

E₃. CORDIAL AND PRIME CORDIAL LABELING OF SOME WHEEL RELATED GRAPHS

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Abstract- A prime cordial labeling of a graph G is a bijection $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ and the induced edge function $f : E(G) \rightarrow \{0, 1\}$ is defined by $f(e=uv) = 1$ if $\gcd(f(u), f(v)) = 1$ and 0 if $\gcd(f(u), f(v)) \geq 2$. Further condition is satisfied that $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ are the number of edges with label 0 and 1 respectively. We show that a wheel graph W_{n+1} is prime cordial. A k -edge labeling of a graph G is a function $f : E(G) \rightarrow \{0, \dots, k-1\}$. Such a labeling induces a labeling on the vertex set $V(G)$ by defining $f(v) := \sum f(e) \pmod{k}$, where the summation is taken over all the edges incident on the vertex v . For an edge labeling f , let $v_f(i)$ (respectively $e_f(i)$) be the number of vertices (respectively edges) receiving the label i . A graph G is said to be E_k -cordial if there is a k -edge labeling f of G such that $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $0 \leq i, j \leq k-1$. A wheel W_{n+1} is the join of the cycle C_n on n vertices and K_1 . A Gear graph $G = G_{2n+1}$ is obtained from wheel graph W_{n+1} by adding a vertex on each edge of cycle C_n . We prove that Gear graph $G = G_{2n+1}$ for $n \geq 3$, and $G^{(k)}$ that is, one point union of k copies of G are E_3 -cordial.

Keywords - labeling, wheel, Gear graph, prime Mathematical subject classification : 05C78

I. INTRODUCTION

Throughout this work all graphs are finite, simple and undirected. Let $V(G)$ and $E(G)$ denote the vertex set and the edge set respectively, of a graph G . Sundaram [7] et al has introduced the notion of prime cordial labeling. A prime cordial labeling of a graph G is a bijection $f : V(G) \rightarrow \{1, 2, \dots, |V(G)|\}$ such that each edge $(e=uv)$ is assigned the label 1 if $\gcd(f(u), f(v)) = 1$ and 0 if $\gcd(f(u), f(v)) \geq 2$. Further condition is satisfied that $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ are the number of edges with label 0 and 1 respectively. The following graphs are proved to be prime cordial. C_n iff $n \geq 6$, P_n if $n \neq 3$ or 5 . $K_{1,n}$ (n odd); the graph obtained by subdividing each edge of $K_{1,n}$ iff $n \geq 3$; bistars, dragons, crowns, triangular snakes iff the snakes has at least ladders 3 triangles. etc. J. Bhaskar Babuji [6] has proved the full binary tree, $S_{n1} : S_{n2}$ for all $n > 2$ has prime cordial labeling. G.V. Ghodsara & J.P. Jena [5] proved that Cycle C_n with one chord, Cycle C_n with twin chords, Cycle C_n with triangle $C_n(1, 1, n-2)$ ($n \neq 7$) are prime cordial.

A k -edge labeling of a graph G is a function $f : E(G) \rightarrow \{0, \dots, k-1\}$. Such a labeling induces a labeling on the vertex set $V(G)$ by defining $f(v) := \sum f(e) \pmod{k}$, where the summation is taken over all the edges incident on the vertex v . For an edge labeling f , let $v_f(i)$ be the number of vertices receiving the label i . Similarly, let $e_f(i)$ be the number of edges of G receiving the label i . These are called the vertex numbers and the edge numbers for the labeling f .

In the year 2000, Cahit and Yilmaz introduced the concept of E_k -Cordial labeling of a graph G . A graph G is said to be E_k -cordial if there is a k -edge labeling f of G such that $|v_f(i) - v_f(j)| \leq 1$ and $|e_f(i) - e_f(j)| \leq 1$ for all $0 \leq i, j \leq k-1$. Such a map is called an E_k -cordial labeling of the graph G . If $k=3$

we get E_k cordial labeling. They proved that following graphs are E_3 -cordial: (1) Paths $P_n, n \geq 3$, (2) the

complete graph $K_n, n \geq 3$, (3) the cycle $C_n, n \geq 3$, (4) the friendship graph C_3^k . (4) the fan $F_n, n \geq 3$, where F_n is obtained by taking $n-3$ concurrent chords in a cycle C_n . (5) They also proved that

The work is partly supported by University Of Mumbai, INDIA the star $S_n = K_2^{(n)}$, $n \geq 2$, is E_k -cordial if and only if $n \equiv 1 \pmod{k}$, for odd k and $n \equiv 1 \pmod{2k}$ for $k \neq 2$, but k even.

