

# OPTIMIZATION OF CHARACTERISTIC PARAMETERS IN COMBUSTION PROCESS OF MEDICAL WASTE USING ANALYTICAL GEOMETRY METHOD

<sup>1</sup>Vladan Mičić, <sup>1</sup>Branko Pejović, <sup>2</sup>Sabina Begić

<sup>1</sup>University of East Sarajevo, Faculty of Technology, Zvornik, Bosnia and Herzegovina

<sup>2</sup>University of Tuzla, Faculty of Technology, Tuzla, Bosnia and Herzegovina

*Abstract- In this paper, for a case of of incineration of two typical medical wastes in the furnace, maximum amounts of both wastes that can be burned at the given conditions were determined, that is, a larger number of limitations that are characteristic for the observed problem. At this, the analytical geometry method was applied, where were included two variables that represent the mass flow of mentioned wastes. A mathematical model with six limitations in linear form is formed on the basis of thermodynamic relations of combustion process of waste. At the end of the paper, a presentation of method at one numerical example from practice was given, where besides required quantities of waste, working area was determined. The solution of problem is obtained using lower heating value of waste. Obtained graphical solution was verified by analytical method.*

**Keywords** - Ecology; Waste incineration; Lower heating value; Graphical methods; Analytical geometry

## I. INTRODUCTION

Problems, in which we find the maximum or minimum of some function of certain number of variables, where variables are subject to certain limitations, are called optimization problems. Solving these problems is current in many areas of technique, industry, economy and so on [1, 2]. Solving them means finding optimal allocation of limited resources compared to given reality. Here, the term resource means labor, instruments of labor, space, time, etc. So, it is about solving problems where certain labor, particular machine or apparatus, or similar, is available. These resources may be used in one or more activities. But all this must be done under certain conditions. In relation to all these limitations, there are many allowable (possible) solutions. Among all these allowable solutions (utilization of resources) it is possible to find one or more solutions that maximize or minimize income, capacity utilization, price, costs, reliability etc. [2, 3]. For each activity, it must be identified its objective, or criterion by which the quality of solution is evaluated. When the verbal description of the goal, or criteria is expressed by mathematical relations, it usually gives a certain function  $F(x)$ , where  $X=(x_1, x_2, \dots, x_n)$ ,  $n$  - dimensional vector.

A set of conditions or limitations under which the best utilization of resources is found, is represented in the form of certain number of equations and inequations in which there are the same unknown components of  $n$  - dimensional vector  $X_i (x_1, x_2, \dots, x_n)$  as well as in the function of criterion. In this way area  $D$  is determined, from where is chosen  $X$  that provides that the function  $F(x)$  gets extreme values. All permissible values of  $X$  in the area of  $D$  are called admissible

solutions, and  $X$  that allows  $F(x)$  to reach extreme values is called an optimal solution.

Translating of verbal description of problem to mathematical symbols is called forming a mathematical model.

Linear programming is a method of operations research that deals with finding optimal solution to problems where relationships between variables are linear [4, 5].

In mathematical model of linear programming problems, limitations are generally written in the form:

$$\begin{aligned} &> \\ a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &< a_{10} \\ &> \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &< a_{20} \\ &> \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &< a_{m0} \\ &x_1, x_2, \dots, x_n \geq 0 \end{aligned} \tag{1}$$

With these limitations, criterion function is usually given in the form of

$$F(x) = c_1x_1 + c_2x_2 + \dots + c_nx_n \tag{2}$$

Simpler linear programming problems are when the criterion function  $F(x)$  is not analysed and extreme values are determined with respect to all functions of limitations. This problem is solved using analytical geometry. In this case, functions of limitations are given in the form of linear inequations that are presented graphically. For the graphical presentation, first the appropriate equations are constructed on the basis of corresponding inequations, and thereafter the area  $D$  is determined [5, 6].

