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PROPOSAL OF MINIMUM AND MAXIMUM VALUES FOR THE POISSON'S RATIO OF ASPHALTIC CONCRETE

[1]Alfred François Messi, [2]Florent Sikali,[3]Bemsibom Chia Toh
[1]Ecole Nationale Supérieure Polytechnique, Yaoundé, Cameroon
[2,3]Soil And Water Investigation S.A, Yaoundé, Cameroon

Abstract*-The Poisson's ratio for asphaltic concretes is a difficult and expensive parameter to measure. It is for this reason that some laboratories and design companies propose standard values to be integrated in the dimensioning of pavement subgrades. This study approaches the subject differently by determining characteristic maximum and minimum values, guaranteed at 90% and 95%. These values were obtained after a statistical exploitation of experimental results obtained from a population of 90 samples. Gauss's law was used after verification of its applicability by appropriate tests. An analysis enabled the comparing of these values to those recommended by some design companies.

***Keywords*–**Asphaltic concrete;Poisson's ratio; dimensioning of pavement subgrades; characteristic values; Gauss's law

I. INTRODUCTION

The first methods for the dimensioning of pavement subgrades were empirical in nature. The advent of heavy vehicles which degraded pavement subgrades after one passage made it necessary to adopt more rational methods of subgrade dimensioning. The general method consists of identifying the different parameters which may have a certain influence, model the behaviour of the pavement subgrade and then propose a pavement structure. The difficulty resides in the fact that the parameters are very many, and some of them difficult and expensive to measure, which often leads certain designers to adopt standard values; the consequence of this method being the risk of over – dimensioning of under – dimensioning. The Poisson's ratio is one of such parameters.

To determine the Poisson's ratio of asphaltic concretes, a probabilistic approach was chosen since it is becoming more and more common, given the generalisation of limit – state building codes. Instead of adopting a nominal standard value for this parameter, the method proposed is to determine the minimum and maximum values guaranteed at certain percentages. For this, the Poisson's ratio was measured from three asphaltic concrete mixes on a sampling of 30 elements per mixture as recommended by quantitative statistics manuals. The data obtained was then treated, enabling the computation of the minimum and maximum values of the Poisson's ratio, guaranteed at 90% and 95%. This work was carried out both at the National Advanced School of Engineering, Yaoundé and the Laboratory "Soil And Water Investigation S.A" of Yaoundé, Cameroon.

II. HOOKE'S LAW AND THE RELATIONSHIP BETWEEN ELASTIC CONSTANTS

In a three – dimensional state of stress, the strains in a point of a linearly elastic body are calculated using the following formulae (1):

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \vartheta\left(\sigma_y + \sigma_z\right)\right] \quad (1)$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \vartheta\left(\sigma_z + \sigma_x\right)\right] \quad (2)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \vartheta\left(\sigma_x + \sigma_y\right)\right] \quad (3)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G};\ \gamma_{xz} = \frac{\tau_{xz}}{G};\ \gamma_{yz} = \frac{\tau_{yz}}{G} \quad (4)$$

where:

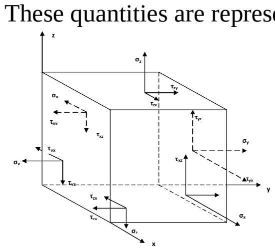
σ_x, σ_y, σ_z : normal stress at the faces perpendicular to the x, y and z axes.

τ_{xy}, τ_{xz}, τ_{yz} : tangential stresses to thex_y, x_z and$y_z$planes

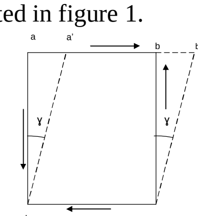
ε_x, ε_y, ε_z : strains in the x, y and z directions.

γ_{xy}, γ_{xz}, γ_{yz} : sliding angles of thex_y, x_z and$y_z$planes

These quantities are represented in figure 1.