Theorem 1 The wheel W_{n+1} is prime cordial for $n \geq 9$.

Proof: Define W_{n+1} as $V(W_{n+1}) = \{v_0, v_1, v_2, \dots, v_n\}$ and $E(W_{n+1}) = \{v_0v_i / i=1, 2, \dots, n\} \cup \{v_i v_{i+1} / i=1, 2, \dots, n\}$ here $i+1$ is taken modulo n . Thus W_{n+1} has $n+1$ vertices and $2n$ edges. We call the edges v_0v_i as p_i and edges $v_i v_{i+1}$ as c_i . Thus pokes P_i are n in number and c_i are the cycle edges n in number. The vertex v_0 is called as hub.

Case i: $n=9$.

$f(v_0)=2, f(v_1)=10, f(v_2)=5, f(v_3)=7, f(v_4)=1, f(v_5)=9, f(v_6)=6, f(v_7)=3, f(v_8)=4, f(v_9)=8$

The cycle edges which are 9 in number, receive the labels as $f(c_1=v_1v_2) = 0, f(c_2) = 1, f(c_3) = 1, f(c_4) = 1, f(c_5) = 0, f(c_6) = 0, f(c_7) = 1, f(c_8) = 0, f(c_9) = 0$. And 9 pokes receive the label as $f(p_1) = 0, f(p_2) = 1, f(p_3) = 1, f(p_4) = 1, f(p_5) = 1, f(p_6) = 0, f(p_7) = 1, f(p_8) = 0, f(p_9) = 0$. Thus $e_f(0, 1) = (9, 9)$.

Case ii: $n=10$. i.e. to show that W_{11} is prime cordial.

$f(v_0)=2, f(v_1)=7, f(v_2)=3, f(v_3)=9, f(v_4)=8, f(v_5)=4, f(v_6)=8, f(v_7)=10, f(v_8)=5, f(v_9)=1, f(v_{10})=11$

The cycle edges which are 10 in number, receive the labels as $f(c_1=v_1v_2) = 1, f(c_2) = 0, f(c_3) = 0, f(c_4) = 0$,

$f(c_5) = 0, f(c_6) = 0, f(c_7) = 0, f(c_8) = 1, f(c_9) = 1, f(c_{10}) = 1$ And 10 spokes receive the label as $f(p_1) = 1, f(p_2) = 1, f(p_3) = 0, f(p_4) = 0, f(p_5) = 0, f(p_6) = 0, f(p_7) = 1, f(p_8) = 1, f(p_9) = 1, f(p_{10}) = 1$

Caseiii $n = 9 + 2x, x = 1, 2, 3, \dots$

We introduce $2x$ new vertices on the cycle C_9 of W_{9+1} whose labeling is given above. Of which x new vertices are introduced between vertex v_3 and

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vertex v_4 as $v_{11}, v_{13}, v_{15}, \dots, v_{9+2x}$ and another x vertices between vertex v_8 and v_9 of cycle C_{9+2x} as $v_{10}, v_{12}, v_{14}, \dots, v_{8+2x}$. The labeling of all these vertices is given by $v_\alpha = \alpha$. The resultant edge numbers are $e_f(0, 1) = (9+x, 9+x)$

Thus the graph W_{n+1} is prime cordial for $n = 9 + 2x$.

Caseiv $n = 10 + 2x, x = 1, 2, 3, \dots$

We introduce $2x$ new vertices on the cycle C_{10} of W_{10+1} whose labeling is given above. We carry the same labeling of C_{10} as above. x vertices of new $2x$ vertices are introduced between v_8 and v_9 as $v_{11}, v_{13}, v_{15}, \dots, v_{9+2x}$ and other x vertices between v_4 and v_5 as $v_{12}, v_{14}, v_{16}, \dots, v_{10+2x}$ with vertex label as $f(v_\alpha) = \alpha$. Note that $e_f(0, 1) = (10+x, 10+x)$

That shows that W_{n+1} is prime cordial.

