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ANOTHER EQUATION FOR DYNAMICS OF VISCOUS FLUID

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Abstract- **This paper presents another equation for dynamics of viscous fluid. It has been derived by using Navier - Stokes equation and by taking into consideration Euler equation integral of ideal fluid dynamics [1].**

Keywords -Euler Equation, fluid, flow.

I. INTRODUCTION

Navier - Stokes' equation for compressible viscous fluid has the following form:

$$\frac{D\vec{v}}{Dt} = \vec{f} - \frac{1}{\rho}\cdot gradp + \nu\cdot\Delta\vec{v} + \frac{\nu}{3}\cdot graddiv\,\vec{v}, \qquad (1)$$

where: $\frac{D\vec{v}}{Dt}$ - first material time derivative of velocity,

f - intensity of the mass forces, p - pressure,

ρ - fluid density, $\nu = \frac{\eta}{\rho}$ - kinematic viscosity,

η - dynamics viscosity, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ - Laplacian operator.

Pressure's function, $\Re_{(p)} = \int_{p_o}^{p} \frac{dp}{\rho_{(p)}}$, could be introduced for barotropic fluid flow, implying:

$$\frac{1}{\rho}\cdot grad\vec{p} = grad\vec{\Re}. \qquad (2)$$

If the conservative mass forces field has the potential U, then:

$$\vec{f} = -grad\vec{U}. \qquad (3)$$

Laplacian operator can be presented:

$$\Delta\vec{v} = graddiv\,\vec{v} - rotrot\,\vec{v}, \qquad (4)$$

equation (1) can be written as:

$$\frac{D\vec{v}}{Dt} = \vec{f} - \frac{1}{\rho}\cdot gradp + \frac{4}{3}\cdot\nu\cdot graddiv\,\vec{v} - rotrot\,\vec{v}. \qquad (5)$$

II POTENTIAL FLOW

Non whirling potential flow can be expressed as:

$$rot\,\vec{v} = 0, \qquad (6)$$

i.e. $rotrot\,\vec{v} = 0$ (7)

yielding:

$$\frac{D\vec{v}}{Dt} = -grad(U+\Re) + \frac{4}{3}\cdot\nu\cdot graddiv\,\vec{v}. \qquad (8)$$

Scalar multiplication of the equation (8) by element of stream line, $d\vec{s}$, yields:

$$\left(\frac{D\vec{v}}{Dt}, d\vec{s}\right) = -\left(grad(U+\Re), d\vec{s}\right) + \frac{4}{3}\cdot\nu\cdot\left(graddiv\vec{v}, d\vec{s}\right) \qquad (9)$$

Member (1) of equation (9) according to [1] , can be written as:

$$d\left(\frac{DI_L}{Dt}\right) = \left[\frac{D\vec{v}}{Dt}, d\vec{s}\right] + \left(\vec{v}, D\vec{v}\right) = \left[\frac{D\vec{v}}{Dt}, d\vec{s}\right] + d\left(\frac{v^2}{2}\right)$$

(10)

i.e.

$$\left[\frac{D\vec{v}}{Dt}, d\vec{s}\right] = d\left(\frac{DI_L}{Dt}\right) - d\left(\frac{v^2}{2}\right). \qquad (11)$$

where I_L - is fluid flow along stream line L. Member (2) can be written as:

$$-\left(grad(U+\Re), d\vec{s}\right) = -d(U+\Re). \qquad (12)$$

Member (3) of equation (9) can be written as:

$$\cdot\frac{4}{3}\cdot\nu\cdot\left(graddiv\,\vec{v}, d\vec{s}\right) = \frac{4}{3}\cdot\nu\cdot d\left(div\,\vec{v}\right) \qquad (13)$$

Replacement (10-13) in equation (9) yields:

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$$d\left(\frac{DI_L}{Dt}\right)-d\left(\frac{v^2}{2}\right)=-d(U+\Re)+\frac{4}{3}\cdot\nu\cdot d\left(div\vec{v}\right), \quad (14)$$

i.e.

$$\frac{DI_L}{Dt}-\frac{v^2}{2}+\Re+U-\frac{4}{3}\cdot\nu\cdot div\,\vec{v}=C_{(t)}\,. \quad (15)$$

