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# ON PROPERTIES OF FUZZY MEALY MACHINES

**S.A. Morye**

Department of Mathematics,Rajaram college, Kolhapur 416 004 Maharashtra, India Email: sam5631@rediffmail.com

**Abstract-** ***The purpose of this paper is to study fuzzy Mealy machines and their (output) subsystems. Apart from usual properties of subsystems of a fuzzy Mealy machine, we characterize them using a class of of fuzzy sets for fixed strings of input and output. Also a class of subsystems of a given fuzzy Mealy machines is obtained with the help of fuzzy points. Cyclic and super cyclic subsystems are also encountered and characterized.***

***Key words***: Subsystem, fuzzy Mealy machine, restricted product.

## I. PRELIMINARIES

In recent studies on fuzzy automaton, various extensions such as,general fuzzy automaton [5, 18], Intutionistic fuzzy automaton [4], Bipolar fuzzy automaton [9], fuzzy pushdown automaton [16, 4] etc are successfully studied. Apart from these extensions various properties of fuzzy finite state machines are extended to these extensions [1, 3, 7, 8, 10, 11, 12, 15]. In [10] subsystem of fuzzy finite state machine is introduced and various issues relating to them are discussed. Since many concepts of fuzzy finite state machine are introduced for fuzzy Mealy machine [2, 8, 15, 17]. It is natural to think about the extension for fuzzy Mealy machine. In [1] fuzzy Mealy and Moore machines are introduced and discussed comparatively. Recall that $X^*$ denote the set of all string of finite length over X, $\lambda$ denotes the empty string and |X| denotes the length of x. This section contains notions in Mealy-type fuzzy finite state machines that are introduced by Liu et al. [14] and Malik et al. [13]. We have also introduced homomorphisms of fuzzy Mealy machines. Few new results on coverings and homomorphisms of Mealy-type fuzzy finite state machines are also reported.

**Definition 1.1** *[13, 14] A fuzzy Mealy machine is a quintuple* $M = (Q,X,Y,\delta,\sigma)$, *where Q is a finite non-empty set called the set of states, X is a finite non-empty set called the set of inputs, Y is a finite non-empty set called the set of outputs,* $\delta$ *is a fuzzy subset of* $Q\times X\times Q$ *called the transition function,* $\sigma$ *is a fuzzy subset of* $Q\times X\times Y$ *called the output function and following condition is satisfied:*

$((\forall q\in Q),(\forall a\in X),(\exists p\in Q,\delta(q,a,p)>0)\Leftrightarrow((\exists b\in Y),\sigma(q,a,b)>0)$.

**Definition 1.2** *[13, 14] Let* $M = (Q,X,Y,\delta,\sigma)$, *be a fuzzy Mealy machine. Then*

*(i) define* $\delta^*: Q\times X^*\times Q\to[0,1]$ *as:* $\forall q,p\in Q,\forall a\in X,$ $\forall x\in X^*$

$$\delta^*(q,\lambda,p)=\begin{cases}1 & \text{if } q=p\\ 0 & \text{if } q\neq p\end{cases}$$

$\delta^*(q,xa,p)=\bigvee_{r\in Q}\{\delta^*(q,x,r)\wedge\delta(r,a,p)\}$ *and*

*(ii)define* $\sigma^*: Q\times X^*\times Y^*\to[0,1]$ *as* $\forall q\in Q,\forall a\in X,$ $\forall x\in X^*,\forall b\in Y,\forall y\in Y^*$ *and*

$$\sigma^*(q,x,y)=\begin{cases}1 & \text{if } x=y=\lambda\\ 0 & \text{if } (x=\lambda,y\neq\lambda)\text{ or }(y=\lambda,x\neq\lambda)\end{cases}$$

$$\sigma^*(q,xa,yb)=\bigvee_{r\in Q}\{\sigma^*(q,x,y)\wedge\delta^*(q,x,r)\wedge\sigma(r,a,b)\}$$
$$=\sigma^*(q,x,y)\wedge\bigvee_{r\in Q}\{\delta^*(q,x,r)\wedge\sigma(r,a,b)\}$$

**Definition 1.3** *[13, 14] Let* $M = (Q,X,Y,\delta,\sigma)$, *be a fuzzy Mealy machine. Then* $\forall q,p\in Q,\forall x,u\in X^*$

$$\delta^*(q,xu,p)=\bigvee_{r\in Q}\{\delta^*(q,x,r)\wedge\delta^*(r,u,p)\}$$

**Definition 1.4** *[13] Let* $M = (Q,X,Y,\delta,\sigma)$, *be a fuzzy Mealy machine, if* $|x|\neq|y|$, *then* $\sigma^*(q,x,y)=0,\forall q\in Q,\forall x\in X^*,$ $\forall y\in Y^*$.

## 2. FUZZY MEALY MACHINES AND HOMOMORPHISMS

In this section, we introduce and discuss various properties of fuzzy Mealy machine.

**Definition 2.1**Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine. Let $q, p \in Q$. Then $p$ is called an immediate successor of $q$, if $\exists \ a \in X$ and $b \in Y$ such that $\delta(q, a, p) \wedge \sigma(q, a, b) > 0$ and p is called successor of $q$, if $\exists \ x \in X^*$ and $y \in Y^*$ such that $\delta^*(q, x, p) \wedge \sigma^*(q, x, y) > 0$.

Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine and $q \in Q$. We shall denote $S(q)$ the set of all successor of $q$. If $T \subseteq Q$, then set of all successor of $T$, denoted by $S(T)$, is defined by the set $S(T) = \bigcup\{S(q) \mid q \in T\}$.

**Theorem 2.2** Let $M = (Q, X, Y, \delta, \sigma)$be a fuzzy Mealy machine. Define a relation $\sim$ on Q as $p \sim q$ if and only if $q$ is successor of $p$. Then $\sim$ is reflexive and transitive.

Clearly $\sim$ is not symmetric.

