d	-724213.16703206	***i***	-4823.2657361591
e	-17.073846940092	***j***	-405113.40542057

Using the IAPWS based correlation, the following of 5^{th} grade is proposed:

$$p^{sat}(t) = \begin{pmatrix} 0.0000028369658919866\cdot t^5 \\ +0.00026384035254176\cdot t^4 \\ +0.027455020266255\cdot t^3 \\ +1.41991306922864\cdot t^2 \\ +44.4562796037644\cdot t \\ +611.2065249735 \end{pmatrix} \quad \{4\}$$

The comparison results are presented on Table III. Given that the IAPWS correlation is considered to be the most accurate among the approximations, the methods proposed be Perry's and ASHRAE, as well as the present method are compared to IAPWS correlation. As shown, the proposed correlation leads to an average error equal to 0.001% (while Perry's and ASHRAE's methods lead to average errors 0.044% and 0.012%, respectively). Additionally, the proposed method is the most efficient in terms of computational time: it requires about 68% less time in relation to IAPWS method, while Perry's and ASHRAE's methods need about 63% and 56% less time, respectively).

III. HUMIDITY RATIO, RELATIVE HUMIDITY & DEGREE OF SATURATION

The **humidity ratio** [kg_w/kg_{da}] of a given condition of moist air is defined as the ration of water vapour mass to the dry air mass. This ratio can alternatively be expressed as a ratio of the mole fractions, x_{wv} and x_{da}, and the ratio of molecular masses, λ_M. The mole fraction of water vapour in the saturated air is equal to the ratio of water vapour partial pressure to the air pressure (which is assumed to be equal to 1*atm*):

$$x_{wv}^{sat}(t) = \frac{p^{sat}(t)}{p_{air}} \xrightarrow{x_{wv}^{sat}+x_{da}^{sat}=1} x_{da}^{sat}(t) = 1 - \frac{p^{sat}(t)}{p_{air}} \quad \{5\}$$

Thus, the humidity ratio of the saturated air is expressed as a function of temperature:

$$W^{sat}(t) = \lambda_M \cdot \frac{x_{wv}^{sat}(t)}{x_{da}^{sat}(t)} = 0.621945\frac{p^{sat}(t)}{p_{air} - p^{sat}(t)} \quad \{6\}$$

The humidity ratio of the saturated air is calculated using the four correlations for saturation pressure. The results are presented on Table IV and show that Perry's method is of lower accuracy and that ASHRAE's method is as accurate as the proposed method. However, in terms of computational time, the proposed method is about 73% faster than the analytical method based on IAPWS correlation, while the average error of the calculations does not exceed 0.001%.

The **relative humidity** is defined as the ratio of the partial pressure of water vapour to the saturation pressure of water vapour under the same temperature:

$$\varphi = \frac{p_{wv}(\varphi,t)}{p_{wv}^{sat}(t)} \quad \{7\}$$

According to the above formula, the humidity ratio of the moist and non-saturated air is express as a function of temperature and relative humidity:

$$W(\varphi,t) = \lambda_M \cdot \frac{p_{wv}(\varphi,t)}{p_{air} - p_{wv}(\varphi,t)} \quad \{4\} \quad \{8\}$$

The correlation above can be inversed to calculate the relative humidity:

$$\varphi(W,t) = \frac{W\cdot p_{air}}{p^{sat}(t)\cdot(W+\lambda_M)} \quad \{9\}$$

The relative humidity must not be confused with the ***degree of saturation***, which is expressed by the following ratio:

$$\mu(W,t) = \frac{W}{W^{sat}(t)} \Leftrightarrow \mu(\varphi,t) = \frac{\varphi}{1 + \frac{(1-\varphi)\cdot W^{sat}(t)}{\lambda_M}} \quad \{10\}$$