On Fig. 1 in the coordinate system  $XOY$  typical example is given for the case of five functions of limitations  $L$  based

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on which the area D is formed. The extreme values of the variables X and Y (their maximum or minimum values) are determined simply from the graphic for each point. Obviously, here is  $Y_B = Y_C$ , while  $X_F = X_E$ . Also, based on graphical, it is possible to determine an analytical solution.

For solving these problems with two variables, graphical method can be effectively applied. Limitations in the form of equations are presented by straight lines, that is, line segments of lines in the first quadrant of the coordinate system, while limitations in the form of in-equations are presented by half planes that are their segments. The importance of graphical methods is that in two-dimensional space the properties of solving the problem are easily observed, and by mathematical proving it can be established that it applies to the area of any dimensions [5, 6].

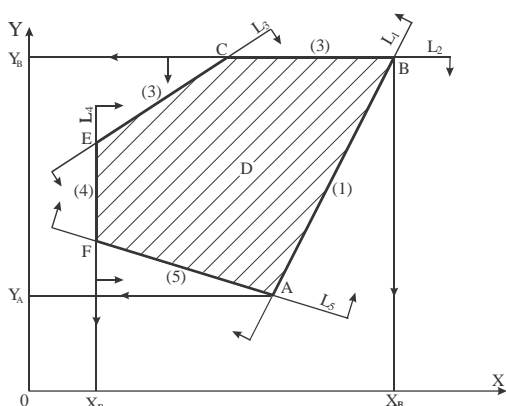


Fig. 1 Solving of simpler problems by graphical method

Further, there will be presented an application of above described analytical method to one practical example in the field of ecology and which relates to the incineration of medical wastes.

**II. PROBLEM SETTING**

In the field of ecology, that is environmental protection, a common case in practice is the incineration of different medical wastes. Two medical wastes that are stored in separate tanks, where there is a possibility of their burning, will be observed. At this, the basic composition and lower calorific value in kJ/kg for both wastes is known. The minimum quantity for each waste to be burned in kg/s is also known, as well as the limitation of capacity of the pump, by which wastes are transported to incinerator in kg/s. In order to achieve efficient decomposition of waste in the process, a specific amount of heat which must be provided in the waste stream, in kJ/kg, is known. And finally, a boundary heat flows in kW, at which the waste incinerator operates, are also known.

**III. THEORETICAL ANALYSIS OF PROBLEM AND ANALYTICAL RELATIONS FOR LIMITATIONS**

Along the possible crashed period, the optimum TCT solution may be encountered at minimum total cost as shown in Fig. 2.

For the problem of burning two medical wastes a practical example will be taken, wherein the two variables  $\dot{m}_1$  and

$\dot{m}_2$  - the quantities of first and second waste, are in unit of time. The main limitations in the observed problem, as has been said are relate to a minimum quantity of both of wastes to be burned ( $\dot{m}_{1min}$ ,  $\dot{m}_{2min}$ ) as well as the limitation concerning the restriction of pump which transports the waste to the incineration furnace – the maximum quantity of each of the two wastes ( $\dot{m}_{1max}$ ,  $\dot{m}_{2max}$ ). In the coordinate system ( $\dot{m}_1$ ,  $\dot{m}_2$ ), these limitations determine the area  $D_0$  in form of rectangle, Fig. 2, which is determined by straight lines (1)-(2) and (3)-(4).

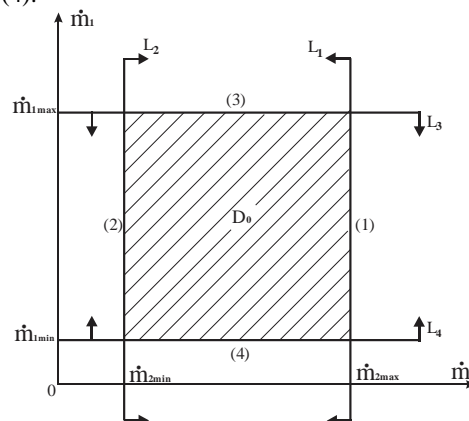


Fig. 2 Basic limitations at the set problem

In order to determine the following specific limitations of the problem, the relationship between quantities of wastes  $\dot{m}_1$  and  $\dot{m}_2$  will be previously determined. At this, the starting point is the fact that this is a slurry waste (in a form of sludge). Exchanged heat during the combustion process of the observed liquid waste can be theoretically analysed from a thermodynamic point of view through several stages [8, 9, 10, 11].