E_3 -cordiality of gear graphs.

Definition 1.1 A gear graph $G = G_{2n+1}, n \geq 3$ is defined as follows: $V(G) = \{v_0, \dots, v_{2n}\}, E(G) = \{c_i = v_0 v_{2i-1} / i = 1, \dots, n\} \cup \{e_i = v_i v_{i+1} / 1, 2, \dots, 2n\}$ Here $i+1$ is taken (modulo $2n$).

The vertex v_0 is called the hub of the gear graph G_{2n+1} . The edges $c_i, 1 \leq i \leq n$, are all the edges joining the hub to alternate vertices of the cycle $C_{2n} = (v_1, \dots, v_{2n}, v_1)$. They are called the spokes of G_{2n+1} . The edges $e_i, 1 \leq i \leq 2n$, are the edges on the cycle C_{2n} .

Clearly, $|V(G)| = 2n+1$ and $|E(G)| = 3n$.

Theorem 1.2 The graph $G = G_{2n+1}$ is E_3 -cordial for all natural numbers $n \geq 3$.

Proof: Case 1: $n = 3x / x = 1, 2, 3, \dots$

Define $f: E(G) \rightarrow \{0, 1, 2\}$ as follows:

The edges on the cycle C_{6x} are labeled as follows:

$$\begin{aligned} f(e_i) &= 0, \text{ for } i \equiv 1, 2 \pmod{6} \\ &= 1, \text{ for } i \equiv 3, 4 \pmod{6} \\ &= 2, \text{ for } i \equiv 5, 6 \pmod{6} \end{aligned}$$

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The $3x$ spokes are labeled as follows:

$$\begin{aligned} f(c_i) &= 0, \text{ for } i \equiv 1 \pmod{3} \\ &= 2, \text{ for } i \equiv 2 \pmod{3} \\ &= 1, \text{ for } i \equiv 3 \pmod{3} \end{aligned}$$

This induces vertex labeling as follows: $f(v_0) = 0$ and

$$\begin{aligned} f(v_i) &= 2, \text{ for } i \equiv 1, 4 \pmod{6} \\ &= 0, \text{ for } i \equiv 2, 3 \pmod{6} \\ &= 1, \text{ for } i \equiv 5, 6 \pmod{6} \end{aligned}$$

One can check that $v_f(0) = 2x+1, v_f(1) = v_f(2) = 2x$ and $e_f(0) = e_f(1) = e_f(2) = 3x$.

Hence f is an E_3 -cordial labeling.

Case 2: $n = 3x+4, x \geq 0$. In this case we give E_3 -cordial labeling as follows:

Type A : We extend the labeling f given in case 1 as follows: For $1 \leq i \leq 6x$ label the first consecutive $6x$ edges e_1, \dots, e_{6x} on the cycle C_{6x+8} and the corresponding $3x$ spokes as in Case 1. The remaining 8 edges on the cycle are labelled as follows:

$$\begin{aligned} f(e_{6x+1}) &= 1, f(e_{6x+2}) = 1, f(e_{6x+3}) = 0, f(e_{6x+4}) = 0, f(e_{6x+5}) \\ &= 2, f(e_{6x+6}) = 1, f(e_{6x+7}) = 1, f(e_{6x+8}) = 2. \end{aligned}$$

The remaining 4 spokes are labelled as follows: $f(c_{3x+1}) = f(c_{3x+4}) = 2; f(c_{3x+2}) = f(c_{3x+3}) = 0$.

One can check that the extra new vertices on the cycle C_{6x+8} receive

the labels as follows: $f(v_{6x+1}) = 2, f(v_{6x+2}) = 2, f(v_{6x+3}) = 1, f(v_{6x+4}) = 0, f(v_{6x+5}) = 2, f(v_{6x+6}) = 0, f(v_{6x+7}) = 1, f(v_{6x+8}) = 0$ and

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$f(v_0) = 1$, that is the hub receives the label 1.

This shows that the label numbers are $v_f(0) = v_f(1) = v_f(2) = 2x+3$ and $e_f(0) = e_f(1) = e_f(2) = 3x+4$. We note that for this Type A labeling $f(v_0) = 1$ and the labeling of type A is an E_3 -cordial labeling.