**Theorem 2.3** Let $M = (Q, X, Y, \delta, \sigma)$be a fuzzy Mealy machine. Let $A, B \subseteq Q$

(1) if $A \subseteq B$ then $S(A) \subseteq S(B)$

(2) $A \subseteq S(A)$

(3) $S(S(A)) = S(A)$

(4) $S(A \cup B) = S(A) \cup S(B)$

(5) $S(A \cap B) \subseteq S(A) \cap S(B)$

**Proof** The proofs of (1), (2), (4) and (5) are straightforward.

(3) By (2) we have $S(A) \subseteq S(S(A))$. Let $q \in S(S(A))$. Then $q \in S(p)$, for some $p \in S(A)$. Thus $p \in S(r)$, for some $r \in A$. Now, $q$ is successor of $p$ and $p$ is successor of $r$, hence by Theorem (2.2), $q$ is successor of $r$. Thus $q \in S(r) \subseteq S(A)$. Hence, $S(S(A)) \subseteq S(A)$.

**Definition 2.4** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine. Let $T \subseteq Q$. Let $\delta'$ and $\sigma'$ be fuzzy subset of $Q \times X \times Q$ and $Q \times X \times Y$ respectively and let $N = (T, X, Y, \delta', \sigma')$. Then $N$ is called a submachine of $M$, if (1) $\delta' = \delta|_{T \times X \times T}$ and $\sigma' = \sigma|_{T \times Y}$ and (2) $S(T) \subseteq T$.

Clearly, if K is a submachine of $N$ and $N$ is a submachine of $M$, then K is a submachine of $M$.

**Definition 2.5** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine. Then $M$ is called strongly connected, if $p \in S(q), \ \forall \ p, q \in Q$.

**Definition 2.6** Let $M_1 = (Q_1, X_1, Y_1, \delta_1, \sigma_1)$ and $M_2 = (Q_2, X_2, Y_2, \delta_2, \sigma_2)$ be a fuzzy Mealy Machines. A triplet $(f, g, h)$ of mappings, $f : Q_1 \longrightarrow Q_2$, $g : X_1 \longrightarrow X_2$ and $h : Y_1 \longrightarrow Y_2$, is called fuzzy Mealy machine homomorphism from $M_1$ to $M_2$, denoted by $(f, g, h) : M_1 \longrightarrow M_2$, if (i) $\delta_1(q_1, x_1, p_1) \leq \delta_2(f(q_1), g(x_1), f(p_1))$ (ii) $\sigma_1^*(q_1, x_1, y_1) \leq \sigma_2^*(f(q_1), g(x_1), h(y_1)), \quad \forall \ q_1, \ p_1 \in Q_1, x_1 \in X_1^*$ and $y_1 \in Y_1^*$. Fuzzy Mealy machine homomorphism $(f, g, h)$ is called strong homomorphism, if $\delta_2(f(q), \ g(x), f(p)) = \delta_1(q, x, p)$ and $\sigma_2^*(f(q), g(x), h(y)) = \sigma_1^*(q, x, y), \quad \forall \ p, q \in Q_1, x \in X_1^*, y \in Y_1^*$.

**Remark 2.7**In above definition (2.6), if $X_1 = X_2$, $Y_1 = Y_2$ and $g, h$ are identity maps, then we simply write $f : M_1 \longrightarrow M_2$ and say that $f$ is a homomorphism or strong homomorphism accordingly.

**Theorem 2.8** Let $(f, g, h) : M_1 \longrightarrow M_2$ be a fuzzy Mealy machine homomorphism. Then

(1) if $p$ is a successor of $q$ in $M_1$, then $f(p)$ is a successor of $f(q)$ in $M_2$.

(2) $S(f(q)) = f(S(q)), \ \forall \ q \in Q_1$, if $(f, g, h)$ is strong.

**Proof** The proof of (1) is straightforward.

(2) $f(p) \in f(S(q)) \Leftrightarrow p \in S(q) \Leftrightarrow \delta_1^*(q, x, p) \wedge \sigma_1^*(q, x, y) > 0$ $\Leftrightarrow \delta_1^*(q, x, p) > 0$ and $\sigma_1^*(q, x, y) > 0 \Leftrightarrow \delta_2^*(f(q), g(x), f(p)) > 0$ and $\sigma_2^*(f(q), g(x), h(y)) > 0 \Leftrightarrow \delta_2^* \ (f(q), g(x), f(p)) \wedge \sigma_2^*(f(q), g(x), h(y)) > 0 \Leftrightarrow f(p) \in S(f(q))$.

**Theorem 2.9**Let $M_1 = (Q_1, X_1, Y_1, \delta_1, \sigma_1)$ and $M_2 = (Q_2, X_2, Y_2, \delta_2, \sigma_2)$ be a fuzzy Mealy Machines and let $(f, g, h) : M_1 \longrightarrow M_2$ be onto homomorphism. If $M_1$ is strongly connected, then $M_2$ is strongly connected.

**Proof** Let $q_2, q_2' \in Q_2$. Then $\exists \quad q_1, q_1' \in Q_1$ such that $f(q_1) = q_2$ and $f(q_1') = q_2'$. Since $M_1$ is strongly connected, we have $q_1 \in S(q_1')$. Then $f(q_1) \in f(S(q_1'))$. By Theorem **??**(2) $f(q_1) \in S(f(q_1'))$, that is $q_2 \in S(q_2')$. Hence, $M_2$ is strongly connected.

## 3. FUZZY SUBSYSTEMS OF FUZZY MEALY MACHINES

In this section the concept of fuzzy subsystem of fuzzy Mealy machine is introduced. Its characterization will be discussed through a fuzzy set defined for fixed strings of input and output. For a fixed state and an element of [0,1] a particular class of fuzzy subsystems will be obtained. Towards the end of the section notions of cyclic and super cyclic fuzzy subsystems will be discussed.

