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SOME PROPERTIES OF CERTAIN SUBCLASS OF UNIVALENT FUNCTIONS USINGRUSCHEWEYGH DERIVATIVE.

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Abstract- ***For analytic functions f (z) normalized with f (0) = 0 and $f'(0) = 1$ in the open unit disk U ,a new class $D_k^*(\beta_1,\beta_2,\ \lambda)$ satisfying someconditions with some complex number β_1,β_2 and some real number λ isintroduced. In the present paper necessary and sufficient condition for f(z) is in the class $D_k^*(\beta_1,\beta_2,\ \lambda)$ is obtained. Also some properties of the same class are obtained.***
Keywords: Analytic and univalent functions; Ruscheweygh derivative; λ convexfunctions; Coefficients bounds; etc.

I. INTRODUCTION

Let A be the class of analytic functions defined on the open unit disk D: = {z C: |z| < 1} and normalized by the conditions f (0) = 0 and f ′ (0) = 1. A function f ∈ A has Taylor's series expansion of the form

$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$(1.1)

Let R(α)denote the subclass of A consisting of functions f (z) which satisfyRe f ′ (z) >α(z ∈ D) for some real α (0 ≤α<1) .A function f (z)∈R(α) is said to be close-to-convex of order α in U (cf.Goodman[1]) .We know that $R(\alpha_1)\subset R(\alpha_2)$ $0 \le \alpha_1 \le \alpha_2 < 1$and R(α) ⊂A by Noshiro-Warshawskitheorem(cf.Duren[2]).

Given two functions f, g∈ A, where f (z) is given by (1.1) and g(z) is given by

$g(z) = z + \sum_{n=2}^{\infty} b_n z^n$

The Hadamard product f and g, denoted by f*g and is defined by

$f*g(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n$

The Ruscheweygh derivative of order k is denoted by $D^k f$ and is defined as follows: If $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$, then

$D^k f(z) = \frac{z}{(1-z)^{k+1}} * f(z)$ where ∗ denotes the Hadamard product of two analytic functions .

We have,

$D^k f(z) = \frac{z}{(1-z)^{k+1}} * f(z) = z + \sum_{n=2}^{\infty} B_n(k) a_n z^n$, k > -1 , z∈ D ,

where

$B_n(k) = \frac{(k+1)(k+2)\dots(k+n-1)}{(n-1)!}$

$\therefore D^k(f)'(z) = 1 + \sum_{n=2}^{\infty} nB_n(k) a_n z^{n-1}$

$$\therefore D^k(f)'(z) - 1 = \sum_{n=2}^{\infty} nB_n(k) a_n z^{n-1}$$

Let $D_k^*(\beta_1,\beta_2,\ \lambda) = \left\{ f \in A : \left| \frac{(D^k f)'(z) - 1}{\beta_1 (D^k f)'(z) + \beta_2} \right| \le \lambda \right\}$(1.2)

where β_1 and β_2 are complex numbers and $D^k f$ representsRuscheweyh derivative of order k and some real number λ. For k $=0$ $(\beta_1,\beta_2,\ \lambda) = L_1 * (\beta_1,\beta_2,\ \lambda) = \left\{ f \in A : \left| \frac{f'(z)-1}{\beta_1 f'(z) + \beta_2} \right| \le \lambda \right\}$

D_0^* was studied by Xiao-Fei Li and An-Ping Wang [3].

Let T denote the subclass of A consisting of functions of the form $f(z) = z - \sum_{n=2}^{\infty} a_n z^n$ $(a_n \ge 0)$. (1.3)

Let $D_k^{**}(\beta_1,\beta_2,\ \lambda) = D_k^*(\beta_1,\beta_2,\ \lambda) \cap T$

2. Properties of the class $D_k^*(\beta_1,\beta_2,\ \lambda)$:

Theorem 2.1: A function f (z) defined by (1.1) is in the class $D_k^*(\beta_1,\beta_2,\ \lambda)$if

$\sum_{n=2}^{\infty} nB_n (1 + \lambda|\beta_1|)|a_n| \le \lambda|\beta_1 + \beta_2|$ (2.1)

Proof:Since

$$\left| \frac{(D^k f)'(z) - 1}{\beta_1 (D^k f)'(z) - \beta_2} \right| = \left| \frac{\sum_{n=2}^{\infty} nB_n(k) a_n z^{n-1}}{(\beta_1 + \beta_2) + \sum_{n=2}^{\infty} nB_n(k)\beta_1 a_n z^{n-1}} \right|$$

$$\le \frac{\sum_{n=2}^{\infty} nB_n(k)|a_n||z^{n-1}|}{|\beta_1 + \beta_2| - \sum_{n=2}^{\infty} nB_n(k)|\beta_1||a_n||z^{n-1}|} \le \frac{\sum_{n=2}^{\infty} n^2|a_n|}{|\beta_1 + \beta_2| - \sum_{n=2}^{\infty} nB_n(k)(k)|\beta_1||a_n|}$$