Case 3 : $n = 3x+5, x \geq 0$. Again we give use Type A labeling which is E_3 -cordial: We extend the same f in the case 1 as follows: For $1 \leq i \leq 6x$ label the first consecutive $6x$ edges e_1, \dots, e_{6x} on the cycle C_{6x+10} and the corresponding $3x$ spokes as in the case 1. The remaining 10 edges on cycle are labelled as follows:

$$\begin{aligned} f(e_{6x+1}) &= 0, f(e_{6x+2}) = 0, f(e_{6x+3}) = 0, f(e_{6x+4}) = 1, \\ f(e_{6x+5}) &= 1, f(e_{6x+6}) = 1, f(e_{6x+7}) = 2, f(e_{6x+8}) = 2, f(e_{6x+9}) = 0, \\ f(e_{6x+10}) &= 2. \end{aligned}$$

The last 5 spokes are labelled as follows : $f(c_{3x+1}) = 2, f(c_{3x+2}) = 1, f(c_{3x+3}) = 0, f(c_{3x+4}) = 2, f(c_{3x+5}) = 1$.

One can check that the labels of the additional 10 cycle vertices are $f(v_{6x+1}) = 1, f(v_{6x+2}) = 0, f(v_{6x+3}) = 1, f(v_{6x+4}) = 1, f(v_{6x+5}) = 2, f(v_{6x+6}) = 2, f(v_{6x+7}) = 2, f(v_{6x+8}) = 1, f(v_{6x+9}) = 0, f(v_{6x+10}) = 2$. Note that $f(v_0) = 0$, that is, the hub receives the label 0. This gives $v_f(0) = 2x+3, v_f(1) = v_f(2) = 2x+4$ and $e_f(0) = 3x+5, e_f(1) = e_f(2) = 3x+5$. Hence this is an E_3 -cordial labeling.

This shows that G_{2n+1} is E_3 -cordial for all $n \geq 3$.

Let $G = G_{2n+1}$. by the one point union G^k of k copies of G , we mean the graph obtained by taking k copies H_1, \dots, H_k of G and identifying their hubs. The common hub is still denoted by v_0 . For the graph H_α , the other vertices are denoted by $v_\alpha, 1, \dots, v_{\alpha, 2n}$. The edges likewise are denoted by $e_{\alpha, 1}, \dots, e_{\alpha, 2n}$ and $c_{\alpha, 1}, \dots, c_{\alpha, n}$.

Theorem 2: $G^{(k)}$ is E_3 -cordial where $G = G_{6x+1}$

Proof: We note that $|V(G^{(k)})| = 1+6kx$ and $|E(G^{(k)})| = 9xk$.

We define a labeling on $G^{(k)}$ as follows:

For $e = e_{\alpha, m} \in E(G^{(k)})$, then define $g(e_{\alpha, m}) = f(e_{\alpha, m}), 1 \leq m \leq k$. Similarly,

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for $e = c_{\alpha, m} \in E(G^{(k)})$, then define $g(c_{\alpha, m}) = f(c_{\alpha, m}), 1 \leq m \leq k$, where f is the labeling constructed in case 1 of Theorem 1.

It is now easy to see that $v_g(0) = 2xk + 1, v_g(1) = v_g(2) = 2xk$ and $e_g(i) = 3xk$,

$i = 0, 1, 2$. Hence g is an E_3 -cordial labeling.

Before proving the main theorem, we need to construct more labeling for $G = G_{2n+1}$ when $n = 3x + 4$ or $3x + 5$. Some of these are not E_3 -cordial labelings.

Alternate labeling for $G_{2n+1}, n = 3x + 4$ or $n = 3x + 5$.

Type B and Type C labelings for G_{6x+9} , that is, for $n = 3x + 4$.

