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AN APPROACH OF A SIMPLE DENTAL BRIDGE AS A FRAME WITH UNIFORM LOAD BY TRANSFER-MATRIX METHOD

Mihaela SUCIU
Department of Mechanical Engineering, Technical University of Cluj-Napoca, Romania
Mihaela.SUCIU@rezi.utcluj.ro

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***Abstract:** In this paper, a frame model for a simple dental bridge will be presented, which will be calculate by TMM, in order to facilitate orthodontic interventions in situ, especially in delicate situations. Applications of mathematical algorithms in bioengineering and especially in orthodontics are in full expansion. In this study, it will be done a similarity between a simple dental bridge and a frame, which is an original idea, as is the application of the Transfer-Matrix Method (TMM) to the analytical calculus of a dental bridge as a frame. The model of frame has two vertical poles, embedded at inferior ends and with an uniformly distributed load of horizontal part of the frame. A simple dental bridge has two pole teeth. They can be assimilated with the two vertical parts of frame and the aggregation elements, together with the bridge body, can be assimilated with the horizontal part of frame. After applying the TMM, we can determine the displacements for aggregation elements. This original idea to assimilate a simple dental bridge as a frame, is an approach that can be applied for different frames and, so, for differently dental bridges, applied for different partial or total edentulous situations. Mathematical algorithm for frame calculus can be very easy to program and this allows the results to be obtained very quickly with immediate application in practice, in orthodontics, which allows the results to be used in situ, especially in special and delicate situations. In the future, we want to be able to present the validation of the theoretical results with experimental results and with those obtained by modeling with Finite Elements Method (FEM).*

***Keywords*:** dental bridge, frame, charge density, Dirac's and Heaviside's functions and operators, state vector, Transfer-Matrix Method (TMM)

I. INTRODUCTION

The applications of mathematical algorithms in bioengineering and especially in orthodontics are in full expansion. A frame is a bent beam that can be studied by various methods, including the Transfer-Matrix Method (TMM). In my study, it will be done a similarity between a simple dental bridge and a frame, which is an original idea, as is the application of TMM to the calculus of a dental bridge as a frame. The study of frames is a very important problem with many practical applications in the fields of life and now in the field of health too, for orthodontic problems, especially in dentistry. Some research on frames and their applications will be mentioned.

In [1] is presented a study for ten-year survival of bridges placed in the General Dental Services in England and Wales. About bending fracture of Co-Cr dental bridges, produced by additive technologies it is in [2] and [3] gives us an experimental approach about bending fracture of Co-Cr dental bridges, produced by additive technologies. In [4] shows an integrated construction and simulation of tool paths for milling dental crowns and bridges. Applications of Transfer-Matrix Method are given in [5]. In [6] a study was made about a comparison of survival and complication rates of tooth-supported fixed dental prostheses (FDPs) and implant-supported FDPs and single crowns. A study about the design implications due to the fatigue of zirconium and dental bridge geometry is presented in [7]. [8] gives us a strength analysis of tree-unit dental bridge framework with the Finite Elements Method. In [9] we have a study of fatigue of zirconium under cyclic loading in water and its implications for the design of dental bridges. [10] gives us a comparison of using different bridge prosthetic designs for partial defect restoration through mathematical modelling. Calculus of strength of materials is presented in [11]. In [12] we have an application of the Transfer-Matrix Method to frame-shear wall systems Research about the buckling calculus of straight bars on elastic environment by TMM for dental implants is shown in [13]. A study of similarity for a dental bridge as a frame with a concentrated vertical load in the middle of horizontal part of frame by TMM is presented in [14]. An analytical calculus of dental bridge with distal extension and single pole tooth assimilated as a beam by TMM is given in [15]. Contributions on the analytical calculus of simple dental bridge assimilated with a beam embedded at both ends by TMM can be seen in [16]. A synthesis of formulas for stress and strain is given in [17] and in [18] is given the Transfer-Matrix Method for frame shear wall structures.

In this study, a frame model for a simple dental bridge will be presented, which will be calculate by TMM, to facilitate orthodontic interventions in situ, especially in delicate situations.