TABLE III: WATER VAPOUR SATURATION PRESSURE

t [^oC]	W&K [*Pa*]	Perry		ASHRAE		NTUA	
		value [*Pa*]	error [%]	value [*Pa*]	error [%]	value [*Pa*]	error [%]
0	611.213	610.118	0.179	611.213	0.000	611.207	0.001
2	705.988	704.920	0.151	705.954	0.005	706.023	0.005
4	813.549	812.526	0.126	813.480	0.009	813.578	0.003
6	935.353	934.394	0.103	935.246	0.011	935.355	0.000
8	1072.988	1072.113	0.082	1072.840	0.014	1072.962	0.002
10	1228.184	1227.414	0.063	1227.995	0.015	1228.138	0.004
12	1402.822	1402.179	0.046	1402.591	0.017	1402.769	0.004
14	1598.944	1598.449	0.031	1598.669	0.017	1598.895	0.003
16	1818.759	1818.433	0.018	1818.440	0.018	1818.726	0.002
18	2064.657	2064.519	0.007	2064.292	0.018	2064.647	0.000
20	2339.215	2339.284	0.003	2338.804	0.018	2339.230	0.001
22	2645.211	2645.501	0.011	2644.753	0.017	2645.250	0.001
24	2985.633	2986.154	0.017	2985.127	0.017	2985.690	0.002
26	3363.687	3364.445	0.023	3363.132	0.017	3363.756	0.002
28	3782.813	3783.807	0.026	3782.207	0.016	3782.882	0.002
30	4246.688	4247.913	0.029	4246.030	0.016	4246.750	0.001
32	4759.247	4760.685	0.030	4758.534	0.015	4759.292	0.001
34	5324.685	5326.311	0.031	5323.915	0.015	5324.707	0.000
36	5947.474	5949.251	0.030	5946.642	0.014	5947.469	0.000
38	6632.370	6634.248	0.028	6631.472	0.014	6632.337	0.000
40	7384.427	7386.344	0.026	7383.460	0.013	7384.371	0.001
42	8209.010	8210.885	0.023	8207.967	0.013	8208.934	0.001
44	9111.800	9113.537	0.019	9110.675	0.012	9111.714	0.001
46	10098.811	10100.294	0.015	10097.597	0.012	10098.725	0.001
48	11176.398	11177.491	0.010	11175.088	0.012	11176.323	0.001
50	12351.270	12351.815	0.004	12349.856	0.011	12351.216	0.000
52	13630.501	13630.314	0.001	13628.975	0.011	13630.476	0.000
54	15021.536	15020.411	0.007	15019.892	0.011	15021.547	0.000
56	16532.211	16529.912	0.014	16530.440	0.011	16532.258	0.000
58	18170.754	18167.019	0.021	18168.850	0.010	18170.834	0.000
60	19945.802	19940.339	0.027	19943.761	0.010	19945.905	0.001
62	21866.409	21858.892	0.034	21864.225	0.010	21866.519	0.001
64	23942.054	23932.127	0.041	23939.727	0.010	23942.153	0.000
66	26182.655	26169.927	0.049	26180.185	0.009	26182.723	0.000
68	28598.576	28582.621	0.056	28595.967	0.009	28598.592	0.000
70	31200.636	31180.991	0.063	31197.895	0.009	31200.588	0.000
72	34000.118	33976.285	0.070	33997.259	0.008	34000.008	0.000
74	37008.782	36980.222	0.077	37005.820	0.008	37008.632	0.000
76	40238.867	40205.004	0.084	40235.824	0.008	40238.734	0.000
78	43703.103	43663.321	0.091	43700.009	0.007	43703.092	0.000
80	47414.720	47368.361	0.098	47411.611	0.007	47415.000	0.001
Average error [%]			**0.044**		**0.012**		**0.001**
Computational time [%]	**100**	**37.5**		**43.8**		**31.3**	

TABLE IV: WATER VAPOUR SATURATION PRESSURE

t [°C]	W&K [kg_w/kg_{da}]	Perry		ASHRAE		NTUA	
		value [kg_w/kg_{da}]	error [%]	value [kg_w/kg_{da}]	error [%]	value [kg_w/kg_{da}]	error [%]
0	0.003774	0.003768	0.180	0.003774	0.001	0.003774	0.001
2	0.004364	0.004357	0.152	0.004364	0.005	0.004364	0.005
4	0.005034	0.005028	0.127	0.005034	0.004	0.005034	0.004
6	0.005795	0.005789	0.104	0.005795	0.000	0.005795	0.000
8	0.006657	0.006651	0.082	0.006656	0.002	0.006656	0.002
10	0.007631	0.007626	0.063	0.007631	0.004	0.007631	0.004
12	0.008732	0.008728	0.046	0.008731	0.004	0.008731	0.004
14	0.009972	0.009969	0.031	0.009972	0.003	0.009972	0.003
16	0.011368	0.011366	0.018	0.011368	0.002	0.011368	0.002
18	0.012937	0.012936	0.007	0.012937	0.000	0.012937	0.000
20	0.014698	0.014698	0.003	0.014698	0.001	0.014698	0.001
22	0.016672	0.016674	0.011	0.016672	0.002	0.016672	0.002
24	0.018883	0.018886	0.018	0.018883	0.002	0.018883	0.002
26	0.021356	0.021361	0.023	0.021356	0.002	0.021356	0.002
28	0.024120	0.024126	0.027	0.024120	0.002	0.024120	0.002
30	0.027207	0.027215	0.030	0.027207	0.002	0.027207	0.002
32	0.030653	0.030662	0.032	0.030653	0.001	0.030653	0.001
34	0.034496	0.034507	0.032	0.034497	0.000	0.034497	0.000
36	0.038783	0.038795	0.032	0.038783	0.000	0.038783	0.000
38	0.043562	0.043575	0.030	0.043561	0.001	0.043561	0.001
40	0.048890	0.048903	0.028	0.048889	0.001	0.048889	0.001
42	0.054830	0.054844	0.025	0.054829	0.001	0.054829	0.001
44	0.061456	0.061469	0.021	0.061455	0.001	0.061455	0.001
46	0.068850	0.068861	0.016	0.068849	0.001	0.068849	0.001
48	0.077107	0.077116	0.011	0.077107	0.001	0.077107	0.001
50	0.086338	0.086342	0.005	0.086338	0.000	0.086338	0.000
52	0.096670	0.096668	0.002	0.096670	0.000	0.096670	0.000
54	0.108253	0.108243	0.009	0.108253	0.000	0.108253	0.000
56	0.121262	0.121242	0.017	0.121262	0.000	0.121262	0.000
58	0.135907	0.135873	0.025	0.135907	0.001	0.135907	0.001
60	0.152437	0.152385	0.034	0.152438	0.001	0.152438	0.001
62	0.171155	0.171080	0.044	0.171156	0.001	0.171156	0.001
64	0.192428	0.192323	0.054	0.192429	0.001	0.192429	0.001
66	0.216711	0.216569	0.066	0.216712	0.000	0.216712	0.000
68	0.244571	0.244380	0.078	0.244571	0.000	0.244571	0.000
70	0.276724	0.276472	0.091	0.276723	0.000	0.276723	0.000
72	0.314092	0.313761	0.105	0.314090	0.000	0.314090	0.000
74	0.357879	0.357444	0.122	0.357877	0.001	0.357877	0.001
76	0.409690	0.409118	0.140	0.409688	0.001	0.409688	0.001
78	0.471712	0.470957	0.160	0.471712	0.000	0.471712	0.000
80	0.547008	0.546004	0.184	0.547014	0.001	0.547014	0.001
Average error [%]			**0.053**		**0.001**		**0.001**
Computational time [%]	**100**	**39.4**		**33.3**		**27.3**	