**Definition 3.1** Let $M = (Q, X, Y, \delta, \sigma)$ be a Fuzzy Mealy Machines. Let $\mu$ be a fuzzy subset of $Q$. Then $\mu$ is called a fuzzy subsystem of $M$, if $\mu(q) \geq \mu(p) \wedge \delta(p, a, q) \wedge \sigma(p, a, b),\ \forall\, q, p \in Q, a \in X$ and $b \in Y$.

If $(Q, X, Y, \delta, \sigma, \mu)$ is a fuzzy subsystem of $M$, then we shall write $\mu$ for $(Q, X, Y, \delta, \sigma, \mu)$.

**Theorem 3.2** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine. Then $\mu$ is a fuzzy subsystem of $M$ if and only if $\mu(q) \geq \mu(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y),\ \forall\, q, p \in Q, x \in X^*, y \in Y^*$.

**Proof** Suppose $\mu$ is a fuzzy subsystem of $M$. Let $q, p \in Q, x \in X^*$ and $y \in Y^*$. We prove the theorem by mathematical induction on $|x| = |y| = n$.

If $n = 0$, then $x = y = \lambda$. Now if $q = p$, then $\mu(p) \wedge \delta^*(q, \lambda, q) \wedge \sigma^*(q, \lambda, \lambda) = \mu(q)$. If $q \neq p$, then $\mu(p) \wedge \delta^*(p, \lambda, q) \wedge \sigma^*(p, \lambda, \lambda) = 0 \leq \mu(q)$. Thus, the theorem is true for $n = 0$.

Assume that the theorem is true for all $u \in X^*$ and $v \in Y^*$ such that $|u| = |v| = n - 1, n > 1$. Let $x = au$ and $y = bv$ where $a \in X, b \in Y$ and $|u| = |v| = n - 1$.

Then $\mu(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(q, x, y) = \mu(p) \wedge \delta^*(p, au, q) \wedge \sigma^*(q, au, bv) = \mu(p) \wedge \{\bigvee_{r \in Q} [\delta(p, a, r) \wedge \delta^*(r, u, q)] \wedge [\delta(p, a, r) \wedge \sigma(r, a, b) \wedge \sigma^*(r, u, v)]\} = \mu(p) \wedge \{\bigvee_{r \in Q} [\delta(p, a, r) \wedge \delta^*(r, u, q)] \wedge [\sigma^*(p, a, b) \wedge \sigma^*(r, u, v)]\} = \{\bigvee_{r \in Q} [\mu(p) \wedge \delta(p, a, r) \wedge \sigma^*(p, a, b)] \wedge [\delta^*(r, u, q) \wedge \sigma^*(r, u, v)]\} \leq \bigvee_{r \in Q} \{\mu(r) \wedge \delta^*(r, u, q) \wedge \sigma^*(r, u, v)\} \leq \mu(q)$. Hence, $\mu(q) \geq \mu(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y)$. The converse is trivial.

The following theorem gives a class of constant fuzzy subsystems for $M$.

**Theorem 3.3** Every constant fuzzy set $\mu$ on $Q$ determines a fuzzy subsystem of $M$.

**Proof** Suppose $\mu$ is constant fuzzy set of $Q$. Then for any $p, q \in Q$, we have $\mu(p) = \mu(q)$. Then for any $a \in X$ and $b \in Y$, clearly $\mu(q) = \mu(p) \geq \mu(p) \wedge \delta(q, a, p) \wedge \sigma(q, a, b)$. Therefore, $\mu$ is a fuzzy subsystem of $M$.

**Theorem 3.4** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine. Let $\mu_1$ and $\mu_2$ be fuzzy subsystems of $M$. Then

(1) $\mu_1 \cap \mu_2$ is a fuzzy subsystem of $M$ and

(2) $\mu_1 \cup \mu_2$ is a fuzzy subsystem of $M$.

**Proof** Since $\mu_1$ and $\mu_2$ are fuzzy subsystem of $M$, for $p, q \in Q, x \in X^*, y \in Y^*$ we have $\mu_1(q) \geq \mu_1(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y)$ and $\mu_2(q) \geq \mu_2(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y)$

1. Therefore, $(\mu_1 \cap \mu_2)(q) = \mu_1(q) \wedge \mu_2(q) \geq (\mu_1(p) \wedge \mu_2(p)) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y)$. Hence, $(\mu_1 \cap \mu_2)$ is a fuzzy subsystem.

2. Therefore, $(\mu_1 \cup \mu_2)(q) = \mu_1(q) \vee \mu_2(q) \geq (\mu_1(p) \vee \mu_2(p)) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y)$. Hence, $(\mu_1 \cup \mu_2)$ is a fuzzy subsystem.

The following example show that the complement of a fuzzy subsystem is not always a fuzzy subsystem.

**Example 3.5** Let $Q = \{p, q\}, X = \{a\}, Y = \{b\}, \delta(r, a, s) = \frac{1}{3}\ \ \forall\ r, s \in Q, \sigma(r, a, b) = \frac{1}{2}\ \forall\ r \in Q$. Let $\mu(q) = \frac{4}{5}$ and $\mu(p) = \frac{1}{2}$. Then $\mu(q) \geq \mu(p) \wedge \delta(p, a, q) \wedge \sigma(p, a, b)$ and $\mu(p) \geq \mu(q) \wedge \delta(q, a, p) \wedge \sigma(q, a, b)$. Then, $\mu$ is a fuzzy subsystem, but $\mu^c = 1 - \mu$ is not.

**Theorem 3.6** Let $M_1 = (Q_1, X_1, Y_1, \delta_1, \sigma_1)$ and $M_2 = (Q_2, X_2, Y_2, \delta_2, \sigma_2)$ be fuzzy Mealy machines. Let $(f, g, h) : M_1 \longrightarrow M_2$ be onto strong homomorphism. If $\mu$ is a fuzzy subsystem of $M_1$, then $f(\mu)$ is a fuzzy subsystem of $M_2$.