II. FRAME MODEL FOR A SIMPLE DENTAL BRIDGE

A dental bridge is a fixed prosthesis. In orthodontics, it is very often used, to compensate for missing one or more teeth. A simple dental bridge is a dental bridge with one missing tooth. Edentulous can be for a single tooth or for several teeth, singular or consecutive. With the help of dental bridges it can be done a morphological restoration of the missing teeth-partial edentulous, or it can be done a protection and re-modelling teeth that have suffered major damages. With dental bridges, it is possible to improve and compensate for the negative influences of partial or total edentulous on the functions of the masticatory apparatus (chewing function, aesthetic function, and phonetic function) of the missing teeth and to prevent complications that may occur because of tooth loss. Dental bridges have the role of compensating for mastication, aesthetic and phonetic functions of absent teeth and to prevent complications that may occur as a result of tooth loss. Dental bridge have: the aggregation elements, the bridge body, which replaces the missing teeth and the poles teeth, pole tooth can be an existing natural tooth or a dental implant. The aggregation elements are dental crowns which the bridge rests on the pole teeth, as in Fig. 1.

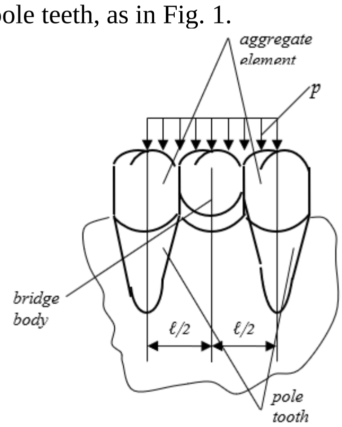

Fig. 1 A simple dental bridge with two poles and a bridge body with uniform load at the superior part

For the analytical calculus of a double embedded frame with TMM, some working hypotheses were introduced.

The frame model for a simple dental bridge was built like this: the two vertical parts of the frame can be assimilated with the two pole teeth and the bridge body together with the aggregation elements can be assimilated with the horizontal part of frame, as in Fig. 2.

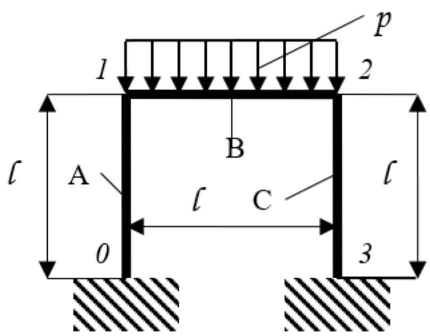

Fig. 2 Frame model for a simple dental bridge with uniform load at horizontal part of frame and embedded at two inferior ends of vertical parts.

The frame was embedded at both inferior ends of the two vertical parts, the embedding being assimilated to the poles embedded into the bone (the pole can be a natural tooth or a dental implant), as in Fig. 2.

The two vertical parts of the frame are considered to be along the symmetry axes of the two poles, having a length equal to the distance between the symmetry axes of three consecutive teeth, *l* (as in Fig 1 and Fig. 2).

The horizontal part is equal in length to the distance between three axes of symmetry for three consecutive teeth, i.e., *l*.

An uniformly distributed force is considered to act on the horizontal part of the frame, between the two axes of the poles.

III. TRANSFER-MATRIX METHOD APPLIED TO CALCULUS FOR A DOUBLE EMBEDDED FRAME WITH UNIFORM LOAD ON THE HORIZONTAL PART

An analytical calculus of a double embedded frame loaded with an uniformly distributed load on the horizontal part through TMM will be presented. Calculus of a frame by TMM is based of theory of Dirac's and Heaviside's functions and operators, [5].

For an uniformly distributed load, the charge density with Heaviside's functions, is (1):

$$p(x) = -p[Y(x) - Y(x-l)] \tag{1}$$

where *Y(x)* and *Y(x-l)* are the Heaviside's functions.

The following notations are introduced:

- with *A* and *C*, we note the vertical parts of frame (the poles);
- with *B* we note the horizontal part;
- the embedded supports are noted with *0* and *3*;
- the left section of the horizontal part of the frame is marked with *1*;
- the right section is noted with *2*, (Fig. 2.);
- *E* is the modulus of longitudinal elasticity (Young modulus);
- *A* is the area of frame transversal section;
- *I* is the moment of inertia;
- $\{D_0\}$, $\{D_1\}$, $\{D_2\}$ and $\{D_3\}$ are the vectors of the displacements in points *0*, *1*, *2* and *3*, in the following form (2), (3), (4) and (5):