IV. SPECIFIC VOLUME

The specific volume [m^3/kg_{da}] of dry air under atmospheric pressure is given as a function of temperature is tabled in Lemmon's study [4]; ASHRAE [1] also provides a general equation of the specific volume of moist air. Using the tabled data by Lemmon's study, a 1st grade polynomial can be written:

$$v_{da}(t) = 0.00283910371072806\, t + 0.77336652663561 \quad \{11\}$$

The comparison between Lemmon's data, the ASHRAE formula and the polynomial expression above is presented on Table V. As the both expressions are of 1st grade, there is no difference on computational time, however the proposed polynomial is much more accurate.

The specific volume of saturated air equal to [2]

$$v^{sat}(t) = \frac{v_{da}(t)}{x_{da}(t)} \quad \{12\}$$

and the specific air of moist non-saturated air is given by the relationship:

$$v(W,t) = v_{da}(t) + \mu(W,t)\cdot\left(v^{sat}(t) - v_{da}(t)\right) = \\ = v_{da}(t)\cdot(1-\mu(W,t)) + \mu(W,t)\cdot v^{sat}(t) \qquad \{13\}$$

As ASHRAE expresses the specific volume as a 1st grade function of two variables:

$$v(W,t) = \frac{0.287042}{101.325}\cdot(t+273.15)\cdot(1+1.607858\,W) \qquad \{14\}$$

The equation {13} is obviously much slower than the equation {14}. As shown on Table VI, both methods lead to similar results (the average deviation is about 0.024%), thus the ASHRAE approximation is preferable.

TABLE V: SPECIFIC VOLUME OF DRY AIR

t [°C]	Lemmon [m^3/kg_{da}]	ASHRAE		NTUA	
		value [m^3/kg_{da}]	error [%]	value [m^3/kg_{da}]	error [%]
-3.15	0.764397	0.764879	0.063	0.764423	0.003
6.85	0.792799	0.793208	0.052	0.792814	0.002
16.85	0.821200	0.821536	0.041	0.821205	0.001
26.85	0.849614	0.849865	0.030	0.849596	0.002
36.85	0.878007	0.878194	0.021	0.877987	0.002
46.85	0.906383	0.906523	0.015	0.906379	0.001
56.85	0.934778	0.934852	0.008	0.934770	0.001
66.85	0.963152	0.963181	0.003	0.963161	0.001
76.85	0.991534	0.991509	0.002	0.991552	0.002
Average error [%]			**0.026**		**0.002**
Computational time [%]	**100**	**100**		**100**	

TABLE VI: SPECIFIC VOLUME OF SATURATED AIR

t [°C]	ASHRAE [m^3/kg_{da}]	NTUA	
		value [m^3/kg_{da}]	error [%]
0	0.78624	0.78580	0.056
5	0.80064	0.80022	0.051
10	0.81503	0.81465	0.047
15	0.82942	0.82907	0.042
20	0.84381	0.84350	0.038
25	0.85820	0.85792	0.033
30	0.87260	0.87234	0.029
35	0.88699	0.88677	0.025
40	0.90138	0.90119	0.021
45	0.91577	0.91562	0.017
50	0.93017	0.93004	0.014
55	0.94456	0.94446	0.010
60	0.95895	0.95889	0.007
65	0.97334	0.97331	0.003
70	0.98773	0.98773	0.000
75	1.00213	1.00216	0.003
80	1.01652	1.01658	0.006
Average error [%]			**0.024**

V. SPECIFIC ENTHALPY & SPECIFIC ENTROPY

According to ASHRAE methodology, the specific enthalpy [kJ/kg_{da}] of moist air is the sum if individual partial enthalpies of the components:

$$h(W,t) = h_{da}(t) + W\cdot h_{wv}^{sat}(t) = 1.006\cdot t + W\cdot(2501+1.86\cdot t) \qquad \{15\}$$