**Proof** Let $p_2, q_2 \in Q_2$ and $x_2 \in X_2^*, y_2 \in Y_2^*$. Since $f$ is onto, there exist $p_1, q_1 \in Q_1$ be such that $f(p_1) = p_2$ and $f(q_1) = q_2$. Also, $g$ and $h$ are onto, therefore there exists $x_1 \in X_1^*$ and $y_1 \in Y_1^*$ such that $g(x_1) = x_2$ and $h(y_1) = y_2$. Suppose also that there is $r_1 \in Q_1$ be such that $f(r_1) = p_2$. Then, $\delta_1^*(p_1, x_1, q_1) = \delta_2^*(f(p_1), g(x_1), f(q_1)) = \delta_2^*(f(r_1), g(x_1), f(q_1)) = \delta_1^*(r_1, x_1, q_1)$. Similarly $\sigma_1^*(p_1, x_1, y_1) = \sigma_1^*(r_1, x_1, y_1)$.

Now, $f(\mu)(p_2) \wedge \delta_2^*(p_2, x_2, q_2) \wedge \sigma_2^*(p_2, x_2, y_2)$

$= \bigvee\{\mu(r_1) | f(r_1) = p_2\} \wedge \delta_2^*(p_2, x_2, q_2) \wedge \sigma_2^*(p_2, x_2, y_2)$

$= \bigvee\{\mu(r_1) \wedge \delta_2^*(p_2, x_2, q_2) \wedge \sigma_2^*(p_2, x_2, y_2) | f(r_1) = p_2\}$

$= \bigvee\{\mu(r_1) \wedge \delta_2^*(f(p_1), g(x_1), f(q_1)) \wedge \sigma_2^*(f(p_1), g(x_1), h(y_1)) | f(r_1) = p_2\}$

$= \bigvee\{\mu(r_1) \wedge \delta_1^*(p_1, x_1, q_1) \wedge \sigma_1^*(p_1, x_1, y_1) | f(r_1) = p_2\}$

$= \bigvee\{\mu(r_1) \wedge \delta_1^*(r_1, x_1, q_1) \wedge \sigma_1^*(r_1, x_1, y_1) | f(r_1) = p_2\}$

$\leq \bigvee\{\mu(q_1) | f(r_1) = p_2)\}$, since $\mu$ is fuzzy subsystem of $M_1$

$\leq \bigvee\{f(\mu)(q_2) | f(r_1) = p_2)\}$.

$= f(\mu)(q_2)$.

Therefore, $f(\mu)$ is a fuzzy subsystem of $M_2$

The following example show that the ontoness is necessary for the above theorem

**Example 3.7** Let $Q_1 = \{p, q\}, Q_2 = \{r, s\}, X = \{a\}, Y = \{b\}, \delta_1(q, a, q) = \delta_1(p, a, p) = \delta_1(p, a, q) = \delta_1(q, a, p) = 1, \sigma_1(t, a, b) = \frac{1}{2}\ \forall\ t \in Q$. and $\delta_2(r, a, s) = \frac{1}{4}, \delta_2(s, a, r) = \frac{1}{7}, \delta_2(r, a, r) = 1 = \delta_2(s, a, s) = 1, \sigma_2(r, a, b) = \frac{1}{2}, \sigma_2(s, a, b) = \frac{1}{8}$. Let $f : Q_1 \longrightarrow Q_2$ defined by $f(q) = f(p) = r$. Then $f$ is not onto. Clearly, $f$ is strong homomorphism. Let $\mu_1$ be a fuzzy subset of $Q_1$ such that $\mu_1(p) = \frac{1}{2}, \mu_1(q) = \frac{2}{3}$. Then $\mu_1$ is fuzzy subsystem of $M_1$, but $f(\mu)$ is not a fuzzy subsystem of $M_2$.

**Theorem 3.8** Let $(f, g, h) : M_1 \longrightarrow M_2$ be a strong homomorphism. If $\mu$ is the fuzzy subsystem of $M_2$. Then $f^{-1}(\mu)$ is a fuzzy subsystem of $M_1$.

**Proof** Let $M_1 = (Q_1, X_1, Y_1, \delta_1, \sigma_1)$ and $M_2 = (Q_2, X_2, Y_2, \delta_2, \sigma_2)$ be fuzzy Mealy machines. Let $p_1, q_1 \in Q_1$ and $x_1 \in X_1^*, y_1 \in Y_1^*$. Then $f(p_1), f(q_1) \in Q_2, g(x_1) \in X_2^*, h(y_1) \in Y_2^*$. Now since $\mu$ is fuzzy subsystem of $M_2$, we have, $\mu(f(p_1)) \geq \mu(f(q_1)) \wedge \delta_2(f(q_1), g(x_1), f(p_1)) \wedge \sigma_2(f(q_1), g(x_1), h(y_1))$. Thus, $\mu(f(p_1)) \geq \mu(f(q_1)) \wedge \delta(q_1, x_1, p_1) \wedge \sigma_1(q_1, x_1, y_1)$. That is, $f^{-1}(\mu)(p_1) \geq f^{-1}(\mu)(q_1) \wedge \delta(q_1, x_1, p_1) \wedge \sigma_1(q_1, x_1, y_1)$. Therefore, $f^{-1}(\mu)$ is a fuzzy subsystem of $M_1$.

**Theorem 3.9** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine and $\mu$ be a fuzzy set of $Q$. Then

(1) if $\mu$ is fuzzy subsystem of $M$, then $N = (Supp(\mu), X, Y, \delta', \sigma')$ is a submachine of $M$, where $\delta' = \delta|_{Supp(\mu) \times X \times Supp(\mu)}$ and $\sigma' = \sigma|_{Supp(\mu) \times Y}$.

(2) $if N_t = (\mu_t, X, Y, \delta_t, \sigma_t)$ is a submachine of $M$, where, $\mu_t = \{q \in Q | \mu(q) \geq t\}$, $\delta_t = \delta|_{\mu_t \times X \times \mu_t}$, and $\sigma_t = \sigma|_{\mu_t \times Y}$, $t \in [0, 1]$, then $\mu$ is a fuzzy subsystem of $M$.

