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Rightside-Left β-Numbers

[1] Ammar Seddiq Mahmood, [2] Shukriyah Sabir Ali
[1] Department of Mathematics, College of Education, University of Mosul, Iraq
[2] Department of Mathematics, College of Computers Sciences and Mathematics, University of Mosul, Iraq

***Abstract-** For any partition $\mu = (\mu_1, \mu_2, ..., \mu_n)$ of a non - negative integer number r there exist a diagram (A) of β - numbers for each e where e is a a positive integer number greater than or equal to two; which introduced by James in 1978.These diagrams (A) play an enormous role in Iwahori-Hecke algebras and q-Schur algebras; as presented by Fayers in 2007. In this paper we introduced a new diagram (A′) by employing the "rightside-left " application on the main diagram (A). We concluded that we can find the successive main diagrams (A′) for the guides b_2, b_3,. . . and b_e depending on the main diagram (A′) for b_1 and set this fact as a rule named Rule (3.1).We depended in our work on the idea of the intersection of the main diagrams (A) given by Mahmood in 2011 and the "upside-down β - numbers" again given by Mahmood in 2013.*

Keywords: β - numbers, Diagram (A), Intersection, Partition, Rightside- Left

I. INTRODUCTION

Let r be a non- negative integer. A partition $\mu = (\mu_1, \mu_2, \dots, \mu_n)$ of r is a sequence of non - negative integers such that $|\mu| = \sum_{i=1}^{n} \mu_i = r$ and $\mu_i \geq \mu_{i+1}$; $\forall i \geq 1$, [1]. For example, μ= (5, 4, 4, 2, 2, 2,1) is a partition of r =20. β - numbers was defined by; see James in [2]: "Fix μ is a partition of r, choose an integer b greater than or equal to the number of parts of μ and define $\beta_i = \mu_i + b - i$, $1 \leq i \leq b$. The set $\{\beta_1, \beta_2, \dots, \beta_b\}$ is said to be the set of β-numbers for μ". For the above example, if we take b =7, then the set of β - numbers is {11,9,8,5,4,3,1}.
Now, let e be a positive integer number greater than or equal to 2, we can represent β - numbers by a diagram called diagram (A).

$$\left.\begin{array}{cccc} \underline{\textbf{run. 1}} & \underline{\textbf{run. 2}} & \underline{\textbf{. . .}} & \underline{\textbf{run. e}} \\ 0 & 1 & \dots & e-1 \\ e & e+1 & \dots & 2e-1 \\ 2e & 2e+1 & \dots & 3e-1 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{array}\right\} \text{diag.(A)}$$

Where every β will be represented by a bead (●) which takes its location in diagram (A). Returning to the above example, diagram (A) of β-numbers for e = 2 and e = 3 is as shown below in diagram 1 and 2 respectively:

e = 2		b =7	
0	1	—	●
2	3	—	●
4	5	●	●
6	7	—	—
8	9	●	●
10	11	—	●

Dig.1

e =3			b =7		
0	1	2	—	●	—
3	4	5	●	●	●
6	7	8	—	—	●
9	10	11	●	—	●

Dig.2

Note: Along this paper, we mean by diagram(A); diagram (A) of β-numbers

This subject has a connection with representation theory of lwahori - Hecke algebras and q-Schur algebras [3].
Also any partition μ of r is called w-regular; w ≥ 2, if there does not exist $i \geq 1$ such that $\mu_i = \mu_{i+w-1} > 0$ and μ is called w-restricted if $\mu_i - \mu_{i+1} < w$; ∀i ≥1.

II. THE INTERSECTION OF β - NUMBERS IN THE MAIN DIAGRAMS

Mahmood in [4] introduced the definition of main diagram(s) (A) and the idea of the intersection of these main diagrams. in this section, we repeat the principals results, as follows: Since the value of b ≥ n; [5], then we deal with an infinite numbers of values of b. Here we want to mention that for each value of b there is a special diagram (A) of β-numbers for it, but there is a repeated part of one's diagram with the other values of b where a "Down –shifted" or "Up-shifted", occurs when we take the following :

(b_1 if b = n), (b_2 if b = n+1), . . . and (b_e if b = n+(e-1)).
Definition (2.1): [4] The values of b_1 , b_2, . . . and b_e are called the guides of any diagram (A) of β-numbers .
For the above example, the guides values are b_1 = 7 and b_2 =8 if e = 2 where μ = (5, 4,4, 2, 2, 2,1), then:

e = 2		b_1 =7		b_1+1(e)		b_1+2(e)	
0	1	—	●	●	●	●	●
2	3	—	●	—	●	●	●
4	5	●	●	—	●	—	●
6	7	—	—	●	●	—	●
8	9	●	●	—	—	●	●
10	11	—	●	●	●	—	—
12	13			—	●	●	●
14	15					—	●

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e = 2		$b_2 = 8$		$b_2+1(e)$		$b_2+2(e)$	
0	1	•	—	•	•	•	•
2	3	•	—	•	—	•	•
4	5	•	•	•	—	•	—
6	7	•	—	•	•	•	—
8	9	—	•	•	—	•	•
10	11	•	—	—	•	•	—
12	13	•	—	•	—	—	•
14	15			•	—	•	—
16	17					•	—

Dig.3 Illustrates the idea of "Down- shifted"

We define any diagram (A) that corresponds any b guides as a "main diagram" or "guide diagram".