On the other hand, using the Lemmon's data for dry air and IAPWS's data for saturated steam, a very accurate relation can be formed:

$$h(W,t) = h_{da}(t) + \mu(W,t)\cdot h_{wv}^{sat}(t) \qquad \{16\}$$

As a part of the present study, an improved polynomial is proposed (of 2nd grade for dry air and of 3rd grade for water vapour):

$$h(W,t) = h_{da}(t) + W \cdot h_{wv}^{sat}(t) = \quad \{17\}$$
$$= \begin{pmatrix} 0.000021930047421211 \cdot t^2 \\ + \ 1.0052997736091 \cdot t \end{pmatrix}$$
$$+ W \cdot \begin{pmatrix} -0.00000654788808 \cdot t^3 \\ -0.00020272387563 \cdot t^2 \\ + \ 1.834669929014 \cdot t \\ + \ 2500.9049361728 \end{pmatrix}$$

The comparison of three approximations (eqs. {16}, {15} and {17}) is presented on Table VII, for a constant humidity ratio $0.010 kg_w/kg_{da}$ and a temperature range $15^oC – 80^oC$. In terms of computational efficiency, the ASHRAE approximation and the proposed polynomial lead to 97% less time compared to the analytical correlation, based on the Lemmon's data for dry air and IAPWS's data for saturated steam. However, in terms of accuracy, the proposed polynomial lead to an average error about 0.002%, which is much lower than the error of ASHRAE approximation. For the calculation of the specific enthalpy as a function of temperature and relative humidity, the combination of the eqs. {8} and {17} is needed.

A similar formula can be written to calculate the specific entropy of moist air [$kJ/kg_{da} \cdot K$]:

$$s(W,t) = s_{da}(t) + W \cdot s_{wv}^{sat}(t) = \quad \{18\}$$
$$= \begin{pmatrix} -0.0000052480873343218 \cdot t^2 \\ +0.00364882621533519 \cdot t \end{pmatrix}$$
$$+ W \cdot \begin{pmatrix} -0.00000029978784 \cdot t^3 \\ +0.000115612414176 \cdot t^2 \\ -0.026638556743152 \cdot t \\ +9.1552031898795 \end{pmatrix}$$

The results of the correlation above are shown on Table VIII; the average error is about 0.058%.

TABLE VII: SPECIFIC ENTHALPY (FOR CONSTANT HUMIDITY RATIO W=0.010kg_w/kg_{da})

t [^oC]	Lemmon and W&K [kJ/kg_{da}]	ASHRAE		NTUA	
		value [kg_w/kg_{da}]	error [%]	value [kg_w/kg_{da}]	error [%]
16	41.3946	41.409	0.034	41.392	0.006
18	43.4430	43.458	0.034	43.441	0.005
20	45.4916	45.506	0.033	45.489	0.005
22	47.5402	47.555	0.032	47.538	0.004
24	49.5889	49.604	0.031	49.587	0.004
26	51.6376	51.653	0.030	51.636	0.003
28	53.6865	53.702	0.029	53.685	0.002
30	55.7355	55.751	0.027	55.735	0.002
32	57.7846	57.800	0.026	57.784	0.001
34	59.8337	59.848	0.025	59.833	0.000
36	61.8830	61.897	0.023	61.883	0.000
38	63.9324	63.946	0.022	63.933	0.001
40	65.9819	65.995	0.020	65.983	0.001
42	68.0315	68.044	0.018	68.032	0.001
44	70.0813	70.093	0.016	70.082	0.002
46	72.1311	72.142	0.015	72.133	0.002
48	74.1811	74.190	0.013	74.183	0.002
50	76.2312	76.239	0.011	76.233	0.002
52	78.2815	78.288	0.009	78.283	0.002
54	80.3318	80.337	0.007	80.334	0.002
56	82.3823	82.386	0.004	82.384	0.002
58	84.4329	84.435	0.002	84.435	0.002
60	86.4837	86.484	0.000	86.485	0.002
62	88.5345	88.532	0.002	88.536	0.002
64	90.5855	90.581	0.005	90.587	0.001
66	92.6367	92.630	0.007	92.638	0.001
68	94.6879	94.679	0.009	94.688	0.001
70	96.7393	96.728	0.012	96.739	0.000
72	98.7909	98.777	0.014	98.790	0.001
74	100.8425	100.826	0.017	100.841	0.001
76	102.8943	102.874	0.019	102.892	0.002
78	104.9462	104.923	0.022	104.943	0.003
80	106.9982	106.9722	0.024	106.9946	0.003
Average error [%]			**0.018**		**0.002**
Computational time [%]	**100**	**2.6**		**2.6**	