Type B: For $1 \leq i \leq 6x$, label the first consecutive $6x$ edges e_1, \dots, e_{6x} on the cycle C_{6x+8} and corresponding $3x$ spokes as in Case 1 of Theorem 1. Corresponding $6x$ vertices on the cycle C_{6x+8} also receive the same labels as in Case 1 of Theorem 1. The remaining 8 edges of the cycle are labeled as:

$f(e_{6x+1}) = 1, f(e_{6x+2}) = 1, f(e_{6x+3}) = 0, f(e_{6x+4}) = 0, f(e_{6x+5}) = 2, f(e_{6x+6}) = 2,$
 $f(e_{6x+7}) = 1, f(e_{6x+8}) = 2.$ The remaining 4 spokes are labelled as: $f(c_{3x+1}) = 1,$

$f(c_{3x+2}) = f(c_{3x+3}) = 0, f(c_{3x+4}) = 2.$ The corresponding 8 vertices on C_{6x+8} receive the labels as follows: $f(v_{6x+1}) = 1, f(v_{6x+2}) = 2, f(v_{6x+3}) = 1, f(v_{6x+4}) = 0, f(v_{6x+5}) = 2, f(v_{6x+6}) = 1, f(v_{6x+7}) = 2, f(v_{6x+8}) = 0.$ Note that $f(v_0) = 0$, that is the hub receives the label 0. It is easy to see that $v_f(0) = v_f(1) = v_f(2) = 2x + 3$ and $e_f(0) = e_f(1) = e_f(2) = 3x + 4$. Hence, this is also an E_3 -cordial labeling with the hub having the label 0.

Type C: For $1 \leq i \leq 6x$ label the first consecutive $6x$ edges e_1, \dots, e_{6x} on the cycle C_{6x+8} and the corresponding $3x$ spokes as in Type B. The remaining 8 edges on the cycle C_{6x+8} are labelled as: $f(e_{6x+1}) = 0, f(e_{6x+2}) = 1, f(e_{6x+3}) = 1, f(e_{6x+4}) = 0, f(e_{6x+5}) = 2, f(e_{6x+6}) = 0, f(e_{6x+7}) = 1, f(e_{6x+8}) = 2.$ The last 4 spokes are labeled as follows: $f(c_{3x+1}) = 0, f(c_{3x+2}) = 1, f(c_{3x+3}) = 2, f(c_{3x+4}) = 2.$

The remaining 8 vertices receive the labels follows: $f(v_{6x+1}) = 2, f(v_{6x+2}) = 1, f(v_{6x+3}) = 0, f(v_{6x+4}) = 1, f(v_{6x+5}) = 1, f(v_{6x+6}) = 2, f(v_{6x+7}) = 0, f(v_{6x+8}) = 0.$ Note that $f(v_0) = 2$, that is the hub receives the label 2. This gives $v_f(0) = v_f(1) = v_f(2) = 2x + 3$ and $e_f(0) = e_f(1) = e_f(2) = 3x + 4$. This shows that this is also an E_3 -cordial labeling.

Remark: Type A, Type B and Type C labeling have the same label numbers and are all E_3 -cordial. The only difference is that in Type A the hub has the label 1, in Type B the hub has the label 0 and in Type C the hub has the label 2

Type B labeling for G_{6x+11} , that is for $n = 3x + 5$.

TYPE B: For $1 \leq i \leq 6x$ label the first consecutive $6x$ edges e_1, \dots, e_{6x} on C_{6x+10} and corresponding $3x$ spokes as in the case 3, Theorem 1. The labels of the vertices involved are the same as before. The remaining 10 edges on the cycle C_{6x+10} are labelled as: $f(e_{6x+1}) = 1, f(e_{6x+2}) = 1, f(e_{6x+3}) = 0, f(e_{6x+4}) = 0, f(e_{6x+5}) = 2, f(e_{6x+6}) = 1, f(e_{6x+7}) = 0, f(e_{6x+8}) = 0, f(e_{6x+9}) = 2, f(e_{6x+10}) = 2.$ The last 5 spokes are labelled as: $f(c_{3x+1}) = 1, f(c_{3x+2}) = 2, f(c_{3x+3}) = 2, f(c_{3x+4}) = 1, f(c_{3x+5}) = 0.$

The labels of the remaining 10 vertices on the cycle are f

$(v_{6x+1}) = 1, f(v_{6x+2}) = 2,$

$f(v_{6x+3}) = 0, f(v_{6x+4}) = 0, f(v_{6x+5}) = 1, f(v_{6x+6}) = 0, f(v_{6x+7}) = 2, f(v_{6x+8}) = 0, f(v_{6x+9}) = 2, f(v_{6x+10}) = 1.$ It is easy to see that $f(v_0)$

$= 0$. This gives $v_f(0) = 2x + 5, v_f(1) = v_f(2) = 2x + 3$ and $e_f(0) = e_f(1) = e_f(2) = 3x + 5$.