a. The transition from the liquid state of the initial temperature  $t_0$  to a state of boiling liquid  $t'$ :

$$Q_a = m \cdot c_a \cdot \Delta t_a = m \cdot c_a \cdot (t' - t_0) \quad (3)$$

where  $c_a$  is corresponding specific heat capacity.

b. The transition of boiling liquid into dry saturated steam (evaporation process at  $p = const.$ , that is  $t = const.$ ):

$$Q_b = m \cdot \Delta h_b = m \cdot (h'' - h') = m \cdot r \quad (4)$$

where  $r$  is latent heat of evaporation.

c. The process of overheating from dry saturated state to a state of superheated steam:

$$Q_c = m \cdot c_c \cdot \Delta t_c = m \cdot c_c \cdot (t_p - t'') \quad (5)$$

where  $c_c$  is corresponding specific heat capacity.

By analysis of relations (3), (4), (5) it can be concluded, theoretically speaking, that the total heat exchange during combustion is directly proportional to mass  $m$ :

$$Q = m \cdot k$$

where  $k \approx const.$

From here follows that the heat flow is proportional to the mass flow rate:

$$\dot{Q} = \dot{m} \cdot k$$

Taking into account both of wastes (1 and 2), the total heat flow (amount of heat per unit of time), it will be:

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 \quad [W] \quad (6)$$

Individual heat flows are linearly dependent on mass flows (the amount of waste per time unit)  $\dot{m}_1$  and  $\dot{m}_2$ , so the relation (6) becomes:

$$\dot{Q} = b_1 \cdot \dot{m}_1 + a_1 \cdot \dot{m}_2 \quad (7)$$

Dividing the relation (7) with  $\dot{Q}$  we obtain a linear dependence in segmental form:

$$\frac{\dot{m}_1}{\dot{Q}/b_1} + \frac{\dot{m}_2}{\dot{Q}/a_1} = 1 \quad (8)$$

that is:

$$\frac{\dot{m}_1}{b} + \frac{\dot{m}_2}{a} = 1 \quad (9)$$

where

$$b = \frac{\dot{Q}}{b_1} \quad \text{and} \quad a = \frac{\dot{Q}}{a_1}$$

For the case where  $\dot{Q} = \text{const}$ , relation (9) can be displayed graphically in the system  $(\dot{m}_1, \dot{m}_2)$ , in the form of straight line, Fig. 3.

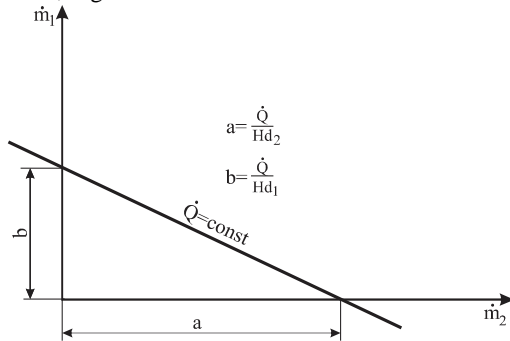


Fig. 3 The linear dependence of mass flows of two wastes

Obviously, for different values of  $\dot{Q}_i = \text{const}$ . according to (9), a set of mutually parallel lines in the coordinate system  $(\dot{m}_1, \dot{m}_2)$  will be obtained. For the case of known heat flux limit values of the waste incinerator,  $\dot{Q}_1 = \dot{Q}_{\text{min}}$ ,  $\dot{Q}_2 = \dot{Q}_{\text{max}}$ , particular limits between straight lines can be obtained (5) i (6), the area of  $D_x$ , according to Fig. 4.

