International Journal of Latest Research in Science and Technology **ISSN (Online):2278-5299**
Volume 4, Issue 6: Page No.126-131, November-December 2015 (special Issue Paper)
http://www.mnkjournals.com/ijlrst.htm
Special Issue on International conference on Methematics Sciences-2015(MSC-2015)
Confernece Held at Sadguru Gadage Maharaj College, Karad ,Maharastra,(India)

SOLUTIONS OF FULLY FUZZY LINEAR EQUATIONS AX+B=C

S. M. Bargir[1] , J. D. Yadav[2], M. S. Bapat[3]
[1]Research Scholar, Department of Mathematics, Shivaji University, Kolhapur, Maharashtra, India Email: bargir1991@gmail.com
[2]Sadguru Gadge Maharaj college, karad, Maharashtra, India Email: jdy1560@yahoo.co.in
[3]Willingdon college, sangli 416415 Maharashtra, India Email: msbapt@gmail.com

Abstract- ***In this paper, we propose a method for computing the solution of fully fuzzy linear equation of the type AX+B = C where A, B and C are Epsilon–Delta fuzzy numbers. Solution of this equation is obtained for the cases: A > 0 and A< 0. Whenever solution of the equation does not exist, applying flexibility to the spreads, we can obtain the solution of region such equations.***

Key words**:** Fully fuzzy linear equations, triangular fuzzy numbers.

I. INTRODUCTION

Fuzzy linear equations have many applications in various fields like operation research, economics, decision theory, statistics, mathematics, engineering sciences, physical sciences, social sciences etc. Fuzzy equations consists fuzzy numbers as coefficients and/or unknown parameters. Since, satisfactory theory for fuzzy arithmetic has not been developed yet, e.g. inverse fuzzy numbers does not exist in fuzzy arithmetic operations like addition and multiplication, the solution of fully fuzzy linear equation becomes computationally difficult [19]. Therefore mathematical equations involving fuzzy parameters F(X) = B can't be solved with usual methods, where F is a linear or polynomial function of fuzzy variable X [21].

Several authors have been proposed the different methods to find the exact or approximate solutions of fuzzy linear equations with some assumptions. Buckley et al. have proposed the model for solving the fuzzy equations based on extension principle [7,8]. The method based on the extension principle is too restrictive and sometimes does not have any solution. Further, classical method, interval arithmetic method and fuzzy Monte Carlo methods presented for solving the equations [9]. But, the obtained solution by Monte Carlo method leads to a large error and it is time consuming. Dug hong et al. contributed the method of interval confidence [13]. Several other resrearchers have given different methods for solution of fuzzy linear equations [1,11,14-17,20,21].

In most of the application the fuzzy numbers in the form of triangular shape are preferred. The arithmetic operations on triangular or trapezoidal fuzzy numbers are computationally easier. Further A. Ban, P. Grzegorzewski etc. have developed methods of triangular or trapezoidal approximation. Hence it is legitimate to assume that coefficients or solution of the equations as triangular fuzzy numbers. We have proposed solution of methods of fuzzy linear equation or fully fuzzy linear equation in which coefficients or solutions are triangular fuzzy numbers. Further while using arithmetic operations particularly for multiplication we have assumed reasonable triangular approximation satisfying certain criteria [12]. Based on this approximation we have obtained approximate solution of the equations sufficiently close to the exact solution and further these methods are computationally simple.

In this paper, we have obtained the solutions of fully fuzzy linear equations of the type AX+B=C where A, B, C and variable X are triangular fuzzy numbers. With the help of better approximation of multiplication, we have established conditions for the solution of the equation. Whenever, solution of equation does not exist, we have provided the solution region using flexibility of spreads.

II. PRELIMINARIES

Through out this paper I stands for the interval [0, 1]. A fuzzy subset A of a set X is a function $A: X \to I$. The set $\{x \in X \mid A(x) \geq \alpha\}$ is called α -level cut or α -cut of A and is denoted by A_α. The strict α -level cut of A is the set $A_{\alpha+} = \{x \in X \,|A(x) > \alpha\}$. Support of A is the set $A_{0+} = \{x \in X \,|A(x) > 0\}$. If $A(x) = 1$ for some $x \in X$ then A is called normal fuzzy set. If each α-cut of A is convex then the fuzzy set A is called convex fuzzy set. Core of fuzzy set A is a set $A_1 = \{x \in X \,|A(x) = 1\}$. We assume X = $\mathbb{R}$.

