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A PROCEDURE FOR USING BOUNDED OBJECTIVE FUNCTION METHOD TO SOLVE TIME-COST TRADE-OFF PROBLEMS

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Abstract- The primary goal of Multi-objective optimization is to optimize simultaneously conflicting objectives (such as time and the cost in construction projects) in order to find acceptable Pareto optimal solutions for the decision maker. This paper presents a procedure for solving continuous linear Time-Cost Trade-off problems based on the Bounded Objective Function Method in combination with the concept of linear membership function in the fuzzy programming. The applicability of the proposed procedure was demonstrated by an application example. A set of the best optimum Time-Cost Trade-off solutions for the problem was obtained.

Keywords - Multi-objective optimization; Continuous Linear Time-Cost Trade-off; Bounded Objective Function Method; Construction Project; membership function

I. INTRODUCTION

Multi-objective optimization (MOP) is a rapidly growing area of research and application in modern day optimization. The primary goal of MOP is to optimize simultaneously several conflicting objectives in order to find acceptable Pareto optimal solutions for the decision maker [1]. The ε-constraint method P(ε) presented by [2], solves MOP by transforming one of the objectives into a constraint. A modification of the ε-constraint method is the Bounded Objective Function Method (BOFM) [3] which minimizes the single most important objective function, while, all other objective functions are used to form additional constraints. One of the MOP problems in construction project is the Time-Cost Trade-off problem (TCT). Crashing the projects schedule causes increasing in the project cost, so, in general, the conflict between time and cost is a fuzzy multi-objective problem. The fuzzy problem in finding the TCT solution can be solved using fuzzy linear programming method [4]. However, the resulted unique solution can be inappropriate. The concept of Membership Functions (MSF) in fuzzy programming method can be used in combination with the BOFM to get a set of solutions and consequently allow the decision maker to choose his appropriate one.

II. PROBLEM

Figure (1) shows a simple representation of the continuous linear relationship between the duration of an activity and its direct costs.

The continuous linear relationship shown in the Fig. 1 between the two points implies that any intermediate duration could also be chosen [5]. It is possible that some intermediate point may represent the ideal or optimal trade-off between time and cost for this activity. The slope of the line connecting the normal point (lower point) and the crash point (upper point) is called the cost slope of the activity (cost slope = [crash cost-normal cost]/ [normal duration-crash duration]).

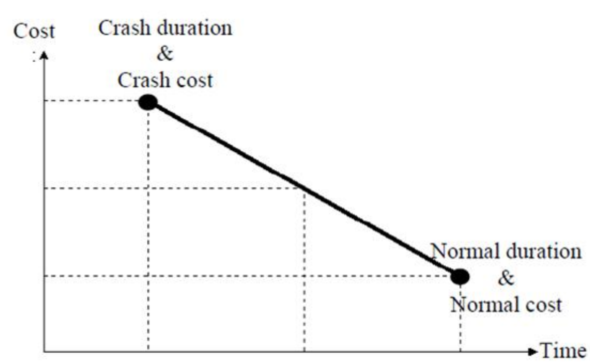

Fig. 1 Continuous Linear Time/Cost Trade-off for an Activity (after Hendrickson [5])

III. THE OPTIMUM TCT SOLUTION

Along the possible crashed period, the optimum TCT solution may be encountered at minimum total cost as shown in Fig. 2.

The optimum duration (D) represents the duration at which minimum total cost is calculated. The disadvantage of this consideration is that, project duration might be inappropriate. It might be better to use this rule to obtain more than one solution along the crashed period as explained in Section VI.

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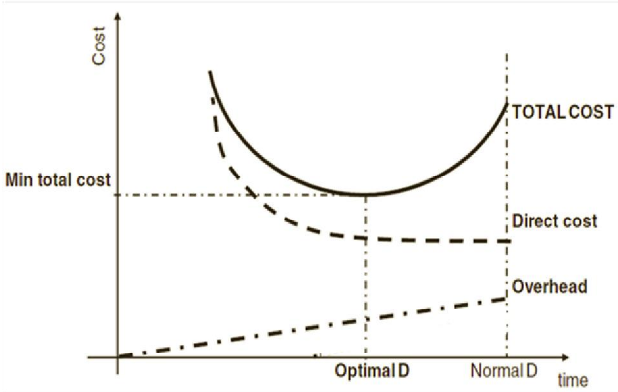

Fig. 2 Optimal duration is considered at the minimum total project cost (after Marco [6])

IV. THE BOUNDED OBJECTIVE FUNCTION METHOD

As explained previously, the ε-constraint method transforms one of the objectives into a constraint. The formulation of Bi-objective problem (such as TCT) using the ε-constraint method is formulated as:

$$P(\varepsilon): \min\ f_1(x) \quad (1)$$

Subject to:

$$f_2(x) \le \varepsilon_2 \quad (2)$$

$$x \in M\ .$$

Here, ε_2 is the upper bound for the second objective function $f_2(x)$; x is the vector of design variables and, M is the feasible domain.