Theorem (2.2): [4] There is e of main diagrams for any partition μ of r.■

The idea of the intersection of any main diagrams is defined by the following:

1- Let τ be the number of redundant part of the partition μof r, then we have $\mu = (\mu_1, \mu_2, \ldots, \mu_n) = (\lambda_1^{\tau_1}, \lambda_2^{\tau_2}, \ldots, \lambda_m^{\tau_m})$ such that $r = \sum_{i=1}^{n} \mu_i = \sum_{j=1}^{m} \lambda_j^{\tau_j}$.

2- We denote the intersection of main diagrams by $\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}$.

3- The intersection result as a numerical value will be denoted by $\#\left(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}\right)$, and it is equal to ϕ in the case of no existence of any bead, or γ in the case that γ common beads exist in the main diagrams.

For the above example where $\mu = (5,4,4,2,2,2,1) = (5, 4^2, 2^3,1)$, r = 20, if e = 2 then there are two guides, the first is $b_1 = 7$ since n = 7 and the second is $b_2 = 8$, the β-numbers are given in table 1:

Table 1. β- Numbers

b_s \ β_i	β_1	β_2	β_3	β_4	β_5	β_6	β_7	β_8
$b_1 = 7$	11	9	8	5	4	3	1	
$b_2 = 8$	12	10	9	6	5	4	2	0

Hence,the main diagrams and their intersection will be as shown in diagram 4:

$b_1 = 7$		$b_2 = 8$		$\bigcap_{s=1}^{2} \mathrm{m.d.}_{b_s}$	
—	•	•	—	—	—
—	•	•	—	—	—
•	•	•	•	•	•
—	—	•	—	—	—
•	•	—	•	—	•
—	•	•	—	—	—
		•	—	—	—

Dig.4

The intersection of the main diagrams (A) for e=2

Notice that, $\#\left(\bigcap_{s=1}^{2} \mathrm{m.d.}_{b_s}\right) = 3$.

The two principle theorems about the idea of the intersection of any main diagrams are:

Theorem (2.3):[4] For any $e \geq 2$, the following holds:

1- $\#\left(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}\right) = \phi$ if $\tau_k = 1, \forall k$ where $1 \leq k \leq m$.

2- Let Ω be the number of parts of λ which satisfies the condition $\tau_k \geq e$ for some k, then:
$\#\left(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}\right) = \left[\sum_{t=1}^{\Omega} \tau_t - \Omega\,(e-1)\right]$.■

Theorem (2.4):[4]

1- Let μ be a partition of r and μ is w-regular, then:

$$\#\left(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}\right) = \begin{cases} \text{value} & \text{if } e < w, \\ \phi & \text{if } e \geq w. \end{cases}$$

2- Let μ be a partition of r and μ is h-restricted, then:

$$\#\left(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}\right) = \begin{cases} \text{value if } e < h \textit{ or } (e = h \text{ and } h < w), \\ \phi \quad \text{if } e > h \textit{ or } (e = h \text{ and } h \geq w). \end{cases}$$ ■

Also, Sarah M. Mahmood in [6] gave the same subject by using a new technique which supported the results of Mahmood in [4].

III. RIGHTSIDE - LEFT β - NUMBERS

In this work, we introduce a new diagram depends on the old diagram (A) with application of " rightside- left". The new diagram has another partition of the origin partition and if we use the idea of the intersection, the partition of the beads will not be the same (or will not be the sum) in $\#\left(\bigcap_{s=1}^{e} \mathrm{m.d.}_{b_s}\right)$ in the normal main diagrams.

From the above example,where $\mu = (5, 4^2, 2^3, 1)$ and e =2, then, we shall have a group of diagram (A') to take the idea of rightside – left as shown in diagram 6 below:

$b_1 = 7$		$b_2 = 8$	
—	•	•	—
—	•	•	—
•	•	•	•
—	—	•	—
•	•	—	•
—	•	•	—
		•	—

Dig.5 (A)

R-L →

$b_1 = 7$		$b_2 = 8$	
•	—	—	•
•	—	—	•
•	•	•	•
—	—	—	•
•	•	•	—
•	—	—	•
		—	•

Dig.6 (A')

Now, if we use the old technique for finding any partition of any diagram (A'), the value of the partition will not be equal to the origin partition? so, we delete any effect of (-) in (A) after the position of β_1, and we start with number 1 for the first (-) a (left to right) in any row exist in (A), and with number 2 for the second (-) and ... etc, and we stop with last (-) before the position β_1 in (A) as shown in diagram 7. Now, to apply "rightside- left " on (A), the new version (A') has the same partition of (A), see diagram 8.