(1.a)



(1.b)

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Figure 1: Representation of stresses and strains. (1.a) Representation of stresses (1.b) Representation of strains

Hooke's law involves the following constants in the formulae linking stresses to strains:

E : Longitudinal modulus of elasticity
ν : Poisson's ratio
G : Transversal modulus of elasticity

These constants are equally called proportionality coefficients and are linked by the formula:

$$G = \frac{E}{2(1+\vartheta)} \quad (5)$$

Under axial loading, there is a longitudinal strain ε_l and a transversal strain ε_T. We therefore have:

$$\nu = \frac{\varepsilon_T}{\varepsilon_l} \quad (6)$$

We also defined E and ν through Lamé's constants λ and μ as follows:

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu} \quad (7)$$

$$\nu = \frac{\lambda}{2(\lambda + \mu)} \quad (8)$$

We can also use the relationship

$$\nu = \frac{1}{2} - \frac{E}{6K} \quad (9)$$

where K is the compressibility modulus.

There are also relationships between the speed of propagation of waves and elastic constants. The propagation equation is obtained by combining the general equation and the elastic behavior equation. If we consider a plane compression wave (P wave) being propagated in the O_x direction, and neglecting volume forces, dynamic equilibrium is given by:

$$\frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial^2 u_x}{\partial x^2} = \rho \frac{\partial^2 u_x}{\partial t^2} \quad (10)$$

This equation has one displacement solution in the form

$$u_x(x,t) = e^{i(kx - \overline{\omega} t)} \quad (11)$$

, which enables the expression of the speed V_P of the P wave as follows:

$$V_p = \sqrt{\frac{\lambda + 2G}{\rho}} \quad (12)$$

In the same way, the propagation of a longitudinal S wave leads to a shear wave speed V_S as follows:

$$V_s = \sqrt{\frac{G}{\rho}} \quad (13)$$

We cab deduce ν from the formula:

$$\nu = \frac{1}{2}\left[1 - \frac{1}{\left({V_p}/{V_s}\right)^2 - 1}\right] \quad (14)$$

III.METHODS FOR DETERMINING THE POISSON'S RATIO

The methods used for the determination of ν are based on the relationships in the preceding paragraph. Some authors suggest that it be determined from the measurement of the three quantities E, G and K. the wave propagation method uses the measurement of ultrasonic P and S waves in the laboratory and the calculation of ν from these formulae.

The ratio can be obtained either by triaxial compression, or by simple compression. Given its cost efficiency, and that the value of this constant for lateritic gravel had been found using the latter method (1), This study was carried out using the simple compression method.

IV.EXPERIMENTATION

The first operation consisted in the mixture of three asphaltic concrete mixes with the following compositions:

Table 1: **constituents of asphaltic concretes**

N°	Proportions of aggregates		Binder content (Pure asphalt of class 60/70)
	Sand 0/5	Gravel 5/10	
1	50%	50%	5,5%
2	50%	50%	6,0%
3	50%	50%	6,5%

Cylindrical test samples of height and diameter 80mm were made using the standard protocol (Duriez moulds) with a pressure of 60kN maintained for 5 minutes. 30 test samples were made per mixture.

The samples were then submitted to simple compression tests with vertical increments of 0,1mm. Axial and lateral strains were deduced from the readings of the three precision gauges mounted at 120° about the samples on magnetic supports.

Two types of tests were carried out: tests to characterize the base materials and tests on the asphaltic concrete samples.

Sand: Sieve analysis, absolute density, apparent density, sand equivalent

Gravel: Sieve analysis, absolute density, apparent density, flatness coefficient, apparent cleanness, Los Angeles, humid Micro – Deval test.

Asphalt: penetrability, softening point by the "marble and ring" method, relative density at 25°C.

Tests on samples: Simple compression tests using an electronic press with vertical increments of 0,1mm.

V. PRESENTATION OF EXPERIMENTAL RESULTS

The results from the characterization tests of the base materials are presented in tables 2 and 3 and in figures 2, 3 and 4.

Table 2: Characteristics of aggregates used for the formulation of asphaltic concrete mixes.

