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STABILITY ANALYSIS IN FUZZY CONTROL

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Abstract- ***In this paper different models of fuzzy control theory are introduced. The concept of stability of fuzzy control model is given and the stability analysis of some control models is discussed.***

***Key words*:** Fuzzy Control, Stability Analysis .

I. INTRODUCTION

The concept of fuzzy control theory was introduced by L.A.Zadeh in 1973.and then explored by Mamdani in 1975. In this paper we have discussed the elements of Static Fuzzy System, Dynamic Fuzzy System, TSK Fuzzy control theory and Discrete-Time Dynamic Fuzzy Systems.

This paper is organized as in section 2, preliminary is introduced, in section 3, TSK Fuzzy Control Theory is presented, in section 4, Discrete-Time Dynamic Fuzzy Systems and Stability Analysis of quality of milk control model is studied and in section 5, a conclusion is discussed.

2. PRELIMINARIES

Suppose an unknown system with a set of inputs $x_1,x_2,\ldots x_n$ and outputs $y_1,y_2,\ldots y_n$ which is either discrete-time series or continuous signals.

The mathematical description can be a mapping or a functional that relates the inputs to the outputs in the form

$$
\begin{aligned}
y_1 &= f_1(x_1,x_2,\ldots x_n),\\
&.\\
&. \qquad\qquad (1)\\
y_n &= f_m(x_1,x_2,\ldots x_n)
\end{aligned}
$$

or

The mathematical description can be a set of differential equations (assuming proper conditions) that relates in the form

$$
\begin{aligned}
y_1 &= f_1(x_1,x_2,\ldots x_n,\dot{x}_1,\dot{x}_2,\ldots \dot{x}_n),\\
&.\\
&. \qquad\qquad (2)\\
y_n &= f_m(x_1,x_2,\ldots x_n,\dot{x}_1,\dot{x}_2,\ldots \dot{x}_n)
\end{aligned}
$$

or

A logical linguistic statement, which can be quantified mathematically in the form

IF (input x_1) AND … AND (input x_n)
THEN (output y_1) AND … AND (output y_m) (3)

The result of the logical linguistic form **(3)** by using fuzzy logic and the mathematical functional model **(1)** is called a Static Fuzzy System and The result of the logical linguistic form **(3)** by using fuzzy logic and the differential equation model **(2)** is called a Dynamic Fuzzy System.

3. TSK FUZZY CONTROL THEORY

Fuzzy control theory known as Takagi- Sugeno- Kang fuzzy control theory was proposed in Takagi, Sugeno (1985). A typical fuzzy rule in TSK fuzzy control theory has the form

IF x_1 is X_1 and x_2 is X_2 and … and x_n is X_n THEN $y = f(x_1,x_2,\ldots,x_n)$

where $X = (x_1,x_2,\ldots,x_n)$ and y are linguistic variables, X_1, X_2, … X_n are fuzzy sets in the antecedent and $y = f(x_1,x_2,\ldots,x_n)$ is a polynomial in the input variable x, but can be appropriately describe the output of the process within the region specified by the antecedent of the rule.

If $f(x_1,x_2,\ldots,x_n)$ is a first order polynomial then the resulting fuzzy model is called first- order TSK fuzzy control.

If f is a constant then the resulting fuzzy model is called zero-order TSK fuzzy control.

The output of a TSK fuzzy control is obtained by the weighted average of the crisp output of fuzzy rules. Graphical interpretation of fuzzy reasoning for a TSK fuzzy control with two rules is given in the following Fig.(1)

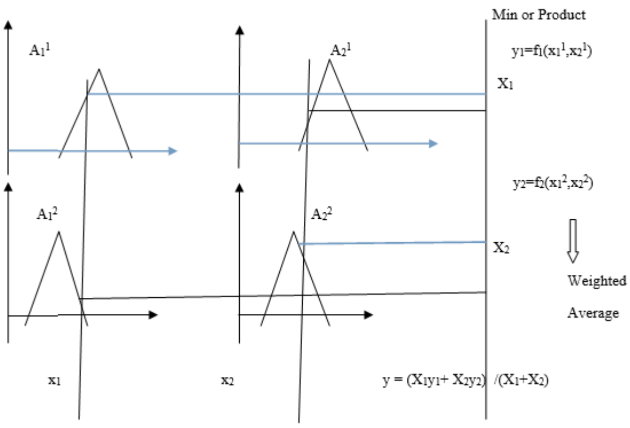


Fig.(1) TSK Fuzzy Inference Process

A finite fuzzy logic implication statements can be described by a set of general fuzzy IF-THEN rules containing only the fuzzy logic AND operation, in the following multi-input single-output form:

R^i : IF (x_1 is X_{i1}) AND …AND (x_n is X_{in})

THEN $y_i = a_{i0} + a_{i1} x_1 + \dots + a_{in} x_n$ i= 1,…,N

The corresponding membership values of y_i are given as $Y(yi) = \sup\{X_1(x_1) \wedge \dots \wedge X_n(x_n)\}$

The final single output y is obtained by weighted average formula as $y = \frac{\sum_{i=1}^{N} Y(yi)\, yi}{\sum_{i=1}^{N} Y(yi)}$ **(4)**

The constant parameters { a_{i0} a_{i1} , … , a_{in}}are determined by **Least Square Method**.