BOFM adds additional constraint which is the lower bound for f_2 (x) such that $L_2 \le f_2(x) \le \varepsilon_2$, where L_2 and ε_2 are the lower and upper bounds for f_2 (x) respectively, and consequently, Inequality (2) becomes as:

$$L_2 \le f_2(x) \le \varepsilon_2 \quad (3)$$

$$x \in M$$

IV. FUZZY PROGRAMMING

Fuzzy multi-objective programming is based on the choice of appropriate MSFs for the objective functions [7]. In fuzzy linear programming, the Bi-objective optimization problem based on the MSF concept can be formulated as:

$$Max\lambda = (\lambda_1 + \lambda_2)/2 \quad (4)$$

Subject to:

$$\lambda_1 = [f_1(\max) - f_1]/[f_1(\max) - f_1(\min)] \quad (5)$$

$$\lambda_2 = [f_2(\max) - f_2]/[f_2(\max) - f_2(\min)] \quad (6)$$

$$0 \le \lambda \le 1 \Leftrightarrow \lambda \in [0,1] \quad (7)$$

Where: λ_1 and λ_2 are the MSF for the first and second objective functions respectively. $f_j(\max)$ and $f_j(\min)$ are The maximum and the minimum values for the j^{th} objective function respectively(j = 1,2).

V. SUGGESTED PROCEDURE

Let the project duration (Z_1) and the project cost (Z_2) be the first and the second objective functions, respectively and let the relationship between the time and the cost of each activity be continuous and linear. Following, are the suggested steps for solution procedure:

Step1

The linear MSF (λ_1) for the project duration objective (Z_1) is constructed based on Equation (5):

$\lambda_1 = [f_1(\max) - f_1]/[f_1(\max) - f_1(\min)]$ Or

$\lambda_1 = [\max Z_1 - Z_1]/[\max Z_1 - \min Z_1]$

- Calculate $\min Z_1$, where Tsci = 0, get $Z_1 = \max Z_1$ (no crashing).
- Calculate $\min Z_1$, where $0 \le Tsci \le Tsc_{max}$, get $Z_1 = \min Z_1$.

Where: Tsci is the number of crashed days for activity i; Tsc_{max} is the maximum possible crashed days for activity i; i = {1...n}; n is the number of project activities.

Step2

The interval ([0, 1]) of λ_1 –values is divided to 10 subintervals [L_k, ε_k]: [0, 0.1], [0.1, 0.2], [0.2, 0.3], [0.3, 0.4], [0.4, 0.5], [0.5, 0.6], [0.6, 0.7], [0.7, 0.8], [0.8, 0.9] and [0.9, 1]. Where L_k is the lower bound of the k^{th} subinterval and ε_k is the upper bound of the k^{th} subinterval (k =1, 2… 10).

Note: The choosing number of the subintervals may differentiate based on the possible crashed period ($\max Z_1 - \min Z_1$). If this period extends, it is better to increase the subintervals number to get more solutions.

Step3

BOFM is used to solve the problem which is formulated as:

$$\text{Minimize } Z_2 \quad (8)$$

Subjected to

$$L_k \prec \lambda_1 \le \varepsilon_k \quad (9)$$

Solving the problem at each λ_1 –subinterval, 10 solutions (Z_1, $\min Z_2$) can be obtained.

Not1: the formulation of the problem in this step meets the rule explained in Section III, however, 10 solutions are obtained.

Note2: the inequality $L_k \le \lambda_1 \le \varepsilon_k$ (in BOFM) is exchanged with $L_k < \lambda_1 \le \varepsilon_k$ to prevent the possibility of resulting the same solution for two neighbour subintervals. For example: the first and the second subintervals are [0, 0.1] and [0.1, 0.2] respectively. If the inequality $L_k \le \lambda_1 \le \varepsilon_k$ is considered, then, $\min Z_2$ subjected to $0 \le \lambda_1 \le 0.1$ may be found at $\lambda_1 = 0.1$ and $\min Z_2$ subjected to $0.1 \le \lambda_1 \le 0.2$ may also be found at $\lambda_1 = 0.1$ and consequently, the same solution is found at the two subintervals.