CHARACTERISTICS STUDIED	VALUES: MINTP 30.003 – R Specifications	VALUES: Gravel of class 5/10	VALUES: Sand of class 0/5
Sieve Analysis % Refuse at 1,56D % Refuse at D % Sieve at d % Sieve at 0,63d % Refuse at 0,5(d+D)	0 < 15 < 15 < 3 33 to 66	0 6,1 16,2 4 56,9	0 0,16 9,0 - 68
Unit weight (t/m^3)	-	2,73	2,850
Apparent density (t/m^3)	-	1,62	1,801
Flatness coefficient (%)	< 20	11,5	-
Sand equivalent	> 60	-	Piston: 68 Visual: 75
Superficialcleanness (%)	< 1	2,5	-
Los Angeles (6/10)	< 35	34,5	-
Humid Micro Deval (6/10)	< 20	22,9	-

Table 3: Characteristics of the asphaltic binder used for the asphaltic concrete mixture

TESTS	Results	MINTP 30.003 – R Specifications
Penetrability at 25°C, 100g, 5s, 1/10 mm	70	60 - 70
Softening point (ASTM marble and ring method in °C)	50,5	43 - 56
Relative density at 25°C T/m^3	1,005	1,00 – 1,10

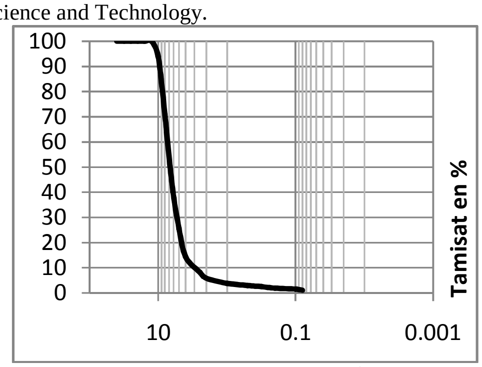

Figure 2: Particle size curve of Gravel

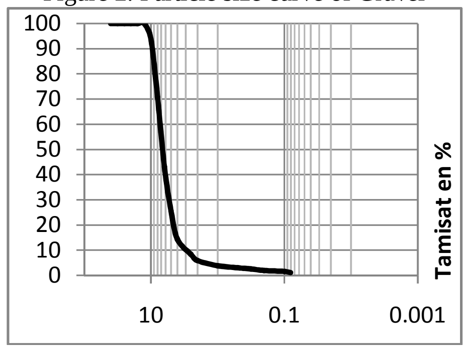

Figure 3: Particle size distribution curve of sand

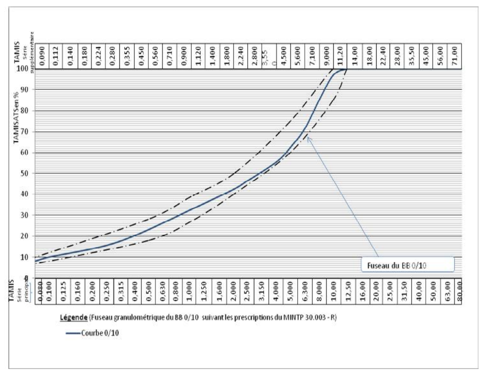

Figure 4: In continuous lines, particle size distribution curve of sand-gravel mixture. We see that it fits well in the range recommended by the MINTP 30-003R for the mixture of asphaltic concretes(1).

The results of the tests on asphaltic concretes are presented as stress – strain curves. The stresses were drawn as a function of axial strains (example figure 5) and radial strain (example figure 6).

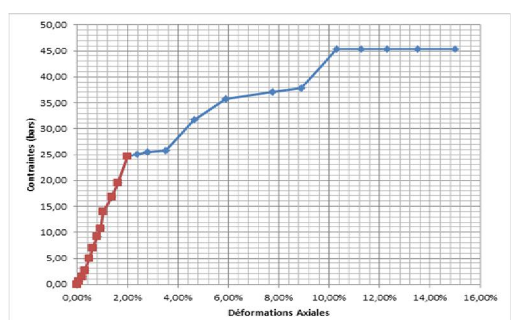

Figure 5: Curve of stress against radial strains for an asphaltic concrete test sample.