$b_1 = 7$	$b_2 = 8$
1 •	• 1
2 •	• 2
• •	• •
3 4	• 3
• •	4 •
5 •	• 5
	• X

Dig.7 (A)

R-L

$b_1 = 7$	$b_2 = 8$
• 1	1 •
• 2	2 •
• •	• •
4 3	3 •
• •	• 4
• 5	5 •
	X •

Dig.8 (A')

Remark: The main diagram (A') in case $b_1 = n$, plays a main role to design all the main diagrams (A') for ($b_2 = n+1$), ... and ($b_e = n+(e-1)$), as follows:

Rule (3.1): Since the main diagram (A') in the case b_1, we can find the successive main diagrams (A') for b_2, b_3, ..., and b_e, as follows:

1) 1^{st} column in the case $b_1 = n \rightarrow$ last column in the case b_2 and to add one (•) in up $\rightarrow$ (e-1) column in the case b_3 and to add one (•) in up $\rightarrow \cdots \rightarrow 2^{nd}$ column in the case b_e and to add one (•) in up of main diagram (A').
2) 2^{nd} column in the case $b_1 \rightarrow 1^{st}$ column in the case b_2 and to add one (-) in down $\rightarrow$ last column in the case b_3 and to add one (•) in up$\rightarrow \cdots \rightarrow 3^{rd}$ column in the case b_e and to add one (•) in up of main diagram (A').

. . . .
. . . .
. . . .

e) last column in the case $b_1 \rightarrow$ (e-1) column in the case b_2 and to add one (-) in down $\rightarrow$ ... $\rightarrow 1^{st}$ column in the case b_e and to add one (-) in down.■

This rule is clarified in diagram 9 For the above example, where $\mu = (5, 4^2, 2^3, 1)$ and e = 3:

$b_1 = 7$
— • —
• • •
— — •
• — •

Dig.2

$b_1 = 7$	$b_2 = 8$	$b_3 = 9$
— • —	• — •	— • •
• • •	• • —	• — •
• — —	— — •	— • •
• — •	— • •	• • —
	— — •	— • —

Di.g. 9

Theorem (3.2): All the results in [4] about the main diagram (A) is the same of the diagram (A') but in rightside – left position.■

One of these results is the intersection of the main diagrams. so, the fact mentioned in theorem (3.2) is clear in diagram 10 comparing it with diagram 4, for our example when $\mu = (5, 4^2, 2^3, 1)$ and e=2 and for e=3, see the two diagrams 11 and 12:

$b_1 = 7$	$b_2 = 8$	$\bigcap_{s=1}^{2} \mathbf{m.d.}_{b_s}$
• —	— •	— —
• —	— •	— —
• •	• •	• •
— —	— •	— —
• •	• —	• —
• —	— •	— —
	— •	— —

Dig.10 The intersection of the main diagrams (A') for e=2

Notice that, $\#(\bigcap_{s=1}^{2} \mathrm{m.d.}_{b_s}) = 3$, in both cases.

$b_1 = 7$	$b_2 = 8$	$b_3 = 9$	$\bigcap_{s=1}^{3} \mathbf{m.d.}_{b_s}$
— • —	• — •	• • —	— — —
• • •	— • •	• — •	— — •
— — •	• — —	• • —	— — —
• — •	• • —	— • •	— — —
	• — —	— • —	— — —

Dig.11 The intersection of the main diagrams (A) for e=3

$b_1 = 7$	$b_2 = 8$	$b_3 = 9$	$\bigcap_{s=1}^{3} \mathbf{m.d.}_{b_s}$
— • —	• — •	— • •	— — —
• • •	• • —	• — •	• — —
• — —	— — •	— • •	— — —
• — •	— • •	• • —	— — —
	— — •	— • —	— — —

Dig.12 The intersection of the main diagrams (A') for e=3

Note that, $\#(\bigcap_{s=1}^{3} \mathrm{m.d.}_{b_s}) = 1$, in both cases.

IV. CONCLUSION

In this paper, a procedure is suggested for the diagram(A') of β-numbers which represents the rightside-left diagram (A) of β-numbers to have the same partition of diagram (A) of β-numbers. Furthermore, a rule for designing all the main diagrams (A') for b_2, b_3, . . . and b_e is setted depending on the main diagram (A') for b_1. And finally, we found out that the intersection of the main diagrams (A') is the same of the main diagrams (A) but in rightside-left position.

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