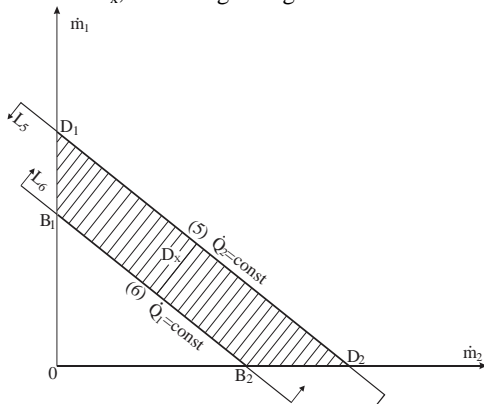


Fig. 4 Special limitations of the set problem

The presented method will be illustrated in a calculation example from practice.

#### IV. CALCULATION EXAMPLE

Practical problem of incineration of two medical wastes in semiliquid state (in form of sludge) will be observed. Both wastes are contaminated with mercury. First waste contains 9 wt % Cd with low heat power  $H_{d1} = 22500$  kJ/kg. Second waste contains 7 wt % Cd with low heat power  $H_{d2} = 18500$  kJ/kg. The above data were determined in the way of applying a standard laboratory method. The minimum amount of each of wastes to be burned is: for the first waste 0.08 kg/s and for the second 0.1 kg/s. Due to limitations of pump capacity, maximum 0.6 kg/s of each waste can be transported to the incinerator. Minimum specific amount of heat that is required in the waste stream to efficiently carried the burning process is  $H_m = 16000$  kJ/kg. Waste incinerator can work with the heat flow in the range 6 to 10 MW.

Given analytical method will set a maximum amount for each waste to be burned. Solution of the problem will be performed in the following order:

a. In specific scale, in the coordinate system X-Y mass flow rates  $\dot{m}_1, \dot{m}_2$  (quantities of first and second waste to be burned) will be entered in kg/s, Fig. 5.

b. The straight lines (1) and (2), relating to the limitation of the minimum amount of waste to be burnt ( $\dot{m}_{1\text{min}} = 0.08$  kg/s,  $\dot{m}_{2\text{min}} = 0.1$  kg/s), will be constructed.

c. The straight lines (3) and (4), relating to the limit of the pump capacity to transport waste ( $\dot{m}_{1\text{max}} = 0.6$  kg/s,  $\dot{m}_{2\text{max}} = 0.6$  kg/s), will be constructed

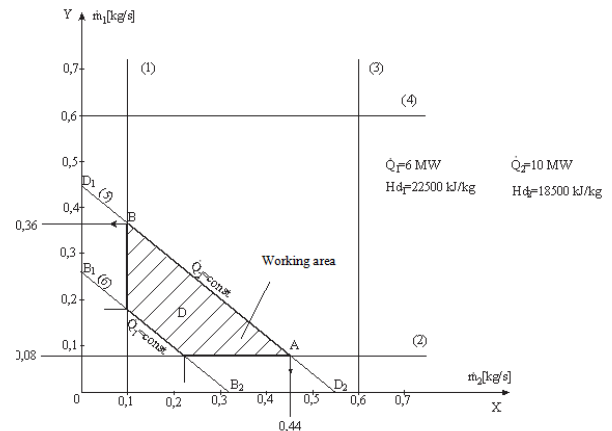


Fig. 5 Determination of maximum quantity of waste at calculation example using analytical method

d. Starting from the fact that the lower heating value of the fuel  $H_d$  refers to combustion in which all combustion products as well as the water contained in the fuel (moisture) are in the form of a gas, incineration of both wastes can be observed individually. In this case, cutoffs on the coordinate axes  $X \equiv \dot{m}_1$ ,  $Y \equiv \dot{m}_2$  can be obtained by dividing the heat flow with the lower heating value of fuel, considering that the lower calorific value for both fuels ( $H_{d1}, H_{d2}$ ), in a given case is greater than the minimum specific amount of heat that is required in the process for its efficient performance  $H_m$ , that is:

$$H_{d1} > H_m, H_{d2} > H_m$$

Dimension of mass flow rate is obtained by dividing of these heat parameters. Taking the above into consideration,

the coordinates of points on axes (points  $D_1$ ,  $D_2$ ,  $B_1$ ,  $B_2$  according to Fig. 5) will be:

$$X_{D_2} = \frac{\dot{Q}_2}{H_{d_2}} = \frac{10 \cdot 10^6 (W)}{18.5 \cdot 10^3 \cdot 10^3 (J/kg)} = 0.5405 \text{ kg/s} \quad (10)$$

$$X_{B_2} = \frac{\dot{Q}_1}{H_{d_2}} = \frac{6 \cdot 10^6 (W)}{18.5 \cdot 10^3 \cdot 10^3 (J/kg)} = 0.3243 \text{ kg/s} \quad (11)$$

$$Y_{D_1} = \frac{\dot{Q}_2}{H_{d_1}} = \frac{10 \cdot 10^6 (W)}{22.5 \cdot 10^3 \cdot 10^3 (J/kg)} = 0.4444 \text{ kg/s} \quad (12)$$

$$Y_{B_1} = \frac{\dot{Q}_1}{H_{d_1}} = \frac{6 \cdot 10^6 (W)}{22.5 \cdot 10^3 \cdot 10^3 (J/kg)} = 0.2667 \text{ kg/s} \quad (13)$$

On the basis of these coordinates, parallel lines (5) and (6) will be constructed, Fig. 5. Now the working area (D) is completely determined.

e. The maximum quantities of individual wastes that can be burnt, are read directly from the graph (points A and B):

$$\dot{m}_{1m} = 0.36 \text{ kg/s}$$

$$\dot{m}_{2m} = 0.44 \text{ kg/s}$$

f. An analytical solution follows, after determining the equation of a straight line (5), given that, according to (8), (9), its cutoffs on coordinate axes are known. Equation of the straight line (5) will be:

$$\frac{\dot{m}_1}{0.4444} + \frac{\dot{m}_2}{0.5405} = 1 \quad (14)$$

Point A is obtained at the intersection of straight lines (2) and (5), so that:

$$\frac{0,08}{0.4444} + \frac{\dot{m}_2}{0.5405} = 1$$

Hence,  $\dot{m}_{2m} = 0.4432 \text{ kg/s}$

Point B is obtained at the intersection of straight lines (1) and (5)

$$\frac{\dot{m}_1}{0.4444} + \frac{0,1}{0.5405} = 1$$

Hence,  $\dot{m}_{1m} = 0.3622 \text{ kg/s}$

As can be seen, there is a very little difference between the graphical and analytical solution.

For a more detailed analysis of the process, it is necessary to know also the equation of straight line (6). Taking into account that the coordinates of the points  $B_1$  and  $B_2$  are known, the equation of straight line (6) will be:

$$\frac{\dot{m}_1}{0.2667} + \frac{\dot{m}_2}{0.3243} = 1 \quad (15)$$

It is obvious that straight lines (5) and (6) are parallel to one another, which is easy to prove.

## V. CONCLUSIONS

A set problem of combustion two medical wastes is efficiently solved by analytical geometry method. This method is especially suited for cases where there is a number of limitation functions in linear form, ie when the lower calorific value of both wastes is greater than the amount of heat that must be provided in the observed process. In addition to the basic task of determining the maximum amount of both wastes that can be burned, permissible

operating area in which the observed process can take place is also determined by the applied method.

For the observed case, the applied method of linear programming can be applied to manage the existing limitations, as well as to more optimize the process. Also, instead of the two variables, it is possible to apply a larger number of them.

When the lower calorific value of waste is less than the heat that must be provided in the process for its efficient operation, in practice the case is solved by adding a certain amount of combustible oil to the waste. This case also can be solved by analytical geometry method.

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