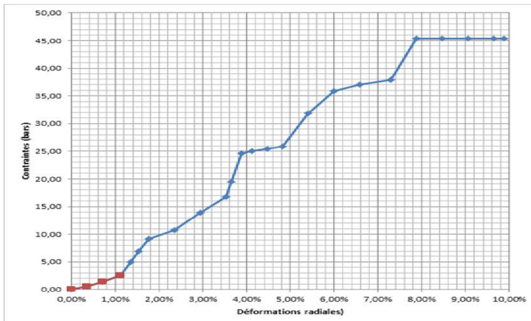

Figure 6: Curve of stress against radial strains for an asphaltic concrete test sample.

This coefficient was thus deduced from regression lines of the curves drawn from experimental values of axial strains drawn against those of radial strains (example figure 7). The gradient of this regression lines corresponds to ν. For example, figure 7 gives a value of ν=0,4264.

It is worth noting that these results corroborate with the literature (2), as the elastic region for asphaltic concretes is usually found for strains less than 3%.

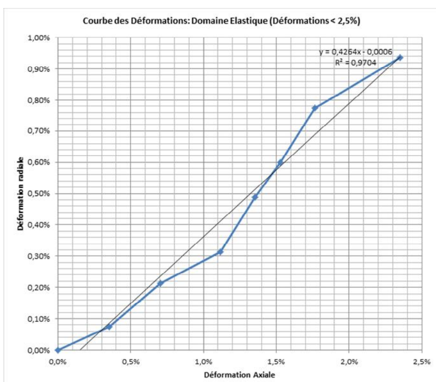

Figure 7: Graph of radial strain against axial strains for an asphaltic concrete sample. Here, ν=0,4264.

VI. EXPLOITATION OF RESULTS

The original idea was to constitute three populations with 30 samples each. Each population represented an asphaltic concrete mix. The results obtained experimentally being very similar, these three populations were grouped together into one population of 90 experimental samples.

The statistical distribution of experimental results obtained is presented as a table (Table4) and as a frequency histogram (figure 8).

Table 4: Distribution per class of experimental values

Class of ν	Center	Frequency
0,32 - 0,34	0,33	3
0,34 - 0,36	0,35	4
0,36 - 0,38	0,37	15
0,38 - 0,40	0,39	31
0,40 - 0,42	0,41	27
0,42 - 0,44	0,43	9
0,44 - 0,46	0,45	1

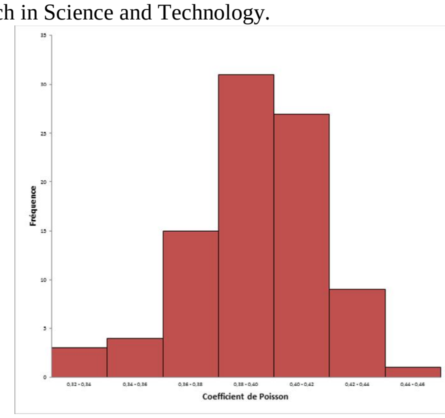

Figure 8: Histogram of experimental values of ν

The normal law or Gauss's law was chosen because it is one of the most common distribution laws in practice. According to statistics, it is actually the law applicable to a statistical variable which is the resultant of a large number of independent causes whose effects add up and none is preponderant over the other. However, the validity of this adjustment was verified by appropriate tests.

On a theoretical plane, the normal random variable X is a continuous variable which can take any value between]-∞ ;+∞[, with a probability density function f(x)

$$f(x) = \frac{1}{\sqrt{2\Pi}\sigma} \exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right] \quad (15)$$

Where m: average and σ : standard deviation

The distribution function which represents the probability for the random variable X to take a value less than x has as expression:

$$F(x) = P(X < x) = \frac{1}{\sqrt{2\Pi}\sigma}\int_{-\infty}^{x} exp\left[-\frac{1}{2}\left(\frac{x-m}{\sigma}\right)^2\right]dx \quad (16)$$

Conversely, the probability for X to be greater than x is equal to 1-F(x).