Temporal function C(t) in a determined moment has a concrete value. Since for potential flow $\vec{v}=grad\varphi$, the material derivative of fluid flow can be written as:

$$\frac{DI_L}{Dt}=\frac{\partial I_L}{\partial t}+\vec{v}\cdot gradI_L=\frac{\partial I_L}{\partial t}+v^2, \quad (16)$$

since $$div\,\vec{v}=divgrad\varphi=\Delta\varphi \quad (17)$$

By replacement of Eqs. (16 - 17) in eq. (15), energy equation for potential flow of viscous compressible fluid can be written as:

$$\frac{\partial I_L}{\partial t}+\frac{1}{2}\cdot v^2+U+\Re-\frac{4}{3}\cdot\nu\cdot\Delta\varphi=C(t), \quad (18)$$

$$\int_L\frac{\partial\vec{v}}{\partial t}\cdot d\vec{s}+\frac{v^2}{2}+U+\Re-\frac{4}{3}\cdot\nu\cdot\Delta\varphi=C(t), \quad (19)$$

$$\int_L\vec{a}\cdot d\vec{s}+U+\Re-\frac{4}{3}\cdot\nu\cdot\Delta\varphi=C(t) \quad (20)$$

Equation (15) can be written:

$$\frac{DI_L}{Dt}-\frac{v^2}{2}+U+\Re-\frac{4}{3}\cdot\nu\cdot\Delta\varphi=C(t) \quad (21)$$

Energy equation for potential flow of ideal fluid (1) is presented the following form [1]:

$$\frac{\partial I_L}{\partial t}+\frac{1}{2}\cdot v^2+U+\Re=C(t) \quad (22)$$

$$\int_L\frac{\partial v}{\partial t}\cdot d\vec{s}+\frac{v^2}{2}+U+\Re=C(t), \quad (23)$$

$$\int_L\vec{a}\cdot d\vec{s}+U+\Re=C(t). \quad (24)$$

Next conclusion can be derived from Eq.(22): during non-stationary flow of the same fluid particles along the streamline or whirling line, sum of energy pressure, potential energy and local time derivative of fluid flow and kinetic energy value, is equal to temporal function C = C(t), being equal C for t = t. [1]

Next conclusion can be derived from Eq.(24): during non-stationary flow of the same fluid particles along streamline or whirling line, sum of kinetic energy, potential energy, energy pressure and local acceleration flow along L curve is equal to temporal function C = C(t), being equal C , for t = t_i. [1]

Next conclusion can be derived from Eq.(23) during non-stationary flow along streamline or whirling line, sum of kinetic energy, potential energy and acceleration flow along L curve is equal to temporal function C = C(t), being equal C for t = t_i. [1]

Analysis of the equations (18 –20) yields the next conclusion: In energy equation, for non-stationary flow of viscous compressible fluid(22-24), energies' member $\frac{4}{3}\cdot\nu\cdot\Delta\varphi$, is supposed to be subtracted, i.e. energy lost due to viscous fluid flow is supposed to be subtracted.

For non-compressible fluidthen, in this case energy lost due to viscosity is equal zero ($\frac{4}{3}\cdot\nu\cdot\Delta\varphi=0$). So, Eq. (18-22) comes to equation of ideal fluid flow. Since energy balance of viscous non-compressible fluid flow cannot be equal to energy balance of ideal fluid flow next conclusion can be made: each flow of viscous, non-compressible fluid is whirling flow. This conclusion has been alsomade in another paper [4].

III WHIRLING FLOW

Scalar multiplication of the equation (8) by element of whirling line, $d\vec{s}$, yields:

$$\left(\frac{D\vec{v}}{Dt},d\vec{s}\right)=-\left(grad\,(U+\Re),d\vec{s}\right)+\frac{4}{3}\cdot\nu\cdot\left(graddiv\,\vec{v},d\vec{s}\right)-$$
$$-\nu\cdot\left(rot\vec{v},d\vec{s}\right)$$

(25)

Whirling flow is presented as:

$$rot\,\vec{v}=2\cdot\vec{\omega}, \quad (26)$$

where $\vec{\omega}$ is angular velocity of fluids particles.