Therefore, if f (z) satisfies the inequality (2.1),

then$\sum_{n=2}^{\infty} nB_n(k)(k)|a_n| \le \frac{\lambda|\beta_1 + \beta_2|}{(1+\lambda|\beta_1|)}$,using this we have

$$\left|\frac{(D^kf)'(z)-1}{\beta_1(D^kf)'(z)+\beta_2}\right| \leq \frac{\sum_{n=2}^{\infty} nB_n(k)|a_n|}{|\beta_1+\beta_2|-\sum_{n=2}^{\infty} nB_n(k)|\beta_1||a_n|} \leq \frac{\frac{\lambda|\beta_1+\beta_2|}{(1+\lambda|\beta_1|)}}{|\beta_1+\beta_2|-\frac{\lambda|\beta_1+\beta_2|}{(1+\lambda|\beta_1|)}|\beta_1|}$$

$$= \frac{\lambda|\beta_1+\beta_2|}{(|\beta_1+\beta_2|)} = \lambda \Rightarrow f(z)\in D_1^*(\beta_1,\beta_2,\lambda)$$

Theorem 2.2: A function f (z) defined by (1.1) is in the class $D_k^{**}(\beta_1,\beta_2,\lambda)$ if and only if

$$\sum_{n=2}^{\infty} nB_n(k)\left(1+\lambda|\beta_1|\right)|a_n| \leq \lambda|\beta_1+\beta_2| \quad (2.1)$$

Proof : In view of theorem (2.1), we need only to prove that necessity.

If f(z) $D_k^{**}(\beta_1,\beta_2,\lambda)$then,

$$\left|\frac{(D^kf)'(z)-1}{\beta_1(D^kf)'(z)+\beta_2}\right| = \left|\frac{\sum_{n=2}^{\infty} nB_n(k)a_nz^{n-1}}{(\beta_1+\beta_2)-\sum_{n=2}^{\infty} nB_n(k)\beta_1a_nz^{n-1}}\right| \leq \lambda$$

Since $|Re\{z\}| \leq |z|$, we have

$$Re\left\{\frac{\sum_{n=2}^{\infty} nB_n(k)a_rz^{n-1}}{(\beta_1+\beta_2)-\sum_{n=2}^{\infty} nB_n(k)\beta_1a_nz^{n-1}}\right\} \leq \lambda$$

The above condition must hold for all values of z;$|z|$= r<1. Upon choosing the value of *z* to be real and let $z \to 1^-$, we get

$$\sum_{n=2}^{\infty} nB_n\left(1+\lambda|\beta_1|\right)|a_n| \leq \lambda|\beta_1+\beta_2|$$

Corollary 2.1: If f (z)$\in D_k^*(\beta_1,\beta_2,\lambda)$

then $|a_n| \leq \frac{\lambda|\beta_1+\beta_2|}{nB_n(k)(1+\lambda|\beta_1|)|a_n|}$(n = 2, 3, …).

Proof: By theorem (2.1) , if f (z)$\in D_k^*(\beta_1,\beta_2,\lambda)$

then $\sum_{n=2}^{\infty} nB_n\left(1+\lambda|\beta_1|\right)|a_n| \leq \lambda|\beta_1+\beta_2|$

$\Rightarrow |a_n| \leq \frac{\lambda|\beta_1+\beta_2|}{nB_n(k)(1+\lambda|\beta_1|)|a_n|}$(n = 2, 3, …).

Theorem:2.3 Let f(z) defined by (1.1) and g(z) defined by (1.2) be in the class $D_k^*(\beta_1,\beta_2,\lambda)$. Then the function h(z) = ξ f(z) + (1-ξ)g(z) = z +$\sum_{n=2}^{\infty} c_nz^n$,where $c_n = \xi a_n$+ (1-ξ)b_n , ($0 \leq \xi \leq 1$) belongs to the class $D_k^*(\beta_1,\beta_2,\lambda)$.

Proof: Since f(z) and g(z) $\in D_k^*(\beta_1,\beta_2,\lambda)$, we have

$$\sum_{n=2}^{\infty} nB_n\left(1+\lambda|\beta_1|\right)|a_n| \leq \lambda|\beta_1+\beta_2|$$

and$\sum_{n=2}^{\infty} nB_n(k)\left(1+\lambda|\beta_1|\right)|b_n| \leq \lambda|\beta_1+\beta_2|$

Clearly, h(z) = □ f(z) + (1-□)g(z) = z +$\sum_{n=2}^{\infty} c_nz^n$,

where$c_n =$ □a_n+(1-$\leq \sum_{n=2}^{\infty} nB_n\left(1+\lambda|\beta_1|\right)|a_n||$□$|$

$+ \leq \lambda|\beta_1+\beta_2|$□$+ \lambda|\beta_1+\beta_2|(1-$□$) = \lambda|\beta_1+\beta_2|$

Hence, $\sum_{n=2}^{\infty} nB_n(k)\left(1+\lambda|\beta_1|\right)|c_n| \leq \lambda|\beta_1+\beta_2|$.

∴h (z) $\in D_k^*(\beta_1,\beta_2,\lambda)$

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