**Proof** 1. Let $p \in S(Supp(\mu))$. Then $p \in S(q)$, for some $q \in Supp(\mu)$. Then $\mu(q) > 0$. Since $p \in S(q)$, $\exists \ x \in X^*, y \in Y^*$ such that $\delta^*(q, x, p) \wedge \sigma^*(q, x, y) > 0$. $\mu$ is fuzzy subsystem, we have $\mu(p) \geq \mu(q) \wedge \delta^*(q, x, p) \wedge \sigma^*(q, x, y) > 0$ Thus, $p \in Supp(\mu)$. Therefore $S(Supp(\mu)) \subseteq Supp(\mu)$. Hence, $N$ is a submachine of $M$.

2. Let $q, p \in Q, x \in X^*, y \in Y^*$. If $\mu(p) = 0$ or $\delta^*(q, x, p) = 0$ or $\sigma^*(q, x, y) = 0$ then $\mu(q) \geq 0 = \mu(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y)$. Suppose, $\mu(p) > 0$, $\delta^*(p, x, q) > 0, \sigma^*(p, x, y) > 0$ and let $\mu(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y) = t$. Then $p \in \mu_t$. Since $N_t$ is submachine of $M$, we have $S(\mu_t) = \mu_t$. Now, $q \in S(p)$ and $S(p) \subseteq S(\mu_t)$ as $p \in \mu_t$. As $S(\mu_t) = \mu_t$, we have $q \in \mu_t$. Hence, $\mu(q) \geq t = \mu(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y)$. Thus, $\mu$ is fuzzy subsystem.

The following example show that a fuzzy subsystem of $M$ need not be a submachine of $M$

**Example 3.10** Let $Q, X, Y, \delta, \sigma$ be defined in Example **??**. Let $\mu(q) = \frac{4}{5}$ and $\mu(p) = \frac{1}{2}$. Then $\mu$ is a fuzzy subsystem. Let $t = \frac{2}{3}$. Let $N_t = (\mu_t, X, Y, \delta_t, \sigma_t)$. Now $\mu(q) \geq t$. Thus, $q \in \mu_t$. Also $\delta(q, a, p) = \frac{1}{3} > 0$ and $\sigma(q, a, b) = \frac{1}{2} > 0$. Thus, $\delta(q, a, p) \wedge \sigma(q, a, b) > 0$. Therefore, $p \in S(q)$. Thus $p \in S(\mu_t)$. But $\mu(p) = \frac{1}{2} < t$. Thus, $p \notin \mu_t$. Hence, $N_t$ is not a submachine of $M$.

We now define a fuzzy subset $\mu$ of $Q$ to characterize it as a fuzzy subsystem for fixed input and output strings as follows:

Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine and $\mu$ be a fuzzy subset of $Q$. For $x \in X^*, y \in Y^*$ define a fuzzy subset $(\mu xy)$ of $Q$ by $(\mu xy)(q) = \bigvee_{p \in Q} \{\mu(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y)\}$, $\forall \ q \in Q$.

**Theorem 3.11**Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine and let $\mu$ be a fuzzy subset of $Q$. Then $\mu$ is a fuzzy subsystem of $M$ if and only if $\mu xy \subseteq \mu, \quad \forall \ x \in X^*, y \in Y^*$.

**Proof** Let $\mu$ be a fuzzy subsystem of $M$. Let $x \in X^*, y \in Y^*, q \in Q$. Then $(\mu xy)(q) = \bigvee_{p \in Q} \{\mu(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y)\} \leq \mu(q)$. Hence, $\mu(xy) \subseteq \mu$.

Conversely, let $q \in Q$ and $x \in X^*, y \in Y^*$. Then $\mu(q) \geq (\mu xy)(q) = \bigvee_{p \in Q} \{\mu(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y)\} \geq \mu(p) \wedge \delta^*(p, x, q) \wedge \sigma^*(p, x, y), \quad \forall \ p \in Q$. Hence, $\mu$ is a fuzzy subsystem of $M$.

**Theorem 3.12** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine. Then for all fuzzy subset $\mu$ of $Q$, $(\mu xy)uv = (\mu xu)yv, \ \forall \ u, x \in X^*, v, y \in Y^*$

**Proof** Let $\mu$ be a fuzzy finite subset of $Q$ and let $x, u \in X^*$ and $y, v \in Y^*$. We use induction on $|u| = |v| = n$ to prove the theorem.

case(i) If $n = 0$, then $u = v = \lambda$. Let $q \in Q$. Then $(\mu xy)\lambda\lambda(q) = \bigvee_{p \in Q} \{(\mu xy)(p) \wedge \delta(p, \lambda, q) \wedge \sigma^*(p, \lambda, \lambda)\} = (\mu xy)(q)$. Hence, $\mu xy\lambda\lambda = (\mu xy) = (\mu x\lambda)y\lambda$.

case(ii) Suppose, that the theorem is true for all $u \in X^*, v \in Y^*$ such that $|u| = |v| = n-1, n > 1$ and for all $\mu$. Let $u' = au \in X^*$ where $a \in X, u' \in X^*$ and $v' = bv \in Y^*$ where $b \in Y, v \in Y^*$ and $|u| = |v| = n - 1$. Let $q \in Q$. Then,

$(\mu xu')yv'(q) = (\mu xau)ybv(q) = (\mu(xa)u)(yb)v(q) = \bigvee_{r \in Q} \{(\mu xayb)(r) \wedge \delta^*(r, u, q) \wedge \sigma^*(r, u, v) \} = \bigvee_{r \in Q} \{ \bigvee_{p \in Q} \{(\mu xy)(p) \wedge \delta(p, a, r) \wedge \sigma^*(p, a, b)\} \wedge \delta^*(r, u, q) \wedge \sigma^*(r, u, v)\} = \bigvee_{p \in Q} \{(\mu xy)(p) \wedge \{ \bigvee_{r \in Q} \{\delta(p, a, r) \wedge \delta^*(r, u, q)\} \wedge \{\sigma^*(p, a, b) \wedge \{\delta(p, a, r) \wedge \sigma^*(r, u, v)\}\}\}\}$

$= \bigvee_{p \in Q} \{(\mu xy)(p) \wedge \delta^*(p, au, q) \wedge \sigma^*(p, au, bv) =$

$\bigvee_{p\in Q}\{(\mu(xy))(p)\wedge\delta^*(p,u',q)\wedge\sigma^*(p,u',v')=(\mu xy)u'v'(q)$.
Hence, $(\mu xu')yv'=(\mu xy)u'v'$.

