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ANOTHER INTEGEAL OF EULER EQUATION FOR FLUID DYNAMICS

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Abstract- ***This paper presents another integration, as well as another solution of Euler equation expressing dynamics of ideal heat unconductive fluid in the case of circular nonstationary fluid flow.***

Keywords - Euler Equation, fluid, flow.

I. INTRODUCTION

Euler equation expressing dynamics of ideal fluid has the following form 1 - 6 :

$$\frac{D\vec{v}}{Dt} = \vec{f} - \frac{1}{\rho}\cdot grad\vec{p} \qquad (1)$$

where: $\frac{D\vec{v}}{Dt}$ - first material time derivative of velocity,
f - intensity of the mass forces,
p - pressure,
ρ - fluid density.

Pressure's function, $\Re_{(p)} = \int_{p_o}^{p} \frac{dp}{\rho_{(p)}}$, could be introducted for barotropic fluid flow, implying:

$$\frac{1}{\rho}\cdot grad\vec{p} = grad\vec{\Re}\,. \qquad (2)$$

If the conservative mass forces field has the potential U, then:

$$\vec{f} = -\,grad\vec{U}\,.$$

The flow of the same fluid particles along the streamline or whirling line $L = L(x,y,z,t)$ is being observed in velocity vectors field, $\vec{v} = \vec{v}(x,y,z,t)$.

II. INTEGRATION OF THE EULER EQUATION FOR FLUID DINAMICS

Scalar multiplication of the equation (1) by element of stream or whriling line, $\vec{\mathbf{ds}}$, yields:

$$\left[\frac{D\vec{v}}{Dt}, d\vec{s}\right] = \left(\vec{f}, d\vec{s}\right) - \left(\frac{1}{\rho}\cdot grad\vec{p}, d\vec{s}\right), \qquad (4)$$

$$\left[\frac{D\vec{v}}{Dt}, d\vec{s}\right] = -\left[grad\left(U+\Re\right), d\vec{s}\right] = -d\left(U+\Re\right). \qquad (5)$$

Generally the scalar product is expressed by equation:

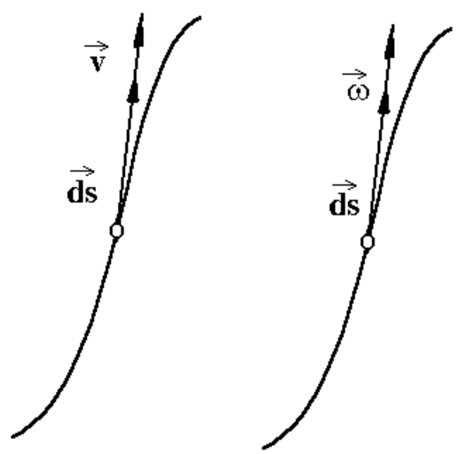

Fig.1. and 2. Streamline and whirlingline

$$\left[grad\left(U+\Re\right), d\vec{s}\right] = d\left(U+\Re\right) - \frac{\partial\left(U+\Re\right)}{\partial t} = d\psi - \frac{\partial\psi}{\partial t}$$

where ψ = ψ(x,y,z,t) - total potential energy, i.e. potential of acceleration.

Fluid flow along the streamline or whirling line is defined as [1]

$$I_L = \int_L \vec{v}\cdot d\vec{s}\,. \qquad (6)$$

Material derivation of Eq.(6) is:

$$\frac{DI_L}{Dt} = \int_L \frac{D\vec{v}}{Dt}\cdot d\vec{s} + \int_L \vec{v}\cdot\frac{D(d\vec{s})}{Dt}. \qquad (7)$$

Total diferential of Eq.(7) is:

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$$d\left(\frac{DI_L}{Dt}\right)=\left[\frac{D\vec{v}}{Dt},d\vec{s}\right]+\left(\vec{v},D\vec{v}\right)=\left[\frac{D\vec{v}}{Dt},d\vec{s}\right]+d\left(\frac{v^2}{2}\right)\quad .\qquad (8)$$

Eq.(7) yields:

$$\left[\frac{D\vec{v}}{Dt},d\vec{s}\right]=d\left(\frac{DI_L}{Dt}\right)-d\left(\frac{v^2}{2}\right).\qquad (9)$$

Replacement of Eq.(9) into Eq.(8) yields:

$$d\left(\frac{DI_L}{Dt}\right)-d\left(\frac{v^2}{2}\right)+d\left(U+\Re\right)=0,\qquad (10)$$

i.e.

$$\frac{DI_L}{Dt}-\frac{v^2}{2}+\Re+U=C_{(t)}.\qquad (11)$$

Temporal function C(t) in a determined moment has a concrete value.