$$\{D_0\} = \begin{Bmatrix} x_0 \\ y_0 \\ \omega_0 \end{Bmatrix} \tag{2}$$

$$\{D_1\} = \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} \tag{3}$$

$$\{D_2\} = \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} \tag{4}$$

$$\{D_3\} = \begin{Bmatrix} x_3 \\ y_3 \\ \omega_3 \end{Bmatrix} \tag{5}$$

The displacements of node *1* and node *2* of the frame must be calculated. For each of the three parts of the frame, after the general approach presented in [5], the following matrices and vectors can be written:
for:

$$p(x)=0 \tag{6}$$

On the part *A* and on the part *C*, we have:

$$\left[M_{00}^{A}\right]=\begin{bmatrix} \frac{12EI}{l^3} & 0 & \frac{6EI}{l^2} \\ 0 & -\frac{EA}{l} & 0 \\ \frac{6EI}{l^2} & 0 & -\frac{4EI}{l} \end{bmatrix}=\left[M_{00}^{C}\right] \tag{7}$$

$$\left[M_{01}^{A}\right]=\begin{bmatrix} \frac{12EI}{l^3} & 0 & \frac{6EI}{l^2} \\ 0 & \frac{EA}{l} & 0 \\ -\frac{6EI}{l^2} & 0 & -\frac{2EI}{l} \end{bmatrix}=\left[M_{01}^{C}\right] \tag{8}$$

$$\left[M_{10}^{A}\right]=\begin{bmatrix} \frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} \\ 0 & \frac{EA}{l} & 0 \\ \frac{6EI}{l^2} & 0 & -\frac{2EI}{l} \end{bmatrix}=\left[M_{10}^{C}\right] \tag{9}$$

$$\left[M_{11}^{A}\right]=\begin{bmatrix} -\frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} \\ 0 & -\frac{EA}{l} & 0 \\ -\frac{6EI}{l^2} & 0 & -\frac{4EI}{l} \end{bmatrix}=\left[M_{11}^{C}\right] \tag{10}$$

and the vectors for the part *A* and the part *C*:

$$\left\{V_{02}^{A}\right\}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}=\left\{V_{02}^{C}\right\} \tag{11}$$

$$\left\{V_{12}^{A}\right\}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}=\left\{V_{12}^{C}\right\} \tag{12}$$

$$\left\{V_{11}^{A}\right\}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}=\left\{V_{11}^{C}\right\} \tag{13}$$

for la part horizontal *B*, for:

$$p(x)=-p \tag{14}$$

we have:

$$\left[M_{00}^{B}\right]=\begin{bmatrix} -\frac{EA}{l} & 0 & 0 \\ 0 & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & -\frac{6EI}{l^2} & -\frac{4EI}{l} \end{bmatrix} \tag{15}$$

$$\left[M_{01}^{B}\right]=\begin{bmatrix} \frac{EA}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & -\frac{2EI}{l} \end{bmatrix} \tag{16}$$

$$\left[M_{10}^{B}\right]=\begin{bmatrix} \frac{EA}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & \frac{6EI}{l^2} \\ 0 & -\frac{6EI}{l^2} & -\frac{2EI}{l} \end{bmatrix} \tag{17}$$

$$\left[M_{11}^{B}\right]=\begin{bmatrix} -\frac{EA}{l} & 0 & 0 \\ 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & -\frac{4EI}{l} \end{bmatrix} \tag{18}$$

- and the vectors for la part *B*:

$$\left\{V_{02}^{B}\right\}=\begin{Bmatrix} 0 \\ -\frac{pl}{2} \\ -\frac{pl^2}{12} \end{Bmatrix} \tag{19}$$

$$\left\{V_{12}^{B}\right\}=\begin{Bmatrix} 0 \\ -\frac{pl}{2} \\ \frac{pl^2}{12} \end{Bmatrix} \tag{20}$$

For the embedded points *0* and *3*, the displacements are *0*, it is known that:

$$\{D_0\}=\{D_3\}=0 \tag{21}$$

or:

$$\{D_0\}=\begin{Bmatrix} x_0 \\ y_0 \\ \omega_0 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}=\{D_3\}=\begin{Bmatrix} x_3 \\ y_3 \\ \omega_3 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{22}$$