TABLE VIII: SPECIFIC ENTROPY (FOR CONSTANT HUMIDITY RATIO W=0.010kg_w/kg_{da})

t [^oC]	Lemmon and W&K [$kJ/kg_{da}K$]	NTUA	
		value [$kJ/kg_{da}K$]	error [%]
16	0.1448	0.1446	0.148
18	0.1513	0.1511	0.138
20	0.1577	0.1575	0.125
22	0.1641	0.1640	0.111
24	0.1705	0.1703	0.096
26	0.1768	0.1767	0.080
28	0.1831	0.1830	0.063
30	0.1894	0.1893	0.047
32	0.1956	0.1955	0.031
34	0.2017	0.2017	0.016
36	0.2079	0.2079	0.002
38	0.2140	0.2140	0.012
40	0.2201	0.2201	0.024
42	0.2261	0.2262	0.035
44	0.2321	0.2322	0.045
46	0.2381	0.2382	0.053
48	0.2440	0.2441	0.059
50	0.2499	0.2501	0.064
52	0.2558	0.2560	0.067
54	0.2616	0.2618	0.068
56	0.2674	0.2676	0.067
58	0.2732	0.2734	0.064
60	0.2790	0.2791	0.060
62	0.2847	0.2848	0.053
64	0.2904	0.2905	0.044
66	0.2960	0.2961	0.033
68	0.3016	0.3017	0.019
70	0.3072	0.3072	0.004
72	0.3128	0.3128	0.014
74	0.3183	0.3182	0.034
76	0.3238	0.3237	0.056
78	0.3293	0.3291	0.081
80	0.3348	0.3344	0.108
Average error [%]			**0.058**
Computational time [%]	**100**	**3.8**	

VI. WET-BULB TEMPERATURE

The wet-bulb temperature is defined as the temperature at which liquid water evaporates into the air to bring it to saturation at exactly this same temperature and total pressure. By solving the following complex equation, the wet-bulb temperature is accurately solved, however the computational time will be obviously high, as a number of iterations is needed to converge.

$$h+\left(W^{sat}(t_{wb})-W\right)\cdot\left(c_{p,w}\cdot t_{wb}\right)=h^{sat}(t_{wb}) \qquad \{19\}$$

where $c_{p,w}$=4.186*kJ/kgK* is the specific heat of water. Instead of the above computationally inefficient relation, a polynomial expression is proposed, to calculate the wet-bulb temperature as a function of temperature and relative humidity; the combination of eqs. {9} and {20} lead to the calculation of wet-bulb temperature as a function of humidity ratio and temperature.

$$t_{wb}(\varphi,t)=\sum_{ii=1}^{36}k_{ii}\cdot\varphi^{l_{ii}}\cdot t^{m_{ii}} \qquad \{20\}$$

The above correlation allows the direct calculation of wet-bulb temperature, without a loop calculation; the coefficients *k*, *l* and *m* are given on Table IX for various values of atmosphere pressure (as a function of altitude), while on Table X and XI the results of eqs. {19} and {20} for a constant relative humidity 50% and 20%, respectively (at the sea level). The results prove that, keeping the average error lower than 0.5% (as the temperature varies between 0^oC and 80^oC, the proposed polynomial approximation needs about 87% less computational time; the accuracy of the polynomial expression {20} is satisfactorily (error <1%) for temperatures higher than 80^oC and lower than 110^oC.