4. DISCRETE-TIME DYNAMIC FUZZY SYSTEMS AND STABILITY ANALYSIS

Definition 1 A typical Single-Input Single-Output (SISO),Discrete-time dynamic fuzzy system is a fuzzy model described by a set of fuzzy IF-THEN rules of the form

R^i : IF x(k) is X_{i1} AND …AND x(k-(n-1)) is X_{in}

THEN $y_i(k) = a_{i0} + a_{i1}\, x(k) + a_{i2}\, x(k-1) + \dots + a_{in}\, x_n(k-(n-1))$, i= 1,…,N

with $x(k+1)=c_k y(k+1)$, k=0,1,2,….,in which

$$y(k+1) = \frac{\sum_{i=1}^{N} wi\, .yi(k+1)}{\sum_{i=1}^{N} wi}, \quad k=0,1,2,\dots \quad \textbf{(5)}$$

Here the fuzzy sets consists of intervals $\{X_j/j=1,\dots,n\}$ with associated fuzzy membership functions $\{X_j(x(k))\}$and $\{w_i/i=1,2,\dots,N\}$is the set of weights satisfying $w_i \geq 0, i=1,2,\dots N$ and $\sum_{i=1}^{N} wi > 0$.

In this definition, in all the rules R^i , i=1,…,N, X_{ij} share the same fuzzy subset X_j and the same membership function X_j for each j=1,…n.

We also note that similar to formula **(5)**, the weights $\{w_i/i=1,2,\dots,N\}$are chosen to be equal to $Y_i(y_i)$ also for simplicity let $c_k = 1$ for all k= 0,1,2,…, and $a_{i0} = 0$ for all i= 1,…,N

We start with a given SISO, discrete-time, dynamic fuzzy system where the fuzzy sets (both intervals and membership functions) and constant coefficients are all known. We want to study under what conditions on those constant coefficients the dynamic fuzzy system is asymptotically stable.

Stability Criterion: The following are Stability Criterion for dynamic system

Theorem 1: Suppose that the MIMO, nonlinear, discrete-time, dynamic system $x(k+1)=f(x(k))$, $x(k) \in R^m$, k= 0,1,2,…has an equilibrium point xe = 0, and that there exists a scalar-valued function V(x(k)) satisfying

(i) V(0)= 0;

(ii) V(x(k)) > 0 for all x(k)≠0;

(iii) $V(x(k))\to\infty$ as $\| x(k) \|_2 \to \infty$; and

(iv) V(x(k+1)) - V(x(k)) < 0 for all x(k)≠0 and for all k= 0,1,2,….

Then this system is asymptotically stable about the equilibrium point 0.

The function V(x(k)) is called a Lyapunov function.

This theorem particularly applies to linear time-invariant state-space system, x(k+1) = Ax(k), $x(k) \in R^m$, k= 0,1,2,… However for linear time-invariant state-space system there is a simple criterion for the determination of asymptotic stability.

Theorem 2: Let λ_j ,j= 1,…,m be eigenvalues of the constant matrix A in the linear time-invariant state-space system, x(k+1) = Ax(k), $x(k) \in R^m$, k= 0,1,2,… The system is asymptotically stable about the equilibrium point 0 iff $|\lambda_j| < 1$ for all j=1,…,m

To apply stability theorem 1 and 2 we formulate SISO discrete-time, dynamic fuzzy system in state-space setting as follows

Let $c_k = 1$ for all k= 0,1,2,…, and $a_{i0} = 0$ for all i= 1,…,N in the system. Define

$x(k)=[x(k),x(k-1),x(k-2),\dots x(k-(n-1))]^T$,

$$A_i = \begin{pmatrix} a_{i1} & a_{i2} & \dots & a_{i,n-1} & a_{in} \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ . & & & & \\ . & & & & \\ . & & & & \\ 0 & 0 & \dots & 0 & 0 \end{pmatrix}$$

We can rewrite the dynamic fuzzy system as

$$x(k+1) = \frac{\sum_{i=1}^{N} wi\, .Aix(k)}{\sum_{i=1}^{N} wi}, \quad k=0,1,2,\dots \quad (6)$$

Corollary 1: In the dynamic system **(6),** let $A= \frac{\sum_{i=1}^{N} wi\, .Ai}{\sum_{i=1}^{N} wi}$ and assume that $\{w_i\}$ are constants independent of k. Then the system is asymptotically stable if and only if all eigen values of A, λ_i , i= 1,…n, satisfying $|\lambda_i| < 1$, i= 1,…,n.