The displacements of points *1* and *2* must be calculated, is the form of vectors of displacement. For this, balance equations must be written in the two points *1* and *2*, as (23):

$$\begin{cases} \left[M_{11}^{A}\right]\cdot\{D_1\}+\left[M_{00}^{B}\right]\cdot\{D_1\}+ \\ +\left[M_{01}^{B}\right]\cdot\{D_2\}+\left\{V_{12}^{B}\right\}+\left\{V_{02}^{B}\right\}=0 \\ \left[M_{10}^{B}\right]\cdot\{D_1\}+\left[M_{11}^{B}\right]\cdot\{D_2\}+ \\ +\left[M_{11}^{C}\right]\cdot\{D_2\}+\left\{V_{12}^{B}\right\}=0 \end{cases} \tag{23}$$

with following observations (24):

$$\begin{cases} \left[M_{00}^{A}\right]=\left[M_{00}^{C}\right] \\ \left[M_{01}^{A}\right]=\left[M_{01}^{C}\right] \\ \left[M_{10}^{A}\right]=\left[M_{10}^{C}\right] \\ \left[M_{11}^{A}\right]=\left[M_{11}^{C}\right] \end{cases} \tag{24}$$

and (25):

$$\begin{cases} \{ V_{02}^{A} \} = \{ V_{02}^{C} \} \\ \{ V_{12}^{A} \} = \{ V_{12}^{C} \} \end{cases} \tag{25}$$

Supports *0* and *3* are embedded, the displacements within them are known as (22). The matrix equations (23) can be written as (26):

$$\begin{cases} \begin{bmatrix} -\frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} \\ 0 & -\frac{EA}{l} & 0 \\ -\frac{6EI}{l^2} & 0 & -\frac{4EI}{l} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} + \begin{bmatrix} -\frac{EA}{l} & 0 & 0 \\ 0 & -\frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & -\frac{6EI}{l^2} & -\frac{4EI}{l} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} + \\ + \begin{bmatrix} \frac{EA}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & -\frac{2EI}{l} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\frac{pl}{2} \\ +\frac{pl^2}{12} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\frac{pl}{2} \\ -\frac{pl^2}{12} \end{Bmatrix} = 0 \\ \begin{bmatrix} \frac{EA}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & -\frac{2EI}{l} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} + \begin{bmatrix} -\frac{EA}{l} & 0 & 0 \\ 0 & -\frac{12EI}{l^3} & \frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & -\frac{4EI}{l} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} + \\ + \begin{bmatrix} -\frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} \\ 0 & -\frac{EA}{l} & 0 \\ -\frac{6EI}{l^2} & 0 & -\frac{4EI}{l} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\frac{pl}{2} \\ \frac{pl^2}{12} \end{Bmatrix} = 0 \end{cases} \tag{26}$$

or (27):

$$\begin{cases} \begin{bmatrix} -\frac{12EI}{l^3} - \frac{EA}{l} & 0 & -\frac{6EI}{l^2} \\ 0 & -\frac{EA}{l} - \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ -\frac{6EI}{l^2} & -\frac{6EI}{l^2} & -\frac{8EI}{l} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} + \\ + \begin{bmatrix} \frac{EA}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & -\frac{2EI}{l} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{pl}{2} \\ 0 \end{Bmatrix} \\ \begin{bmatrix} \frac{EA}{l} & 0 & 0 \\ 0 & \frac{12EI}{l^3} & -\frac{6EI}{l^2} \\ 0 & \frac{6EI}{l^2} & -\frac{2EI}{l} \end{bmatrix} \begin{Bmatrix} x_1 \\ y_1 \\ \omega_1 \end{Bmatrix} + \\ + \begin{bmatrix} -\frac{EA}{l} - \frac{12EI}{l^3} & 0 & -\frac{6EI}{l^2} \\ 0 & -\frac{12EI}{l^3} - \frac{EA}{l} & \frac{6EI}{l^2} \\ -\frac{6EI}{l^2} & \frac{6EI}{l^2} & -\frac{8EI}{l} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \\ \omega_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{pl}{2} \\ -\frac{pl^2}{12} \end{Bmatrix} \end{cases} \tag{27}$$

(27) gives a linear system of six equations with six unknowns.