TABLE IX: WET-BULB TEMPERATURE CALCULATION COEFFICIENTS

ii	l_{ii}	m_{ii}	k_{ii}									
			0m (101.325kPa)		200m (98.945kPa)		500m (95.461kPa)		1000m (89.875kPa)		1500m (84.556kPa)	
1	1	6	-2.76790849324	$\cdot 10^{-11}$	-2.7912240735524	$\cdot 10^{-11}$	-2.80384856812762	$\cdot 10^{-11}$	-2.81341765197842	$\cdot 10^{-11}$	-2.80782757676099	$\cdot 10^{-11}$
2	0	7	-2.085785506	$\cdot 10^{-13}$	-2.07707187156183	$\cdot 10^{-13}$	-2.05280145053345	$\cdot 10^{-13}$	-2.00007171969922	$\cdot 10^{-13}$	-1.93267632309204	$\cdot 10^{-13}$
3	0	6	8.27722286316	$\cdot 10^{-11}$	8.20674431689693	$\cdot 10^{-11}$	8.07065112397343	$\cdot 10^{-11}$	7.8013861477356	$\cdot 10^{-11}$	7.48141943699274	$\cdot 10^{-11}$
4	0	5	-1.36190746545769	$\cdot 10^{-8}$	-1.34108131904988	$\cdot 10^{-8}$	-1.31165145737188	$\cdot 10^{-8}$	-1.25641165143048	$\cdot 10^{-8}$	-1.19374313571458	$\cdot 10^{-8}$
5	1	5	1.48997464204914	$\cdot 10^{-8}$	1.50585810091982	$\cdot 10^{-8}$	1.52556289271802	$\cdot 10^{-8}$	1.55167117337251	$\cdot 10^{-8}$	1.56879916205882	$\cdot 10^{-8}$
6	2	5	-6.8863081481931	$\cdot 10^{-9}$	-7.09100603874058	$\cdot 10^{-9}$	-7.34773595913535	$\cdot 10^{-9}$	-7.74188179417568	$\cdot 10^{-9}$	-8.08976760551129	$\cdot 10^{-9}$
7	0	4	1.08986854276442	$\cdot 10^{-6}$	1.06300299015354	$\cdot 10^{-6}$	1.02882784998957	$\cdot 10^{-6}$	9.67041948199234	$\cdot 10^{-7}$	8.9941044999617	$\cdot 10^{-7}$
8	1	4	-3.50688355343236	$\cdot 10^{-6}$	-3.50282744188509	$\cdot 10^{-6}$	-3.54544313764484	$\cdot 10^{-6}$	-3.60075800031308	$\cdot 10^{-6}$	-3.63510473558723	$\cdot 10^{-6}$
9	2	4	4.0995110230012	$\cdot 10^{-6}$	4.1071615247129	$\cdot 10^{-6}$	4.21795522641787	$\cdot 10^{-6}$	4.38573879022505	$\cdot 10^{-6}$	4.53034271917594	$\cdot 10^{-6}$
10	3	4	-1.41814163354963	$\cdot 10^{-6}$	-1.41367174349066	$\cdot 10^{-6}$	-1.46372014306391	$\cdot 10^{-6}$	-1.54223204690331	$\cdot 10^{-6}$	-1.61373827370243	$\cdot 10^{-6}$
11	0	3	-2.14288228908424	$\cdot 10^{-5}$	-1.97669147588791	$\cdot 10^{-5}$	-1.75257503651183	$\cdot 10^{-5}$	-1.35955207823654	$\cdot 10^{-5}$	-9.42692320216243	$\cdot 10^{-6}$
12	1	3	4.04850127314613	$\cdot 10^{-4}$	4.01648474915019	$\cdot 10^{-4}$	4.02027467284217	$\cdot 10^{-4}$	4.00706015289015	$\cdot 10^{-4}$	3.96805719761134	$\cdot 10^{-4}$
13	2	3	-9.70337488518371	$\cdot 10^{-4}$	-9.68848221427034	$\cdot 10^{-4}$	-9.773179327765	$\cdot 10^{-4}$	-9.87393127759681	$\cdot 10^{-4}$	-9.9204475657797	$\cdot 10^{-4}$
14	3	3	8.69796943303159	$\cdot 10^{-4}$	8.72117008500905	$\cdot 10^{-4}$	8.80161004061759	$\cdot 10^{-4}$	8.90227797482768	$\cdot 10^{-4}$	8.95776926877257	$\cdot 10^{-4}$
15	4	3	-2.88904439017472	$\cdot 10^{-4}$	-2.90897872490299	$\cdot 10^{-4}$	-2.92550542957406	$\cdot 10^{-4}$	-2.94233062643627	$\cdot 10^{-4}$	-2.94472213272994	$\cdot 10^{-4}$
16	0	2	-4.43896746706535	$\cdot 10^{-3}$	-4.46727407456611	$\cdot 10^{-3}$	-4.51677461380338	$\cdot 10^{-3}$	-4.60139326626203	$\cdot 10^{-3}$	-4.6885833846191	$\cdot 10^{-3}$
17	1	2	3.40625085992046	$\cdot 10^{-3}$	3.90635749698053	$\cdot 10^{-3}$	4.62173099076558	$\cdot 10^{-3}$	5.93424378061707	$\cdot 10^{-3}$	7.40211818107226	$\cdot 10^{-3}$
18	2	2	-6.23924966052324	$\cdot 10^{-3}$	-7.96700605690312	$\cdot 10^{-3}$	-1.07772969234503	$\cdot 10^{-2}$	-1.59116441658142	$\cdot 10^{-2}$	-2.16311858789472	$\cdot 10^{-2}$
19	3	2	5.1175180301907	$\cdot 10^{-2}$	5.42691010310108	$\cdot 10^{-2}$	5.98877205235375	$\cdot 10^{-2}$	6.99546718331025	$\cdot 10^{-2}$	8.09310479721427	$\cdot 10^{-2}$
20	4	2	-8.11055006768809	$\cdot 10^{-2}$	-8.40313924628142	$\cdot 10^{-2}$	-8.94474251700091	$\cdot 10^{-2}$	-9.89903596459569	$\cdot 10^{-2}$	-1.09195553430054	$\cdot 10^{-1}$
21	5	2	3.7339072265255	$\cdot 10^{-2}$	3.84270912318236	$\cdot 10^{-2}$	4.03617995370135	$\cdot 10^{-2}$	4.37323773418451	$\cdot 10^{-2}$	4.72883458630627	$\cdot 10^{-2}$
22	0	1	6.83640878206276	$\cdot 10^{-1}$	6.80218638428452	$\cdot 10^{-1}$	6.75268676577559	$\cdot 10^{-1}$	6.66968232984235	$\cdot 10^{-1}$	6.58601303799074	$\cdot 10^{-1}$
23	1	1	3.4534970673524	$\cdot 10^{-1}$	3.45401307608234	$\cdot 10^{-1}$	3.45346648703158	$\cdot 10^{-1}$	3.45077862333989	$\cdot 10^{-1}$	3.44760235621283	$\cdot 10^{-1}$
24	2	1	3.21395842970335	$\cdot 10^{-1}$	3.37063519111409	$\cdot 10^{-1}$	3.63949549832963	$\cdot 10^{-1}$	4.07217279258021	$\cdot 10^{-1}$	4.47342865458618	$\cdot 10^{-1}$
25	3	1	-5.14758342211161	$\cdot 10^{-1}$	-5.00148904908928	$\cdot 10^{-1}$	-4.73731325482335	$\cdot 10^{-1}$	-4.09490481954519	$\cdot 10^{-1}$	-3.15677584046504	$\cdot 10^{-1}$
26	4	1	-1.45380007113002	$\cdot 10^{0}$	-1.58087497878533	$\cdot 10^{0}$	-1.81448371020299	$\cdot 10^{0}$	-2.25670537101486	$\cdot 10^{0}$	-2.77254302745938	$\cdot 10^{0}$
27	5	1	3.31420258149949	$\cdot 10^{0}$	3.48704890219482	$\cdot 10^{0}$	3.80477710035709	$\cdot 10^{0}$	4.38762758304024	$\cdot 10^{0}$	5.04328247215791	$\cdot 10^{0}$
28	6	1	-1.70084554227042	$\cdot 10^{0}$	-1.77365917509534	$\cdot 10^{0}$	-1.90617236950317	$\cdot 10^{0}$	-2.14590630581722	$\cdot 10^{0}$	-2.41115385375618	$\cdot 10^{0}$
29	0	0	-6.03698064601361	$\cdot 10^{0}$	-6.13340257462673	$\cdot 10^{0}$	-6.29329055716588	$\cdot 10^{0}$	-6.56702683225686	$\cdot 10^{0}$	-6.8502973120053	$\cdot 10^{0}$
30	1	0	6.82630149494756	$\cdot 10^{0}$	6.9592063405452	$\cdot 10^{0}$	7.18225687917138	$\cdot 10^{0}$	7.56748847074584	$\cdot 10^{0}$	7.9703939887901	$\cdot 10^{0}$
31	2	0	-3.14994093282345	$\cdot 10^{0}$	-3.27141520605275	$\cdot 10^{0}$	-3.49787864377798	$\cdot 10^{0}$	-3.9044193602441	$\cdot 10^{0}$	-4.35219638594608	$\cdot 10^{0}$
32	3	0	1.26631637553059	$\cdot 10^{1}$	1.33752758101142	$\cdot 10^{1}$	1.47188992795482	$\cdot 10^{1}$	1.72713510994394	$\cdot 10^{1}$	2.02750379038848	$\cdot 10^{1}$
33	4	0	-4.91519836713173	$\cdot 10^{1}$	-5.23849321420389	$\cdot 10^{1}$	-5.84030017981054	$\cdot 10^{1}$	-6.98365419512324	$\cdot 10^{1}$	-8.32647989952505	$\cdot 10^{1}$
34	5	0	1.09704683999231	$\cdot 10^{2}$	1.16572265942301	$\cdot 10^{2}$	1.29289585039927	$\cdot 10^{2}$	1.53215410701567	$\cdot 10^{2}$	1.8098086534204	$\cdot 10^{2}$
35	6	0	-1.14202159832997	$\cdot 10^{2}$	-1.20783907643188	$\cdot 10^{2}$	-1.3293143276387	$\cdot 10^{2}$	-1.55587604237336	$\cdot 10^{2}$	-1.81607615563265	$\cdot 10^{2}$
36	7	0	4.33985668272991	$\cdot 10^{1}$	4.57201938154749	$\cdot 10^{1}$	4.9990492003656	$\cdot 10^{1}$	5.79010936204886	$\cdot 10^{1}$	6.69127663627473	$\cdot 10^{1}$