Thus this is not an E_3 -cordial labeling.

E_3 -cordiality of $G_{2n+1}^{(k)}$ for $n = 3x + 4, 3x + 5$.

For a labeling f by $v_f(0, 1, 2)$ we mean the triple $(v_f(0), v_f(1), v_f(2))$. A similar notation will be used for the edge numbers.

Theorem 3 : If $G = G_{6x+9}$ or $G = G_{6x+11}$, then $G^{(k)}, k = 1, 2, \dots$ is E_3 -cordial.

Proof: Case 1: For $G = G_{6x+9}$ the graph $G^{(k)}$ has $(6x+8)k+1$ vertices and $(9x+12)k$ edges. Let the copies of G be H_1, \dots, H_k . Assign the labeling of Type A to $H_{3t+1}, t = 1, 2, \dots$, assign the labeling of Type B to $H_{3t+2}, t = 1, 2, \dots$ and assign the labeling of Type C to $H_{3t}, t = 1, 2, \dots$. Call the resulting labeling g .

k	Label of the Hub	$V_g(0, 1, 2)$	$e_g(0, 1, 2)$
1	1	$(2x+3, 2x+3, 2x+3)$	$(3x+4, 3x+4, 3x+4)$
2	1	$(4x+5, 4x+6, 4x+6)$	$(6x+8, 6x+8, 6x+8)$
3	0	$(6x+9, 6x+8, 6x+8)$	$(9x+12, 9x+12, 9x+12)$
3p	0	$(p(6x+8)+1, p(6x+8), p(6x+8))$	$(p(9x+12), p(9x+12), p(9x+12))$
3p+1	1	$(p(6x+8), p(6x+8), p(6x+8)+2x+3, 2x+3, 2x+3)$	$(p(9x+12), p(9x+12), p(9x+12)+3x+4, 3x+4, 3x+4)$
3p+2	1	$(p(6x+8), p(6x+8), p(6x+8)+4x+5, 4x+6, 4x+6)$	$(p(9x+12), p(9x+12), p(9x+12)+6x+8, 6x+8, 6x+8)$

This table clearly shows that $G^{(k)}$ is E_3 -cordial for every natural number k .

Case 2 : $G = G_{6x+11}$. $G^{(k)}$ has $(6x+10)k+1$ vertices and $(9x+15)k$ edges. Let the copies of G be H_1, \dots, H_k . Assign the labeling of Type A to $H_{3t+1}, t = 1, 2, \dots$, assign the labeling of Type B to H_{3t+2} and to $H_{3t}, t = 1, 2, \dots$. Call the resulting labeling g . The following table gives the label numbers of the resulting labeling.

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k	Label of the Hub	$V_g(0,1,2)$	$e_g(0,1,2)$
1	0	$(2x+3, 2x+4, 2x+4)$	$(3x+5, 3x+5, 3x+5)$
2	0	$(4x+7, 4x+7, 4x+7)$	$(6x+10, 6x+10, 6x+10)$
3	0	$(6x+11, 6x+11, 6x+10)$	$(9x+15, 9x+15, 9x+15)$
3p	0	$(p(6x+10)+1, p(6x+10), p(6x+10))$	$(p(9x+15), p(9x+15), p(9x+15))$
3p+1	0	$(p(6x+10)+1, p(6x+10), p(6x+10))$ $+(2x+3, 2x+4, 2x+4)$	$(p(9x+15), p(9x+15), p(9x+15))$ $+(3x+5, 3x+5, 3x+5)$
3p+2	0	$(p(6x+10), p(6x+10), p(6x+10))$ $+(4x+7, 4x+7, 4x+7)$	$(p(9x+15), p(9x+15), p(9x+15))$ $+(6x+10, 6x+10, 6x+10)$

Again this table shows that $G^{(k)}$ is E_3 -cordial.

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