Member (4) of Eq. (25) can be written as:

$$\nu\cdot\left(rotrot\,\vec{v},d\,\vec{s}\right)=2\cdot\nu\cdot\left(rot\,\vec{\omega},d\,\vec{s}\right). \quad (27)$$

We try to find function $\Im(x,y,z)$, whose total differential is Eq.(27), *i.e.*,

$$d\Im(x,y,z)=2\cdot\nu\cdot\left(rot\,\vec{\omega},d\,\vec{s}\right), \text{ then,}$$

$$\left(rot\,\vec{\omega},d\,\vec{s}\right)=P(x,y,z)\cdot dx+Q(x,y,z)\cdot dy+R(x,y,z)\cdot dz$$

(28)

Since:

$$\left(rot\,\vec{\omega}, d\,\vec{s} \right) = \left(\frac{\partial \omega_z}{\partial y} - \frac{\partial \omega_y}{\partial z} \right) \cdot dx + \left(\frac{\partial \omega_x}{\partial z} - \frac{\partial \omega_z}{\partial x} \right) \cdot dy + \left(\frac{\partial \omega_x}{\partial y} - \frac{\partial \omega_y}{\partial x} \right) \cdot dz$$
(29)

where:

$P(x,y,z) = \left(rot\,\vec{\omega} \right)_x = \left(\frac{\partial \omega_z}{\partial y} - \frac{\partial \omega_x}{\partial z} \right)$, projection $rot\,\vec{\omega}$ to x - axis,

$Q(x,y,z) = \left(rot\,\vec{\omega} \right)_y = \left(\frac{\partial \omega_x}{\partial z} - \frac{\partial \omega_z}{\partial x} \right)$, projection $rot\,\vec{\omega}$ to y - axis,

$R(x,y,z) = \left(rot\,\vec{\omega} \right)_z = \left(\frac{\partial \omega_x}{\partial y} - \frac{\partial \omega_y}{\partial x} \right)$, projection $rot\,\vec{\omega}$ to z - axis.

Conditions for the function $\Im(x,y,z)$ to be total differential are:

$$\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} = 0, i.e., \frac{\partial}{\partial y}\left(\frac{\partial \omega_z}{\partial y} - \frac{\partial \omega_y}{\partial z} \right) - \frac{\partial}{\partial x}\left(\frac{\partial \omega_x}{\partial z} - \frac{\partial \omega_z}{\partial x} \right) = 0$$
(30)

$$\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} = 0, i.e., \frac{\partial}{\partial z}\left(\frac{\partial \omega_z}{\partial y} - \frac{\partial \omega_y}{\partial z} \right) - \frac{\partial}{\partial x}\left(\frac{\partial \omega_x}{\partial y} - \frac{\partial \omega_y}{\partial x} \right) = 0$$
(31)

$$\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y} = 0, i.e., \frac{\partial}{\partial z}\left(\frac{\partial \omega_x}{\partial z} - \frac{\partial \omega_z}{\partial x} \right) - \frac{\partial}{\partial y}\left(\frac{\partial \omega_x}{\partial y} - \frac{\partial \omega_y}{\partial x} \right) = 0$$
(32)

We introduced new vectors function:

$$\vec{\psi} = P(x,y,z) \cdot \vec{i} + Q(x,y,z) \cdot \vec{j} + R(x,y,z) \cdot \vec{k} = \quad (33)$$
$$= \psi_x \cdot \vec{i} + \psi_y \cdot \vec{j} + \psi_z \cdot \vec{k} = rot\,\vec{\omega}$$

whose rotor is:

$$rot\,\vec{\psi} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \cdot \vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right) \cdot \vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{yz} \right) \cdot \vec{k}$$
(34)

if the condition (29 - 31) is fulfilled, then:

$rot\,\vec{\psi} = 0$, (35)

where: $\vec{\psi} = rot\,\vec{\omega}$. (36)

Existence of potential vector function, or rotor's angular velocity is confirmed by Eq. (36).