Step4

At some subintervals, two or more equal $\min Z_2$ can be encountered for different Z_1 values as shown in Fig. 3. To choose $\min Z_2$ and the corresponding less available Z_1 value, an additional step must be conducted as:

$$\text{Minimize} Z_1 \quad (10)$$

Subjected to

$$Z_2 = \min Z_2 \quad (11)$$

Where $\min Z_2$ is the resulted minimum cost at each subinterval in the previous step using Equation (8). The obtained 10 solutions represent the 10 best (minimum) values of project Cost and the corresponding 10 values of project Duration along the whole crashed period.

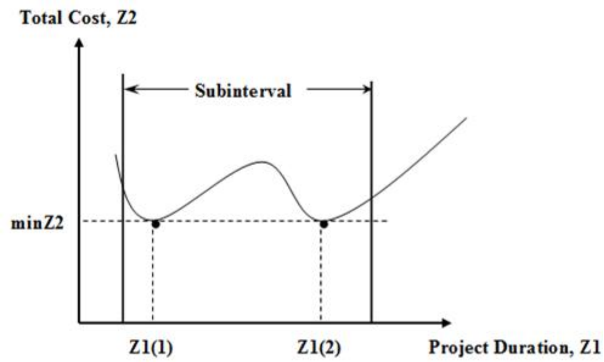

Fig. 3 minZ_2 is obtained at Z_1(1) and at Z_1(2)

VI. APPLICATION EXAMPLE

In this example, a data from a highway project is used. The project refers to the upgrading of an existing two-way highway subdivided to a four-lane. In this application, a 100 m road section length is considered for simplicity. The project network is illustrated in Fig. 4 and the data for the 29 activities is presented in Table 1.

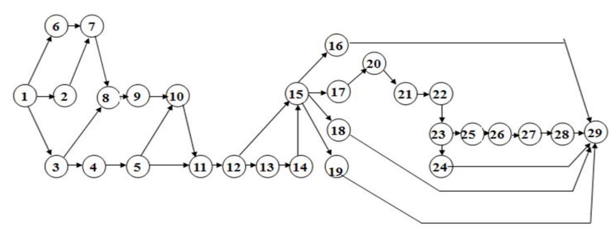

Fig. 4 Project Network

TABLE I ACTIVITIES DATA

Activity	Normal duration,	Crash duration,	Normal Cost, unit	Crash Cost, unit	Dependences	Lags (+) Or Leads (-)
Service road A						
1-Rock excavation	5	4	2030	2300	-	0
2- Embankment construction	8	6	1020	1510	1(FS)	-3
3- Sub base and base layers	8	6	1700	2090	1(FS) 2(FS)	0 0
4- Asphalt layer	4	3	590	730	3(FS)	0
5- Temporary marking and signing	2	-	90	-	4(SS)	+1
Service road B						
6-Earth and semi-rock excavation	4	3	910	1100	1(FS)	0
7-Embankment construction	2	-	250	-	2(FS) 6(FS)	0 -1
8-Subbase and base layers	7	5	1490	1830	3(FS) 7(FS)	0 0
9-Asphalt layer	4	3	520	750	4(FS) 8(FS)	0 0
10-Temporary marking and signing	2	-	90	-	5(FS) 9(FF)	0 +1
Main road						
11-Traffic diversion	1	-	50	-	5(FS) 10(FF)	0 0
12-Rock excavation	8	6	3260	3710	11(FS)	0
13-Earth and semi-rock excavation—existing pavement removal	5	3	1140	1720	12(SS)	+2
14- Sub grade stabilization, retaining wall/culvert construction	4	3	300	450	13(SS)	+2
15-Embankment construction	8	5	1020	1430	12(FS) 14(FS)	-4 -2
16-Drainage pipe construction	9	6	790	1180	15(FS)	-6
17-Drainage layer	13	11	3340	4060	15(SS)	+4
18-Planting at roadway verges	9	7	470	830	15(FS)	+4
19- El. installations at roadway verges	6	4	460	810	15(FS)	0
20- Ditches	6	5	1280	1430	17(SS)	+3
21- Sub base layer	14	10	1090	1560	20(SS)	+2
22- Base layer	14	9	900	1400	21(SS)	+2
23- Median island (New Jersey)	14	11	2220	2690	22(FS)	-9
24- Elect. installation in median island	3	-	230	-	23(SS)	+6
25- Asphalt layer #1	6	4	1590	1990	23(FS)	-4
26- Asphalt layer #2	10	8	2630	3240	25(SS)	+4

27- Friction course overlay	8	6	2060	2660	26(FS)	0
28 -Final marking and signing	10	8	320	610	27(FS)	-3
29- Traffic restoration	1	-	50	-	28(FS)	0

An external constraint is set for the completion time of the service roads. In particular, finish time of activity 11 is 23 days after the beginning of the project. The indirect project cost is 150 units per day. Further, a penalty at a rate of 200 units per day of delay applies after the 80th day while a bonus of 100 units per day is given for project completion before the 80th day. 7 working days per week is considered.