Transformation of Euler equation also yields Eq.(8):

$$\frac{\partial\vec{v}}{\partial t}+grad\left(\frac{v^2}{2}+U+\Re\right)=\vec{v}\times rot\vec{v}\ .\qquad (12)$$

Scalar multiplication of Eq.(12) by $\mathbf{d\vec{s}}$ yields:

$$\frac{\partial\vec{v}}{\partial t}\cdot d\vec{s}+grad\left(\frac{v^2}{2}+U+\Re\right)\cdot d\vec{s}=\left(\vec{v}\times rot\vec{v}\right)\cdot d\vec{s}\ .\qquad (13)$$

Since vector $\vec{v}\times rot\vec{v}$ is colinear with $\mathbf{d\vec{s}}$ the right side of Eq.(13) is equal zero, i.e.:

$$\frac{\partial\vec{v}}{\partial t}\cdot d\vec{s}+d\left(\frac{v^2}{2}+U+\Re\right)=0,$$

$$d\left[\int_L\frac{\partial\vec{v}}{\partial t}\cdot d\vec{s}\right]+d\left(\frac{v^2}{2}+U+\Re\right)=0,\ .....(14)$$

$$\int_L\frac{\partial\vec{v}}{\partial t}\cdot d\vec{s}+\frac{v^2}{2}+U+\Re=C(t).$$

Material time derivative of velocitz vector is:

$$\frac{D\vec{v}}{Dt}=\frac{\partial\vec{v}}{\partial t}+\frac{1}{2}\cdot gradv^2-\vec{v}\times rot\vec{v}\ .\qquad (15)$$

Replacement of Eq.(15) into Eq.(9) yields:

$$\left[\frac{D\vec{v}}{Dt},d\vec{s}\right]=\frac{\partial\vec{v}}{\partial t}\cdot d\vec{s}+\frac{1}{2}\cdot\left(gradv\right)^2-\left(\vec{v}\times rot\vec{v}\right)\cdot d\vec{s}=$$

$$=\int_L\frac{\partial\vec{v}}{\partial t}\cdot d\vec{s}+d\left(\frac{v^2}{2}\right)=d\left(\frac{DI_L}{Dt}\right)-d\left(\frac{v^2}{2}\right)\quad .\qquad (16)$$

Eq.(16) yields:

$$\int_L\frac{\partial\vec{v}}{\partial t}\cdot d\vec{s}=\frac{DI_L}{Dt}-v^2\ .\qquad (17)$$

Material derivation of fluid flow $I_L = I_L(x,y,z,t)$ can be expressed as:

$$\frac{DI_L}{Dt}=\frac{\partial I_L}{\partial t}+\vec{v}\cdot gradI\ .\qquad (18)$$

According to Calvin theoreme:

$$\frac{DI_L}{Dt}=\int_L\vec{a}\cdot d\vec{s}+\frac{v^2}{2},\qquad (19)$$

where $\vec{\mathbf{a}}$ is fluid particles acceleration.

Replacement of Eqs.(17)-(19) into Eq.(11) yields another integral of Euler equation for fluid dynamics, which may be expressed in following forms:

$$\frac{DI_L}{Dt}-\frac{v^2}{2}+U+\Re=C(t),\qquad (20)$$

$$\frac{\partial I_L}{\partial t}+\vec{v}\cdot gradI-\frac{v^2}{2}+U+\Re=C(t),\qquad (21)$$

$$\int_L\frac{\partial v}{\partial t}\cdot d\vec{s}+\frac{v^2}{2}+U+\Re=C(t),\qquad (22)$$

$$\int_L\vec{a}\cdot d\vec{s}+U+\Re=C(t).\qquad (23)$$

If potential function is also temporal dependent then energy is being subtracted at the left side of Eqs.(20-23) $\int_L\frac{\partial\psi}{\partial t}\cdot dt$.

Next conclusion can be derived from Eq.(20): during non-stationary flow of the same fluid particles along the streamline or whirling line, sum of energy pressure, potential energy and local time derivative of fluid flow and kinetic energy value, is equal to temporal function C = C(t), being equal C for t = t.

Next conclusion can be derived from Eq.(22) during non-stationary flow of the same fluid particles along streamline or whirling line, sum of kinetic energy, potential energy, energy pressure and local acceleration flow along L curve is equal to temporal function C = C(t), being equal C , for $t = t_i$.

Next conclusion can be derived from Eq.(23) during non-stationary flow along streamline or whirling line, sum of kinetic energy, potential energy and acceleration flow along L curve is equal to temporal function C = C(t), being equal C for $t = t_i$.