TABLE X: WET-BULB TEMPERATURE (φ=50%)

t [°C]	ASHRAE [°C]	NTUA	
		value [°C]	error [%]
21	14.845	14.594	1.688
22	15.665	15.416	1.591
23	16.486	16.238	1.502
24	17.306	17.060	1.421
25	18.127	17.883	1.345
26	18.947	18.706	1.275
27	19.768	19.529	1.210
28	20.590	20.353	1.149
29	21.412	21.178	1.092
30	22.234	22.003	1.038
31	23.057	22.830	0.987
32	23.881	23.657	0.940
33	24.706	24.485	0.895
34	25.531	25.314	0.852
35	26.358	26.144	0.812
36	27.185	26.975	0.774
37	28.014	27.807	0.737
38	28.844	28.641	0.703
39	29.674	29.475	0.670
40	30.506	30.311	0.638
41	31.339	31.148	0.608
42	32.173	31.987	0.580
43	33.009	32.826	0.553
44	33.845	33.667	0.527
45	34.683	34.509	0.502
46	35.522	35.352	0.479
47	36.362	36.196	0.456
48	37.203	37.042	0.434
49	38.046	37.888	0.414
50	38.889	38.736	0.394
51	39.734	39.585	0.376
52	40.580	40.435	0.358
53	41.427	41.286	0.341
54	42.275	42.138	0.326
55	43.125	42.991	0.311
56	43.975	43.845	0.296
57	44.826	44.699	0.283
58	45.679	45.555	0.271
59	46.532	46.411	0.259
60	47.386	47.269	0.248
61	48.241	48.127	0.238
62	49.097	48.985	0.229
63	49.954	49.844	0.220
64	50.812	50.704	0.212
65	51.671	51.565	0.205
66	52.530	52.425	0.199
67	53.390	53.287	0.194
68	54.251	54.148	0.189
69	55.112	55.010	0.186
70	55.975	55.872	0.183
71	56.837	56.735	0.180
72	57.701	57.597	0.179
73	58.565	58.460	0.179
74	59.429	59.323	0.179
75	60.294	60.186	0.180
76	61.160	61.049	0.182
77	62.026	61.912	0.184
78	62.892	62.774	0.188
79	63.759	63.637	0.192
80	64.626	64.499	0.198
Average error [%]			**0.554**
Computational time [%]	**100**	**12.2**	