Example 1 Consider the discrete-time, linear, dynamic fuzzy system described by

R^1: IF x(k-1) is X_a THEN $x_1(k+1)=x(k)-0.5x(k-1)$,

R^2: IF x(k-1) is X_b THEN $x_2(k+1)=x(k)-0.5x(k-1)$,

Where $X_a = X_b = [-1,1]$ with membership values $X_a(u) = \frac{1-u}{2}$, $-1 \leq u \leq 1$ and $X_b(v) = \frac{v+1}{2}$, $-1 \leq v \leq 1$and with initial conditions x(1)=0.90 and x(0)= -0.70

It can be easily verified that the two subsystem matrices $A_1=\begin{bmatrix} 1 & -0.5 \\ 1 & 0 \end{bmatrix}$, $A_2=\begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix}$ have all eigen values with absolute values strictly less than 1,so the two subsystems are asymptotically stable.

Example2. The Quality of Milk in dairy plant is studied by considering three input parameters with different linguistic variables. Here the Total Count(TC)(x_1), FAT(x_2) and Acidity(x_3) are taken as three input parameters with different linguistic variables as follows

Total Count (TC) ≅{ Low (**L**), Medium (**M**), High (**H**), Very High (**VH**)}

FAT ≅{ Low (**L**), Normal (**N**), High (**H**)}

Acidity ≅{ Low (**L**), Normal (**N**), High (**H**)}

These linguist variables can be represented by a fuzzy set as given below

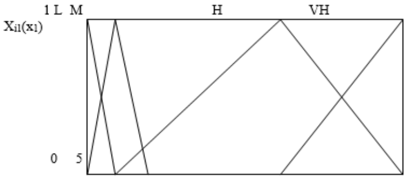
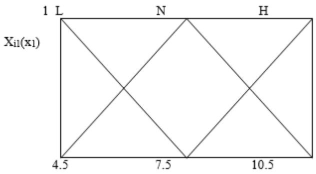


Fig.(2) Total Microorganism Count **(TC)**($\times 10^5$ spc/ml)



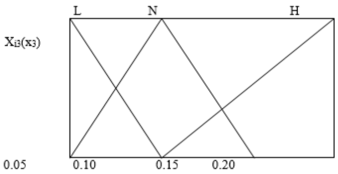


Fig.(3) FAT (%) **Fig.(4) Acidity (%)**

and Grade of Milk Quality(y) is taken as output parameter. The input-output relations (implications) of the quality of milk are taken in the form of fuzzy IF-THEN rules as

R^i : IF x_1^i is X_{i1} And x_2^i is X_{i2} And x_3^i is X_{i3} THEN $y^i = a_{i0} + a_{i1}x_1 + a_{i2}x_2 + a_{i3}\,x_3$ (7)

i= 1,2,…5, where a_{i0}, a_{i1}, a_{i2}, a_{i3} are coefficient parameters. The final output y for the inputs x_1, x_2 and x_3 is given by

$$y = \frac{\sum_1^5 (Xi1(x1j) \wedge Xi2(x2j) \wedge Xi3(x3j))\, yi}{\sum_1^5 Xi1(x1j) \wedge Xi2(x2j) \wedge Xi3(x3j)} \qquad (8)$$

Table 1.Suppose that three sets of Input-Output Data for the quality of milk are given below

Input Data			Output Data
TC (x_1^j)	FAT (x_2^j)	Acidity (x_3^j)	Grade of Milk Quality
LOW (0-5)	LOW(4.5-7.5)	LOW(0-0.10)	0.75
LOW (0-5)	NORMAL(6-9)	HIGH(0.10-0.20)	0.50
MEDIUM (0-10)	LOW(4.5-7.5)	NORMAL(0.075-0.125)	0.50
HIGH (5-50)	NORMAL(6-9)	HIGH(0.10-0.20)	0.50
VERYHIGH (30-50)	NORMAL(6-9)	LOW(0-0.10)	0.50

The coefficient parameters { a_{i0} , a_{i1},a_{i2}, a_{i3} a_{i3} } are determined by using the given data, and Least-Squares Method .

We have the following outputs

$y^1 = 0.0298 + 0.0744x_1+0.1547x_2+ 0.0223\,x_3$

$y^2 = 0.0188 - 0.0079x_1+0.0861x_2+ 0.0212\,x_3$

$y^3 = 0.4992 - 0.0000x_1-0.0002x_2- 0.0199\,x_3$

The three subsystem matrices are

$$A2 = \begin{pmatrix} -0.007 & 0.0861 & 0.0212 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$A3 = \begin{pmatrix} 0 & -0.0002 & 0.0199 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$A1 = \begin{pmatrix} 0.0744 & 0.1577 & 0.0223 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

have eigen values with absolute values strictly less than 1, so the three subsystems are asymptotically stable.

5. CONCLUSION

It is observed that some subsystems of the TSK Fuzzy Model for Quality of Milk are asymptotically stable by using the Stability Criterion. Some different Stability Criterion can be used for more effective control.

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