III. ANALYSIS OF INTEGRAL OF EULER EQUATION FOR FLUID DYNAMICS

For stationary flow

During stationary flow ($\frac{\partial}{\partial t}=0$) Eq.(21 and 22) as well as other integrals (20, 21 and 22) comes to Bernulli integral of Euler equation (24), where C constant for $C(t=t_i)$ is equal

$$\frac{v^2}{2}+U+\Re=C(t=t_i)=C_{i.}\ .\qquad (24)$$

Equation (21) then comes to:

$$\vec{v}\cdot gradI-\frac{v^2}{2}+U+\Re=C_i\ .\qquad (25)$$

According to Eqs.(24-25) one may conclude:

$$\vec{v} \cdot gradI - \frac{v^2}{2}, i.e... \vec{v} = gradI \,, \qquad (26)$$

and that is possible when vector $\vec{v}$ and gradv are colinear only (Fig.3)

During stationary flow streamline, i.e. whirling lines with trajectories of fluid particles overlap each other, i.e. the velocity is tangent on the fluid particles trajectories. Vectors v and gradI are perpendicular on the plane (α_i) being perpendicular on the streamline (Fig.3). On the base of Eq.(26) one may conclude that fluid flow is potential function of velocity vector during stationary flow. As a matter of fact the plane of cross section of the flow tube is equipotential plane of fluid flow at the moment of observation.

For potential flow

Starting from

$$rot\vec{v} = 0 \qquad (27)$$

for non whirling, potential flow, which comes to:

$$v_x\, dx + v_y\, dy + v_z\, dz = d\varphi, \qquad (28)$$

i.e. a function $\varphi = \varphi(x,y,z,t)$ does exist, representing potential of the velocity:

$$\vec{v} = gead\varphi \,. \qquad (29)$$

Due to independence of gradient calculation and time diferentiation, the sequence of these operations can be changed, so:

$$\frac{\partial \vec{v}}{\partial t} \cdot d\vec{s} = \frac{\partial}{\partial t} grad\varphi \cdot d\vec{s} = \left(grad \frac{\partial \varphi}{\partial t} \right) \cdot d\vec{s} = d\left(\frac{\partial \varphi}{\partial t} \right). \qquad (30)$$

Cauchey-Lagrange integral can also reached on the base of Eq.(11) in the following manner:

$$\frac{dI_L}{dt} = \frac{d}{dt}\left(\int_L grad\varphi \cdot d\vec{s} \right) = \frac{d}{dt}\int_L d\varphi = \frac{d\varphi}{dt} = \frac{\partial \varphi}{\partial t} + \left(grad\varphi \right)^2 . \qquad (31)$$

Replacement of Eqs.(31) and (27) into (11) yields Cauchey-Lagrange integral of Euler equation:

$$\frac{\partial \varphi}{\partial t} + \frac{1}{2} \cdot \left(grad\varphi \right)^2 + U + \Re = C(t) \,. \qquad (32)$$

On the base of Eq.(29) the Eq.(11) may be written as:

$$\frac{\partial I_L}{\partial t} + \frac{1}{2} \cdot \left(gradI_L \right)^2 + U + \Re = C(t) \,. \qquad (33)$$

During potential flow along L streamline, potential function of fluid particles velocity

$\varphi = \varphi(x,y,z,t)$ is equal to fluid flow along L curve, i.e. $I_L = \varphi(x,y,z,t)$.

IV. CONCLUSIONS

Another approach of integration of Euler equation for fluid dynamics has been presented in the paper through introduction of fluid flow. The equations are applicable along streamline or whirling lines, whereas in the case of stationary flow are applicable for flow tube to.

The paper also presents a manner of transformation of the derived equation into well known integrals of Euler equation, like those of Bernulli and Cauchey-Lagrange.

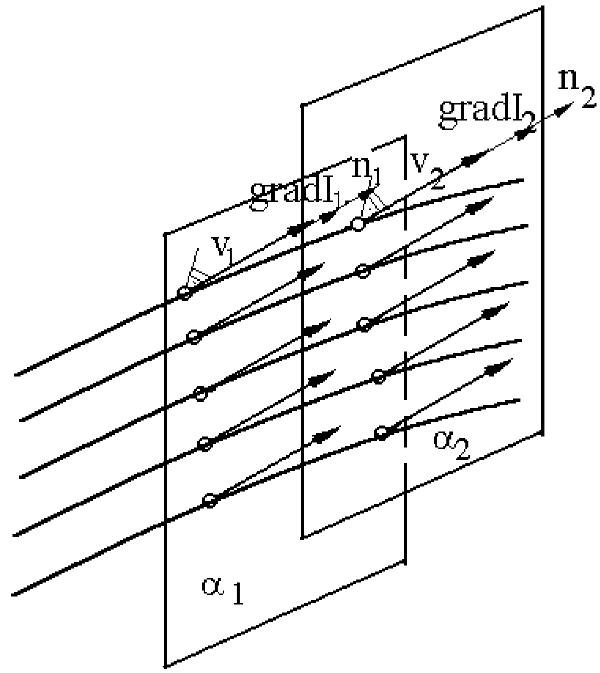

Fig. 3. Flow in the two adjacent cross-section

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