The philosophy of limit state design introduced the notion of characteristic values which are values guaranteed by a probability of surpassing, or of not being reached. Let X_{max} be the superior characteristic value and X_{min} the inferior characteristic value and y the probability threshold. We have:

$$P(X > X_{min}) = y \quad (17)$$

$$P(X < X_{max}) = y \quad (18)$$

We show that with the normal law,

$$X_{max} = m + ks \quad (19)$$

$$X_{min} = m - ks \quad (20)$$

k: factor depending on the probability threshold.

We consider X_{max} if the parameter intervenes in an unfavorable way with respect to security and X_{min} in the contrary case.

The data were treated with the help of the SPSS software package which enabled the adjustment of the histogram and the normal curve, the computation of m and σ and the validity tests for the choice of the normal law.

The SPSS package (Statistical Package for Social Sciences) is a statistical analysis software package. In addition to statistical analysis, data management and data documentation are two other characteristics of this software package.

With this package, the following histogram was constructed:

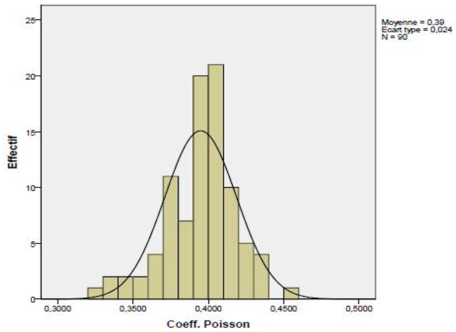

Figure 9: Histogram of Poisson’s ratio

Kolmogorv – Smirnov’s test is a hypothesis test to determine if a sample follows a known law. After carrying out the Kolmogorov – Smirnov test on this population, the following results were obtained:

Table 5: Results of the Kolmogorov – Smirnov test on the population

N		Poisson’s ratio
		90
Normal parameters	Average	0,394873
	Standard deviation	0,0238146
Most extremedifferences	Absolute	0,107
	Positive	0,053
	Negative	-0,107
Kolmogorov-Smirnov’s Z		1,020
Bilateralasymptoticsignificance		0,250

According to the results presented in the above table, the results obtained for the Poisson’s ratio show strongly normal tendencies (bilateral asymptotic significance of 25%).

With the help of the SPSS software, other tests were carried out on this data to verify the closeness to the normal law. Hereafter are presented the diagrams of theoretical cumulative probabilities as a function of real cumulative probabilities (P-P diagram) for the Poisson’s ratio (Figure 10). We can retain as mean value m=0,39 and standard deviation σ=0,024.

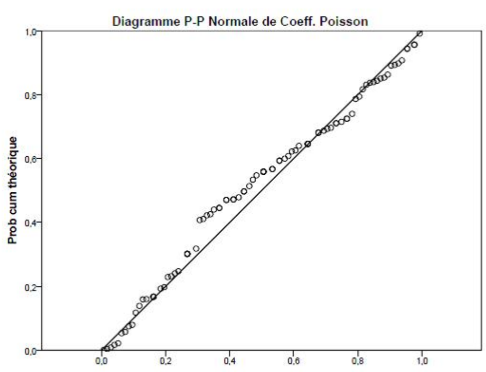

Figure 10: Normal P – P diagram for the Poisson’s ratio.

The thresholds of 90% and 95% were retained for the computation of the characteristic values presented in the following table:

Table 6: characteristic values

Value of ν	Probabilitythreshold	
	90% (k=1,28)	95% (k=1,24)
ν_{max}	0,42	0,43
ν_{min}	0,36	0,35

VII. DATA ANALYSIS AND CONCLUSION

After comparing the results obtained to those used by some design companies and laboratories, we retain that the default value recommended by the library of the Alizé III software package is 0,25. This value was equally adopted in the 1980s by the National Civil Engineering Laboratory of Cameroon, which today recommends ν=0,45. We can remark that most laboratories in Cameroon use either ν=0,35 or ν=0,45, which corresponds approximately to the minimal and maximum characteristic values guaranteed at 95%. The problem posed is to know in which case either of these values offers more security.

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