TABLE XI: WET-BULB TEMPERATURE (φ=20%)

t [°C]	ASHRAE [°C]	NTUA	
		value [°C]	error [%]
21	9.927	9.895	0.316
22	10.577	10.545	0.299
23	11.224	11.193	0.283
24	11.869	11.838	0.268
25	12.512	12.481	0.254
26	13.153	13.122	0.240
27	13.792	13.761	0.227
28	14.430	14.399	0.215
29	15.066	15.036	0.203
30	15.701	15.671	0.191
31	16.335	16.306	0.180
32	16.969	16.940	0.170
33	17.601	17.573	0.160
34	18.233	18.206	0.150
35	18.865	18.839	0.141
36	19.497	19.471	0.132
37	20.129	20.104	0.124
38	20.761	20.737	0.116
39	21.393	21.370	0.108
40	22.025	22.003	0.101
41	22.659	22.637	0.094
42	23.292	23.272	0.087
43	23.927	23.908	0.081
44	24.563	24.544	0.075
45	25.199	25.181	0.070
46	25.837	25.820	0.065
47	26.475	26.459	0.060
48	27.115	27.100	0.055
49	27.756	27.742	0.051
50	28.399	28.386	0.047
51	29.043	29.031	0.043
52	29.689	29.677	0.039
53	30.336	30.325	0.035
54	30.984	30.975	0.032
55	31.635	31.626	0.028
56	32.287	32.278	0.025
57	32.940	32.933	0.022
58	33.595	33.589	0.018
59	34.252	34.247	0.015
60	34.911	34.907	0.012
61	35.572	35.569	0.008
62	36.234	36.232	0.004
63	36.898	36.897	0.001
64	37.563	37.565	0.003
65	38.231	38.234	0.008
66	38.900	38.904	0.012
67	39.571	39.577	0.017
68	40.243	40.252	0.021
69	40.917	40.928	0.027
70	41.593	41.607	0.032
71	42.271	42.287	0.038
72	42.950	42.969	0.044
73	43.631	43.653	0.051
74	44.313	44.339	0.057
75	44.997	45.026	0.065
76	45.682	45.715	0.072
77	46.369	46.406	0.080
78	47.057	47.099	0.089
79	47.747	47.794	0.097
80	48.438	48.490	0.107
Average error [%]			**0.094**
Computational time [%]	**100**	**12.2**	

VII. CONCLUSIONS

Exact analytical expressions for all psychrometric properties of moist air have been derived using the complex equations of state for dry air and water vapour. However, the efficiency of these expressions in terms of computational time is low, making difficult the fast modelling of energy systems, working with moist air.

Utilizing the specialities of each thermodynamic parameter, the analytical expression of each of them can be simplified, producing a comprehensive and computationally efficient psychrometric model, the accuracy of it is about equal to the model based on analytical methods. Thus, the modelling of apparatuses, such as evaporative coolers and cooling towers is much faster, without any significant deviation in comparison with models using the analytical psychrometric equations. The greates deviation between the proposed approximation and the widely used ASHRAE's approximation in terms of computational efficiency is observed during the wet-bulb calculation; this thermodynamic parameter is critical for the operation of evaporative devices.

In this paper, the analytical expressions were substituted by simpler polynomial equations, based on the analytical ones, which were characterized by an R^2 value greater than 0.999. Using the polynomial equation, a significant reduction of computational time was achieved, without significantly affecting to the produced results. Especially and in order to evaluate the generic computational efficiency of the proposed approximation, an evaporative cooler utilizing the novel Maisotsenko cycle was modelled [7]. Under the same conditions, the new approximation leads to about 65% less computational time than using the analytical psychrometric correlations (based on Lemmon and IAPWS formulations), while the modelling results were almost equal for both psychrometric approximation.

Conclusively, the proposed approximation can help engineers and system designers model more effectively novel devices and focus in depth on how the moist air properties characterize the performance of them.

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NOMENCLATURE

c_p	specific heat [kJ/kgK]
h	specific enthalpy [kJ/kg_{da}]
p	pressure [Pa]
s	specific entropy [$kJ/kg_{da}K$]
t	temperature [^oC]
T	temperature [K]
v	specific volume [m^3/kg_{da}]
W	humidity ratio [kg_w/kg_{da}]
x	mole fraction [-]
λ_M	water molecular mass to dry air molecular mass ratio [-]
μ	degree of saturation [-]
φ	relative humidity [%]

Subscripts

air	moist air
da	dry air
db	dry-bulb (refers to temperature)
w	water
wb	wet-bulb (refers to temperature)
wb	water vapour

Superscript

sat	saturation