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FIXED POINT THEOREM FOR SEQUENCE OF SELF MAPPING IN COMPLETER METRIC SPACE

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Abstract- ***In 1978 Kishorimohan Ghosh and S. K. Chatterjea [1] have investigated fixed point theorem in metric space for two self mapping. In the present paper a fixed point theorem for sequence of self mapping in the complete metric space has been proved.***

Keywords : Complete metric space, contraction, fixed point, self mapping.

I. INTRODUCTION

In 1974 M. Sen Gupta [2] has proved that in a complete metric space (M, d) if there exists two operators T_1 and T_2 mapping M into itself and satisfying the relation.

$d(T_1x, T_2x) \leq \alpha\, d(x, T_1 x) + \beta\, d(y, T_2 y) + \gamma\, d(x, y)$ _____(1.1)

Or

$d(T_1x, T_2y) \leq \alpha\, d(y, T_1 x) + \beta\, d(x, T_2y) + \gamma\, d(x, y)$ _____(1.2)

For x, y in M, where α, β, γ are non-negative real numbers and $\alpha + \beta + \gamma < 1$, then T_1 and T_2 have a unique common fixed point.

In 1978 Kinshorimohan Ghosh and S. K. Chatterjea [1] have investigated the following theorem.

Theorem : Let (x, d) be metric space. T_1 and T_2 be two self mapping for which there exists non-negative real numbers α_i (i = 1. 2…5) and $\sum_{i=1}^{5} \alpha_i < 1$ such that

$d(T_1x, T_2y) \leq \alpha_1 d(x, y) + \alpha_2 d(x, T_1 x) + \alpha_3 d(y, T_2y) + \alpha_4 d(x, T_2 y) +$
$\alpha_5 d(y, T_1 x)$

For all $x, y \in X$.

For any $x_0 \in X$, the sequence
$X_1 = T_2 X_0$, $X_2 = T_2 X_1$ ………
$X_2n = T_2 (X_{2n-1})$, $X_{2n+1} = T_1 (X_{2n})$ ……….
has a subsequence converging to $u \in x$ then T_1 and T_2 have a unique common fixed point u. ____________ (1.3)

In the present paper we have extended the above fixed point theorem for sequence of mapping in complete metric space.

Theorem : Let (x, d) be complete metric space, T_i and T_j be two sequences of self mapping for which there exists non-negative real numbers. α_i (I = 1, 2 ….. 5)

with $0 \leq \alpha_i < 1$ and $\sum_{i=1}^{5} \alpha_I < 1$ such that

$d(T_i x, T_j y) \leq \alpha_1 d(x, y) + \alpha_2 d(x, T_i x) + \alpha_3 d(y, T_j y) + \alpha_4 d(x, T_j y) +$

$\alpha_5 d(y, T_i x)$

For all $x, y \in X$

For any $x_0 \in x$ the sequence $X_1 = T_i X_0$, $X_2 = T_2X_1$ …………… $X_{2n} = T_2 (X_{2n-1})$
$X_{2n+1} = T_1 (x_{2n})$ ….. has a
Subsequence converging to unique common fixed point u. ____________ (1.4)

Proof of (1.4) :

We will prove the above theorem by considering the following three steps.

i) First we will show that $\{X_n\}$ is a Cauchy sequence.
ii) Existence of fixed point.
iii) Uniqueness of fixed point.

(i) Let X_0 be any point of X and consider the sequence
$X_1 = T_1 X_0$, $X_2 = T_2 X_1$, $X_3 = T_1 X_2$
$X_2n = T_2\ X_{2n-1}$, $X_{2n+1} = T_1 X_{2n}$

We have for $x, y \in X$

$d(T_i x, T_j y) \leq \alpha_1 d(x, y) + \alpha_2 d(x, T_i x) + \alpha_3 d(y, T_j y) + \alpha_4 d(x, T_j y) +$
$\alpha_5 d(y, T_i x)$ _________________ (A)