Our aim is now to use the characterization Theorem (3.11) to find a particular class of fuzzy subsystems of $M$, we begin with classes of fuzzy sets

**Definition 3.13** Let $M=(Q,X,Y,\delta,\sigma)$ be a fuzzy Mealy machine and $\mu$ be a fuzzy subset of $Q$. Define fuzzy subsets $\mu XY$ and $\mu X^*Y^*$ of $Q$ by

$(\mu XY)(p)=\bigvee_{a\in X,b\in Y,r\in Q}\{\mu(r)\wedge\delta(r,a,p)\wedge\sigma(r,a,b)\}\quad\forall\, p\in Q$

and

$(\mu X^*Y^*)(p)=\bigvee_{u\in X^*,v\in Y^*,r\in Q}\{\mu(r)\wedge\delta^*(r,u,p)\wedge\sigma^*(r,u,v),\}\quad\forall\, p\in Q$.

Note that

(1) $(\mu XY)\subseteq(\mu X^*Y^*)$

(2) $(\mu XY)=0$ and $(\mu X^*Y^*)=0$ if there exists $r\in Q$ such that $\mu(r)=0$, and

(3) $(\mu xy)\subseteq(\mu X^*Y^*)\ \ \forall\ x\in X^*,y\in Y^*$.

**Theorem 3.14** Let $M=(Q,X,Y,\delta,\sigma)$ be a fuzzy Mealy machine $t\in[0,1]$ and $q\in Q$. Then $(q_tXY)(p)=\bigvee_{a\in X,b\in Y}\{t\wedge\delta(q,a,p)\wedge\sigma(q,a,b)\},\quad\forall\ p\in Q$ and $(q_tX^*Y^*)(p)=\bigvee_{u\in X^*,v\in Y^*}\{t\wedge\delta^*(q,u,p)\wedge\sigma^*(q,u,v)\}\ \forall\, p\in Q$.

One can note that for arbitrary fuzzy subset of $Q$, $\mu X^*Y^*$ is not necessarily a fuzzy subsystem of $M$, but for $\mu=q_t$ for any $q\in Q$ and $t\in(0,1]$, $(q_tX^*Y^*)$ is a fuzzy subsystem of $M$. Thus we have following theorem

**Theorem 3.15** Let $M=(Q,X,Y,\delta,\sigma)$ be a fuzzy Mealy machine. Let $t\in(0,1]$ and $q\in Q$. Then the following hold

(1) $q_tX^*Y^*$ is a fuzzy subsystem of $M$

(2) $Supp(q_tX^*Y^*)=S(q)$

**Proof** 1. Let $x\in X^*$ and $y\in Y^*$. Then for any $r\in Q$, we have

$((q_tX^*Y^*)(xy))(r)=\bigvee_{p\in Q}\{(q_tX^*Y^*)(p)\wedge\delta^*(p,x,r)\wedge\sigma^*(p,x,y)\}=$

$=\bigvee_{p\in Q}\{\bigvee_{u\in X^*,v\in Y^*}\{t\wedge\delta^*(q,u,p)\wedge\sigma^*(q,u,v)\}\wedge\delta^*(p,x,r)\wedge\sigma^*(p,x,y)\}$

$=\bigvee_{p\in Q,u\in X^*,v\in Y^*}\{t\wedge\delta^*(q,u,p)\wedge\sigma^*(q,u,v)\wedge\delta^*(p,x,r)\wedge\sigma^*(p,x,y)\}$

$=\bigvee_{p\in Q,u\in X^*,v\in Y^*}\{t\wedge\{\delta^*(q,u,p)\wedge\delta^*(p,x,r)\}\wedge\{\sigma^*(q,u,v)\wedge\{\delta^*(q,u,p)\wedge\sigma^*(p,x,y)\}\}$

$=\bigvee_{u\in X^*,v\in Y^*}\{t\wedge\delta^*(q,ux,r)\wedge\sigma^*(q,ux,vy)\}$

$\le\bigvee_{u'\in X^*,v'\in Y^*}\{t\wedge\delta^*(q,u',r)\wedge\sigma^*(q,u',v')\}$

$\le(q_tX^*Y^*)(r)$.

Thus, $((q_tX^*Y^*)(xy))\subseteq(q_tX^*Y^*)$. Hence, $(q_tX^*Y^*)$ is a fuzzy subsystem of $M$, by Theorem(3.11).

2. $p\in S(q)\Leftrightarrow\ \exists\ x\in X^*,y\in Y^*$ such that $\delta^*(q,x,p)\wedge\sigma^*(q,x,y)>0\Leftrightarrow\bigvee_{x\in X^*,y\in Y^*}\{t\wedge\delta^*(q,x,p)\wedge\sigma^*(q,x,y)\}>0\Leftrightarrow(q_tX^*Y^*)(p)>0\Leftrightarrow p\in Supp(q_tX^*Y^*)$.