$\psi = grad\Im(x,y,z)$, (37)

i.e.

$$\frac{\partial \Im}{\partial x} = P(x,y,z); \frac{\partial \Im}{\partial y} = Q(x,y,z); and\ \frac{\partial \Im}{\partial z} = R(x,y,z)$$
(38)

Equation (26) can be now written in the form:

$$\nu \cdot \left(rotrot\,\vec{v}, d\,\vec{s} \right) = 2 \cdot \nu \cdot \left(rot\,\vec{\omega}, d\,\vec{s} \right) = 2 \cdot \nu \cdot d\left(\Im((x,y,z)) \right)$$
(39)

If the condition (27) is fulfilled, then function $\Im(x,y,z)$ is potential function ofvectors' function $\psi(x,y,z)$, *i.e.* function $\Im(x,y,z)$ is potential function of rotors' angular velocity.

Replacement of Eqs. (10 - 13) and (38) in Eq. (25) can by written as:

$$\frac{DI_L}{Dt} - \frac{v^2}{2} + \Re + U - \frac{4}{3} \cdot \nu \cdot \Delta\varphi - 2 \cdot \nu \cdot \Im((x,y,z)) = C_{(t)}$$
(40)

$$\frac{\partial I_L}{\partial t} + \frac{1}{2} \cdot v^2 + U + \Re - \frac{4}{3} \cdot \nu \cdot \Delta\varphi - 2 \cdot \nu \cdot \Im((x,y,z)) = C(t)$$
,(41)

$$\int_L \frac{\partial \vec{v}}{\partial t} \cdot d\vec{s} + \frac{v^2}{2} + U + \Re - \frac{4}{3} \cdot \nu \cdot \Delta\varphi - 2 \cdot \nu \cdot \Im((x,y,z)) = C(t)$$
,(42)

$$\int_L \vec{a} \cdot d\vec{s} + U + \Re - \frac{4}{3} \cdot \nu \cdot \Delta\varphi - 2 \cdot \nu \cdot \Im((x,y,z)) = C(t)$$
(43)

For nonstationary whirling flow of viscous compressible fluid itcan be concluded (40-43):
sum of energy pressure, potential energy and local time derivative of fluid flow and kinetic energy value, minus energies' member $\frac{4}{3} \cdot \nu \cdot \Delta\varphi$, minus energies' member $2 \cdot \nu \cdot \Im((x,y,z))$ due to potential function of rotors' angular velocity. is equal to temporal function C = C(t), being equal C for t = t.

Sumof kinetic energy, potential energy, energy pressure and local acceleration flow along L curve minusenergies' member $\frac{4}{3} \cdot \nu \cdot \Delta\varphi$, energy lost due to viscous fluid flow is supposed to be subtracted, minus energies' member $2 \cdot \nu \cdot \Im((x,y,z))$ due to potential function of rotors' angular velocity is equal to temporal function C = C(t), being equal C , for t = t_i.

Sum of kinetic energy, potential energy and acceleration flow along L curve minus energies' member $\frac{4}{3} \cdot \nu \cdot \Delta\varphi$, energy lost due to viscous fluid flow is supposed to be subtracted, minus energies' member $2\cdot\nu\cdot\Im((x,y,z))$ due to potential function of rotors' angular velocity is equal to temporal function C = C(t), being equal C for t = t $_i$.

IV CONCLUSION

This paper presents another equation for dynamics of viscous fluid. For non-stationary potential viscous fluid flow ,analysis of the equations (18 – 20) yields the next conclusion: In energy equation, for non-stationary flow of viscous compressible fluid (22-24), energies' member $\frac{4}{3}\cdot\nu\cdot\Delta\varphi$, is supposed to be subtracted, i.e. energy lost due to viscous fluid flow is supposed to be subtracted.
For non-compressible fluid then, in this case energy lost due to viscosity is equal zero ($\frac{4}{3}\cdot\nu\cdot\Delta\varphi=0$). So, Eq. (18-22) comes to equation of ideal fluid flow. Since energy balance of viscous noncompressible fluid flow cannot be equal to energy balance of ideal fluid flow next conclusion can be made: each flow of viscous, non-compressible fluid is whirling flow. This conclusion has been also made in another paper [4].

For nonstationary whirling flow of viscous compressible fluid it can be concluded (40-43):
sum of energy pressure, potential energy and local time derivative of fluid flow and kinetic energy value, minus energies' member $\frac{4}{3}\cdot\nu\cdot\Delta\varphi$, minus energies' member $2\cdot\nu\cdot\Im((x,y,z))$ due to potential function of rotors' angular velocity. is equal to temporal function C = C(t), being equal C for t = t.

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