By interchanging x with y and T_i with T_j we get

$d(T_j y, T_i x) \leq \alpha_1 d(y, x) + \alpha_2 d(y, T_j y) + \alpha_3 d(x, T_i x) + \alpha_4 d(y, T_i x) +$
$\alpha_5 d(x, T_j y)$ _________________ (B)

Now adding (A) and (B) we have

$d(T_i x, T_j y) + d(T_j y, T_i x) \leq \alpha_1 d(x, y) + \alpha_1 d(y, x) + \alpha_2 d(x, T_i x) + \alpha_2 d(y, T_j y)$
$+ \alpha_3 d(y, T_j y) + \alpha_3 d(x, T_i x) + \alpha_4 d(x, T_j y) + \alpha_4 d(y, T_i x) +$
$+ \alpha_5 d(y, T_i x) + \alpha_5 d(x, T_j y)$

Now by symmetric property we have
$d(x, y) = d(y, x)$

$\therefore$ 2d $(T_i x, T_j y) \le 2\alpha_1 d(x, y) + (\alpha_2 + \alpha_3) d(x, T_i x) + (\alpha_3 + \alpha_2) d(y, T_j y) +$

$(\alpha_4 + \alpha_3) d(x, T_j y) + (\alpha_5 + \alpha_4) d(y, T_i x)$

$2d(T_i x, T_j y) \le 2\alpha_1 d(x, y) + (\alpha_2 + \alpha_3) \{d(x, T_i x) + d(y, T_j y)\} + (\alpha_4 + \alpha_5)$

$\{d(x, T_j y) + d(y, T_i x)\}$

$\therefore d(T_i x, T_j y) \le \alpha_1 d(x, y) + \left(\frac{\alpha_2 + \alpha_3}{2}\right)\{d(x, T_i x) + d(y, T_j, y)\} + \left(\frac{\alpha_4 + \alpha_5}{2}\right)$

$\{d(x, T_j y) + d(y, T_i x)\}$

Put $x = x_0$ and $y = x_1$ we have

$d(T_i x_0, T_j x_1) \le \alpha_1 d(x_0, x_1) + \left(\frac{\alpha_2 + \alpha_3}{2}\right)\{d(x_0, T_i x_0) + d(x_1, T_j x_1)\} + \left(\frac{\alpha_4 + \alpha_5}{2}\right)$

$d\{x_0, T_j x_1\} + d\{x_1, T_i x_0\}$

Now since $T_i x_0 = x_1$ and $T_j x_1 = x_2$ we have

$d(x_1, x_2) \le \alpha_1 d(x_0, x_1) + \left(\frac{\alpha_2 + \alpha_3}{2}\right)\{d(x_0, x_1) + d(x_1 x_2)\} + \left(\frac{\alpha_4 + \alpha_5}{2}\right)\{d(x_0, x_2) + d(x_1, x_1)\}$

Now since $d(x_1, x_1) = 0$ and $d(x_0, x_2) \le d(x_0, x_1) + d(x_1, x_2)$

We have

$d(x_1, x_2) \le \alpha_1 d(x_0, x_1) + \left(\frac{\alpha_2 + \alpha_3}{2}\right)\{d(x_0, x_1) + d(x_1 x_2)\} + \left(\frac{\alpha_4 + \alpha_5}{2}\right)$

$\{d(x_0, x_2) + d(x_1, x_2)\}$

$2d(x_1, x_2) - (\alpha_2 + \alpha_3) d(x_1, x_2) - (\alpha_4 + \alpha_5) d(x_1, x_2) \le$

$2\alpha_1 d(x_0, x_1) + (\alpha_2 + \alpha_3) d(x_0, x_1) + (\alpha_4 + \alpha_5) d(x_0, x_1)$

$\therefore 2 - (\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5) d(x_1, x_2) \le (2\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5) d(x_0, x_1)$

$$d(x_1, x_2) \le \frac{2\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5}{2 - (\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)} d(x_0, x_1)$$

$\therefore d(x_1, x_2) \le r\, d(x_0, x_1)$

Where $r = \frac{2\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5}{2 - (\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5)}$