**Theorem 3.16** Let $M=(Q,X,Y,\delta,\sigma)$ be a fuzzy Mealy machine. Let $\mu$ be a fuzzy subset of $Q$ and $q\in Q$. Then the following are equivalent

(1) $\mu$ is a fuzzy subsystem of $M$

(2) $q_tX^*Y^*\subseteq\mu,\ \forall\, t\in[0,1]$ such that $t\le\mu(q)$

(3) $q_tXY\subseteq\mu,\ \forall\, q_t\subseteq\mu,\ \forall\, t\in[0,1]$ such that $t\le\mu(q)$

**Proof** 1. $\Rightarrow$ 2. Let $q\in Q,t\in[0,1]$ such that $t\le\mu(q)$. Then for $p\in Q$, we have

$(q_tX^*Y^*)(p)=\bigvee_{u\in X^*,v\in Y^*}\{t\wedge\delta^*(q,u,p)\wedge\sigma^*(q,u,v)\}\le\bigvee_{u\in X^*,v\in Y^*}\{\mu(q)\wedge\delta^*(q,u,p)\wedge\sigma^*(q,u,v)\}\le\mu(p)$, since $\mu$ is fuzzy subsystem. Hence, $q_tX^*Y^*\subseteq\mu$.

2. $\Rightarrow$ 3. Clear, due to $q_tXY\subseteq q_tX^*Y^*$.

3. $\Rightarrow$ 1. let $p,q\in Q$ and $a\in X,b\in Y$. If $\mu(q)=0$ or $\delta(q,a,p)=0$ or $\sigma(q,a,b)=0$ then $\mu(p)\ge0=\mu(p)\wedge\delta(q,a,p)\wedge\sigma(q,a,b)$. Suppose $\mu(q)\neq0$ and $\delta(q,a,p)\neq0$ and $\sigma(q,a,b)\neq0$. Let $\mu(q)=t$. Thus, by the hypothesis, $q_tXY\subseteq\mu$. Then $\mu(p)\ge(q_tXY)(p)=\bigvee_{u\in X,v\in Y}\{t\wedge\delta(q,u,p)\wedge\sigma(q,a,v)\}\ge t\wedge\delta(q,a,p)\wedge\sigma(q,a,b)=\mu(q)\wedge\delta^*(q,a,p)\wedge\sigma(q,a,b)$. Hence, $\mu$ is a fuzzy subsystem of $M$.

**Corollary 3.17** Let $M=(Q,X,Y,\delta,\sigma)$ be a fuzzy Mealy machine and $\mu$ be a fuzzy subsystem of $M$. Then for any $q\in Q$, we have

(1) $\mu\supseteq q_{\mu(q)}XY$. and

(2) $\mu\supseteq q_{\mu(q)}X^*Y^*$.

**Definition 3.18** Let $M=(Q,X,Y,\delta,\sigma)$ be a fuzzy Mealy machine and $\mu$ be a fuzzy subsystem of $M$. Then $\mu$ is called cyclic if $\exists\ q\in Q,t\in(0,1]$ with $t\le\mu(q)$ such that $\mu\le q_tX^*Y^*$. In this case we call $q_t$ a generator of $\mu$.

The Theorem (3.16) enable to characterize cyclic fuzzy subsystems as:

**Theorem 3.19** Let $M=(Q,X,Y,\delta,\sigma)$ be a fuzzy Mealy machine. and $\mu$ be a fuzzy subsystem of $M$. Then $\mu$ is cyclic if and only if $\exists\ q \in Q$ and $t \in (0,1]$ such that $\mu = q_t X^* Y^*$, whenever $t \leq \mu(q)$.

**Theorem 3.20** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine. Suppose the fuzzy subsystem $\mu$ of $M$ is cyclic with generator $q_t$, $q \in Q$ and $t \in (0,1]$. Then

(1) $\mu(q) = t$,

(2) $\mu(q) \geq \mu(p),\ \forall\, p \in Q$

(3) for any fuzzy subsystem $\mu'$ of $M$ such that $\mu' \subseteq \mu$, if $\mu'(q) \geq \mu'(r),\ \forall\, r \in Q$, we have $\mu' = q_{\mu'(q)} X^* Y^*$.

**Proof** 1. Since $\mu = q_t X^* Y^*$, we have $\mu(q) = (q_t X^* Y^*)(q) = \bigvee_{x \in X^*, y \in Y^*} \{t \wedge \delta^*(q,x,q) \wedge \sigma^*(q,x,y)\} = t \wedge (\bigvee_{x \in X^*, y \in Y^*} \{\delta^*(q,x,q) \wedge \sigma^*(q,x,y)\}) = t \wedge 1 = t$.
2. Let $p \in Q$. Since $\mu = q_t X^* Y^*$, we have $\mu(p) = (q_t X^* Y^*)(p) = \bigvee_{x \in X^*, y \in Y^*} \{t \wedge \delta^*(q,x,p) \wedge \sigma^*(q,x,y)\} = \bigvee_{x \in X^*, y \in Y^*} \{\mu(q) \wedge \delta^*(q,x,p) \wedge \sigma^*(q,x,y)\} = \mu(q) \wedge (\bigvee_{x \in X^*, y \in Y^*} \{\delta^*(q,x,p) \wedge \sigma^*(q,x,y)\}) \leq \mu(q)$
3. Let $p \in Q$. Since $\mu' \subseteq \mu$ we have $\mu'(p) \leq \mu(p)$. Then $\mu'(p) = \mu'(p) \wedge \mu(p)$. Also since $\mu = q_t X^* Y^*, \mu(p) = (q_t X^* Y^*)(p) = \bigvee_{x \in X^*, y \in Y^*} \{t \wedge \delta^*(q,x,p) \wedge \sigma^*(q,x,y)\} = \bigvee_{x \in X, y \in Y} \{\mu(q) \wedge \delta^*(q,x,p) \wedge \sigma^*(q,x,y)\}$. Hence, $\mu'(p) = \mu'(p) \wedge \mu(p) = \bigvee_{x \in X^*, y \in Y^*} \{\mu'(p) \wedge \mu(q) \wedge \delta^*(q,x,p) \wedge \sigma^*(q,x,y)\} =$
$\bigvee_{x \in X^*, y \in Y^*} \{\mu'(p) \wedge \delta^*(q,x,p) \wedge \sigma^*(q,x,y)\}$, since $\mu'(p) \leq \mu'(q) \leq \mu(q) \leq \bigvee_{x \in X^*, y \in Y^*} \{\mu'(q) \wedge \delta^*(q,x,p) \wedge \sigma^*(q,x,y)\} = (q_{\mu'(q)} X^*\ Y^*)(p)$. Hence $\mu' \subseteq q_{\mu'(q)} X^* Y^*$. Thus, $\mu' = q_{\mu'(q)} X^* Y^*$, by above corollary.