The integer linear programming model presented by Adel et al. [8] is used. The two objective functions of the model are the project Duration:

$$Z_1 = (t_{fn} - t_{s0}) * \alpha \quad (10)$$

The total project Cost:

$$Z_2 = \sum Cni + \sum (Csli * Tsci) + indC + PC - BC \quad (11)$$

Where; t_{fn}: the finish time of the last activity (n), t_{s0}: the start time of the first activity (0); α is the Real time factor, which is equal to the ratio between the number of days per week (7days) to the number of working days per week, Csli: cost slope for activity i, Tsci**:** number of crashed days for each activity i.

∑ Cni is the sum of normal costs of all activities; indC is the total indirect cost, PC is the penalty cost, and BC is the bonuses cost.

The four steps of the suggested procedure are applied as follows:

Step1

The MSF of the project duration objective (Equation 5) is constructed as:

- Model running at no crashing (Tsci = 0), so, maxZ_1 = 85 days.

- Model running at crashing, so, minZ_1 = 70 days. Consequently, the membership function can be written as: $\lambda_1 = (85 - Z_1)/ (85 - 70)$.

Step2

λ_1-interval ([0, 1]) is divided to 10 subintervals as explained previously.

Step 3 and Step 4

The problem is solved by obtaining minZ_2 subjected to $L_k < \lambda_1 \leq \varepsilon_k$ and then minZ_1 subjected to Z_2 = minZ_2 for each subinterval as explained previously.

The model solutions are plotted in Fig. 5 and presented in Table 2. Fig. 6 shows the resulted total costs against the exact resulted values of the membership function (λ_1).

Fig. 5 Time-Cost Trade-off solutions

Fig. 6 λ_1- Total Cost Trade-off solutions

TABLE 2 TIME-COST TRADE-OFF SOLUTIONS

λ_1-subint.	(Z_1) , days	λ_1	Direct Cost , unit	IndC, unit	BC , unit	PC , unit	minZ_2 , unit
λ_1= 0	85 maxZ_1	0	33610	12750	0	1000	47360
0 < $\lambda_1 \leq$ 0.1	84	0.06	33710	12600	0	800	47110
0.1 < $\lambda_1 \leq$ 0.2	82	0.2	33960	12300	0	400	46660
0.2 < $\lambda_1 \leq$ 0.3	81	0.26	34055	12150	0	200	46405
0.3 < $\lambda_1 \leq$ 0.4	79	0.4	34260	11850	100	0	46010
0.4 < $\lambda_1 \leq$ 0.5	78	0.46	34417	11700	200	0	45917
0.5 < $\lambda_1 \leq$ 0.6	76	0.6	34707	11400	400	0	45707
0.6 < $\lambda_1 \leq$ 0.7	75	0.66	35007	11250	500	0	45757

$0.7 < \lambda_1 \leq 0.8$	74	0.73	35021	11100	600	0	45521
$0.8 < \lambda_1 \leq 0.9$	73	0.86	35321	10950	700	0	45571
$0.9 < \lambda_1 \leq 1$	71	0.93	35931	10650	900	0	45681
$\lambda_1 = 1$	70 minZ_1	1	36231	10500	1000	0	45731

For each subinterval, the obtained solution is a Parito optimum. According to whole set of solutions, it is noted that, solutions with $Z_1 \geq 75$ days are dominated by the solution with Z_1= 74 days. This means that solution at Z_1 = 74 day is better than those with $Z_1 \geq 75$ days according to both Z_1 and Z_2. In practice, the project team might not be able to execute the crashing process based on a certain solution, therefore, the presence of other solutions is very important and any solution must not be excluded. Among these solutions, the decision maker can search for his appropriate one.

VII. CONCLUSIONS

In this paper, a procedure for getting a set of solutions for continuous linear TCT problem in construction projects is suggested. The procedure uses the BOFM which is an extension of the popular ε-constraint method in combination with fuzzy linear MSF concept. Using this procedure, a set of optimum TCT solutions along the whole possible crashed period can be obtained as was demonstrated through the presented application.

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