Similarly we can show that

$d(x_2, x_3) \le r\, d(x_2, x_1)$

$\le r \cdot r\, d(x_0, x_1)$

$\therefore d(x_2, x_3) \le r^2 d(x_0, x_1)$

By induction we can prove that

$d(x_n, x_{n+1}) \le r^n d(x_{0+}x_1)$

Hence

$d(x_n, x_{n+p}) \le d(x_n, x_{n+1}) + d(x_{n+1}, x_{n+2}) + \ldots\ldots$

$\ldots\ldots + d(x_{n+p-1}, x_{n+p})$

$\therefore d(x_n, x_{n+p}) \le (r^n + r^{n+1} + \ldots\ldots\ldots\ldots\ldots + r^{n+p-1}) d(x_0, x_1)$

$\therefore d(x_n, x_{n+p}) \le \frac{r^n(1 - r^{n+p-1})}{(1-r)} d(x_0, x_1)$

$\to 0$ as $n \to \infty$ since $r < 1$

and owning to the assumption $\Sigma\, \alpha_i < 1$

$\therefore d(x_n, x_{n+p}) \to 0$ as $n \to \infty$

So we have $\{x_n\}$ is a Cauchy sequence since subsequence $\{Xn_k\}$ of this sequence $\{X_n\}$ converges to u

$\lim_{n \to \infty} X_n = u \in x$

ii) Now we will prove that u is fixed point of T_i and T_j i.e. we will prove that

$T_i u = u, \quad T_j u = u$

Now first Consider

$d(T_i u, u) \le d(T_i u, x_{2n}) + d(x_{2n}, u)$

$= d(T_i u, T_j x_{2n-1}) + d(x_{2n}, u)$

$d(T_i u, u) \le \alpha_1 d(u, x_{2n-1}) + \alpha_2 d(u, T_i u) + \alpha_3 d(X_{2n-1}, X_{2n}) + \alpha_4 d(u, X_{2n}) + \alpha_5 d(x_{2n-1}, T_i u) + d(x_{2n}, u)$

As $n \to \infty$ we have

$d(T_i u, u) \le (\alpha_2 + \alpha_5) d(u, T_i u)$

$\therefore \lim_{n \to \infty} X_n = u$

$\Rightarrow \lim_{n \to \infty} X_{2n-1} = u$ and $d(u, u) = 0$

$\therefore d(T_i u, u) \le (\alpha_2 + \alpha_5) d(T_i u, u)$

$\therefore d(T_i u, u) - (\alpha_2 + \alpha_5) d(T_i u, u) \le 0$

$\{1 - (\alpha_2 + \alpha_5)\} d(T_i u, u) \le 0$

Which is possible if $d(T_i u, u) = 0$

Since $1 - (\alpha_2 + \alpha_5) \ne 0$

Similarly we can prove that $T_j u = u$

$\Rightarrow T_i$ and T_j have common fixed point u

iii) Now consider the uniqueness of fixed point u. If possible let there be another

fixed point v of Ti and T_j

$\therefore T_i v = v$ and $T_j v = v$

Then $d(u, v) = d(T_i u, T_j v)$

$\therefore d(u, v) \le \alpha_1 d(u, v) + \alpha_2 d(u, T_i u) + \alpha_3 d(v, T_j v) + \alpha_4 d(u, T_j v) + \alpha_5 d(v, T_i u)$

$\therefore d(u, v) \le \alpha_1 d(u, v) + \alpha_2 d(u, u) + \alpha_3 d(v, v) + \alpha_4 d(u, v) + \alpha_5 d(u, v)$

$\therefore d(u, v) - \alpha_1 d(u, v) - \alpha_4 d(u, v) - \alpha_5 d(u, v) \le 0$

$\therefore d(u, v) = 0 = d(v, v)$

and $d(u, v) = d(v, u)$

$\Rightarrow [1 - (\alpha_1 + \alpha_4 + \alpha_5)] d(u, v) \le 0$

Which is possible if $d(u, v) = 0$

Since $\{1 - (\alpha_1 + \alpha_4 + \alpha_5)\} \ne 0$

$\therefore d(u, v) = 0$

$\Rightarrow v = u$

Hence T_i and T_j have unique common fixed point u.

Hence the theorem

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