**Definition 3.21** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine and $\mu$ a fuzzy subsystem of $M$. Then $\mu$ is called super cyclic, if $q_{\mu(q)}$ is its generator $\forall\, q \in Q$.

**Theorem 3.22** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine and $\mu$ a fuzzy subsystem of $M$. Then $\mu$ is called super cyclic if and only if $\mu = q_{\mu(q)} X^* Y^*, \quad \forall\, q \in Q$.

**Theorem 3.23** If $\mu$ is super cyclic, then $\mu$ is constant.

**Proof** Since $\mu$ is super cyclic, for any $p \in Q$ we have $\mu = p_{\mu(p)} X^* Y^*$. Also, we have $\mu(p) \geq \mu(r),\ \forall\ r \in Q$. This implies that $\mu(p) = \mu(r),\ \forall\ p, r \in Q$. Therefore, $\mu$ is constant.

**Corollary 3.24** Every super cyclic fuzzy subsystem of a fuzzy Mealy machine $M$ is cyclic.

The following example show that a constant fuzzy subsystem $\mu$ of $M$ need not be (super) cyclic fuzzy subsystem.

**Example 3.25** Let $Q = \{p, q\}, X = \{a\}, Y = \{b\}, \delta(q,a,q) = \delta(p,a,p) = \frac{1}{2}, \delta(p,a,q) = \delta(q,a,p) = \frac{1}{3}, \sigma(r,a,b) = 1 \quad \forall\ r \in Q$. Let $\mu(q) = \mu(p) = \frac{3}{4}$. Then $\mu(q) \geq \mu(p) \wedge \delta(p,a,q) \wedge \sigma(p,a,b)$ and $\mu(p) \geq \mu(q) \wedge \delta(q,a,p) \wedge \sigma(q,a,b)$. Hence, $\mu$ is a fuzzy subsystem and $\mu$ is constant. Now,
$(q_1 X^* Y^*)(p) = \bigvee_{x \in X, y \in Y} \{1 \wedge \delta^*(q,x,p) \wedge \sigma^*(q,x,y)\} = \frac{1}{3} < \frac{3}{4} = \mu(p)$. Therefore, $\mu$ is not cyclic.

**Theorem 3.26** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine and $\mu$ be a fuzzy subsystem of $M$. Suppose $Supp(\mu) = Q$. If $\mu$ is super cyclic, then $M$ is strongly connected.

**Proof** Let $p, q \in Q$. Then $(q_{\mu(q)} X^* Y^*)(p) = \bigvee_{x \in X, y \in Y} \{\mu(q) \wedge \delta^*(q,x,p) \wedge \sigma^*(q,x,y)\ \} > 0$, since $\mu$ is super cyclic $\mu = (q_{\mu(q)} X^* Y^*)$ and $Supp(\mu) = Q$. Hence, $\delta^*(q,x,p) \wedge \sigma^*(q,x,y)\} > 0$, for some $x \in X^*, y \in Y^*$. Thus $p \in S(q)$. Hence, $M$ is strongly connected.

**Theorem 3.27** Let $M = (Q, X, Y, \delta, \sigma)$ be a fuzzy Mealy machine and $\mu$ a fuzzy subsystem of $M$. Then $\mu$ is super cyclic if and only if $\forall\ p, q \in Q,\ \exists\ x \in X^*, y \in Y^*$ such that $\delta^*(p,x,q) \wedge \sigma^*(p,x,y) \geq \mu(p)$.

**Proof** Suppose that $\mu$ is super cyclic. Then $\mu$ is constant by Theorem (3.23). Suppose $\exists\ p, q \in Q,\ \forall\ x \in X^*, y \in Y^*, \delta^*(p,x,q) \wedge \sigma^*(p,x,y) < \mu(p)$. Then
$(p_{\mu(p)} X^* Y^*)(q) = \bigvee_{x \in X, y \in Y} \{\mu(p) \wedge \delta^*(p,x,q) \wedge \sigma^*(p,x,y)\} < \mu(p)$.
Thus, $p_{\mu(p)} X^* Y^* \neq \mu$. which is contradiction to $\mu$ is super cyclic. Conversely, Suppose that $\forall\ p, q \in Q,\ \exists\ x \in X^*, y \in Y^*$ such that $\delta^*(p,x,q) \wedge \sigma^*(p,x,y) \geq \mu(p)$. Then $\forall\ p, q \in Q,\ \exists\ x \in X^*, y \in Y^*$ such that $\mu(q) \geq \mu(p) \wedge \delta^*(p,x,q) \wedge \sigma^*(p,x,y) = \mu(p)$. Similarly $\mu(p) \geq \mu(q)$. Hence, $\mu$ is constant. Now,
$(p_{\mu(p)} X^* Y^*)(q) = \bigvee_{x \in X, y \in Y} \{\mu(p) \wedge \delta^*(p,x,q) \wedge \sigma^*(p,x,y)\} = \mu(p) = \mu(q)$. Thus, $p_{\mu(p)} X^* Y^* = \mu$. Hence, $\mu$ is super cyclic.

## 4. CONCLUSION

In this paper the results of fuzzy finite state machine are successfully extended for fuzzy Mealy machines. We introduced successor, submachines, subsystem, homomorphism and (super) cyclic subsystems for Fuzzy Mealy machines. Along with various properties, we have characterized subsystems and (super) cyclic subsystems. Three classes, based on constants fuzzy sets, fuzzy input-output sets and fuzzy points, of subsystems are also obtained.

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