The unknowns are the six displacements, three for node *1* and three for node *2*. The linear system is (28):

$$\begin{cases} -\left(\frac{12EI}{l^3} + \frac{EA}{2l}\right)x_1 - \frac{6EI}{l^2}\omega_1 + \frac{EA}{l}x_2 = 0 \\ -\left(\frac{12EI}{l^3} + \frac{EA}{l}\right)y_1 - \frac{6EI}{l^2}\omega_1 + \frac{12EI}{l^3}y_2 - \frac{6EI}{l^2}\omega_2 = \frac{pl}{2} \\ -\frac{6EI}{l^2}x_1 - \frac{3EI}{2l^2}y_1 - \frac{8EI}{l}\omega_1 + \frac{6EI}{l^2}y_2 - \frac{2EI}{l}\omega_2 = 0 \\ -\frac{EA}{l}x_1 - \left(\frac{12EI}{l^3} + \frac{EA}{l}\right)x_2 - \frac{6EI}{l^2}\omega_2 = 0 \\ -\frac{12EI}{l^3}y_1 - \frac{6EI}{l^2}\omega_1 - \left(\frac{12EI}{l^3} + \frac{EA}{l}\right)y_2 + \frac{6EI}{l^2}\omega_2 = \frac{pl}{2} \\ \frac{6EI}{l^2}y_1 - \frac{2EI}{l}\omega_1 - \frac{6EI}{l^2}x_2 + \frac{6EI}{l^2}y_2 - \frac{8EI}{l}\omega_2 = -\frac{pl^2}{12} \end{cases} \tag{28}$$

that can be written in the general form (29):

$$\begin{cases} m_{11}x_1 + m_{12}y_1 + m_{13}\omega_1 + m_{14}x_2 + m_{15}y_2 + m_{16}\omega_2 = a_1 \\ m_{21}x_1 + m_{22}y_1 + m_{23}\omega_1 + m_{24}x_2 + m_{25}y_2 + m_{26}\omega_2 = a_2 \\ m_{31}x_1 + m_{32}y_1 + m_{33}\omega_1 + m_{34}x_2 + m_{35}y_2 + m_{36}\omega_2 = a_3 \\ m_{41}x_1 + m_{42}y_1 + m_{43}\omega_1 + m_{44}x_2 + m_{45}y_2 + m_{46}\omega_2 = a_4 \\ m_{51}x_1 + m_{52}y_1 + m_{53}\omega_1 + m_{54}x_2 + m_{55}y_2 + m_{56}\omega_2 = a_5 \\ m_{61}x_1 + m_{62}y_1 + m_{63}\omega_1 + m_{64}x_2 + m_{65}y_2 + m_{66}\omega_2 = a_6 \end{cases} \tag{29}$$

The determinant is (30):

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} & m_{26} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} & m_{36} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} & m_{46} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} & m_{56} \\ m_{61} & m_{62} & m_{63} & m_{64} & m_{65} & m_{66} \end{bmatrix} \tag{30}$$

or (31):

$$\begin{bmatrix} m_{11} & 0 & m_{13} & m_{14} & 0 & 0 \\ 0 & m_{22} & m_{23} & 0 & m_{25} & m_{26} \\ m_{31} & m_{32} & m_{33} & 0 & m_{35} & m_{36} \\ m_{41} & 0 & 0 & m_{44} & 0 & m_{46} \\ 0 & m_{52} & m_{53} & 0 & m_{55} & m_{56} \\ 0 & m_{62} & m_{63} & m_{64} & m_{65} & m_{66} \end{bmatrix} \tag{31}$$

The vector of free terms is (32):

$$\begin{Bmatrix} 0 \\ a_2 \\ 0 \\ 0 \\ a_5 \\ a_6 \end{Bmatrix} \tag{32}$$

Solving the linear system (28) or (29) allows finding all displacements in nodes *1* and *2*. TMM is very easy to program. This will allow to obtain results quickly and apply them in practice, in orthodontics, even in situ, in special situations, to obtain dental bridge of maximum resistance.

IV. CONCLUSIONS

In this work we present an original and interesting analytical calculus for a model of a frame for a simple dental bridge. The simple dental bridge has two poles assimilated with the two vertical parts of the frame, embedded at inferior ends and with an horizontal part assimilated with the two aggregation elements between which the body of the dental bridge is located. The horizontal part is loaded with a load uniformly distributed along its entire length using the TMM.

This original idea to assimilate a simple dental bridge as a frame, is an approach that can be applied for different frames and, so, for differently dental bridges, applied for different partial or total edentulous situations.

The mathematical algorithm for frame calculus can be very easy to program and this allows the results to be obtained very quickly with immediate application in practice, in orthodontics, which allows the results to be used in situ, especially in special and delicate situations. In the future, we want to be able to present the validation of the theoretical results with experimental results and with those obtained by modeling with Finite Elements Method (FEM).

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