A fuzzy number A is a fuzzy subset of the set of real numbers $\mathbb{R}$ with membership function $A : \mathbb{R} \to I$ such that A is normal, convex , upper semi-continuous with bounded support . If left hand curve and right hand curve are straight lines then the fuzzy number A is called trapezoidal fuzzy number. If the core is singleton set then the trapezoidal fuzzy number is called triangular fuzzy number. Many authors have represented triangular fuzzy number A as A = (a, m, b). This representation is interpreted as membership function

$$A(x) = \begin{cases} \dfrac{x-a}{m-a} & \text{if } a < x \leq b \\ \dfrac{b-x}{b-m} & \text{if } b < x \leq c \\ 0 & \text{if } x > 0 \end{cases}$$

The ε-δ fuzzy number is a triangular fuzzy number. The membership function of the triangular fuzzy number is of the form.

$$r(x)=\begin{cases}\dfrac{x-(r-\varepsilon)}{\varepsilon} & \text{when } r-\varepsilon<x\le r\\ \dfrac{(r+\delta)-x}{\delta} & \text{when } r<x\le r+\delta\\ 0 & \text{otherwise}\end{cases}.$$

We denote the above triangular fuzzy number by $r_{\varepsilon,\delta}$ where ε and δ are left and right spreads of the fuzzy number. Note that (a, m, b) = $m_{m-a,b-m}$ and $m_{\varepsilon,\delta}=(m-\varepsilon,m,m+\delta)$

Thus the two notations are equivalent. The notation $r_{\varepsilon,\delta}$ is useful in arithmetic operation. If the left and the right spreads of ∈-δ fuzzy number are equal then it is called symmetric fuzzy number denoted by r_{ε}. If $r-\varepsilon>0$ then $r_{\varepsilon,\delta}$ is positive fuzzy number and if $r+\delta<0$ then $r_{\varepsilon,\delta}$ is negative fuzzy number. We define Arithmetic operations on ∈-δ fuzzy numbers as follows

The addition of ∈-δ fuzzy numbers is given by $(r_{\varepsilon_1,\delta_1}+s_{\varepsilon_2,\delta_2})=(r+s)_{\varepsilon_1+\varepsilon_2,\delta_1+\delta_2}$

Negation of ε-δ fuzzy numbers $r_{\varepsilon,\delta}$ is $-(r_{\varepsilon,\delta})=(-r)_{\delta,\varepsilon}$.

The subtraction of ∈-δ fuzzy numbers is given by $(r_{\varepsilon_1,\delta_1}-s_{\varepsilon_2,\delta_2})=(r-s)_{\varepsilon_1+\delta_2,\varepsilon_2+\delta_1}$. Multiplication or division of triangular fuzzy number is not triangular. Hence we introduce the triangular approximation of multiplication [4] as follows,

If $r_{\varepsilon_1,\delta_1}>0$ and $s_{\varepsilon,\delta}>0$ then

$$r_{\varepsilon_1,\delta_1}\square s_{\varepsilon,\delta}=(rs)_{r\varepsilon+s\varepsilon_1-\frac{1}{2}\varepsilon\varepsilon_1,\,r\delta+s\delta_1+\frac{1}{2}\delta\delta_1}.$$

If $r_{\varepsilon_1,\delta_1}<0$ and $s_{\varepsilon,\delta}>0$ then

$$r_{\varepsilon_1,\delta_1}\square s_{\varepsilon,\delta}=(rs)_{-r\varepsilon+s\varepsilon_1+\frac{1}{2}\delta\varepsilon_1,\,-r\varepsilon+s\delta_1-\frac{1}{2}\varepsilon\delta_1}$$

If $r_{\varepsilon_1,\delta_1}>0$ and $s_{\varepsilon,\delta}<0$ then

$$r_{\varepsilon_1,\delta_1}\square s_{\varepsilon,\delta}=(rs)_{r\varepsilon-s\delta_1+\frac{1}{2}\varepsilon\delta_1,\,r\delta-s\varepsilon_1-\frac{1}{2}\delta\varepsilon_1}$$

If $r_{\varepsilon_1,\delta_1}<0$ and $s_{\varepsilon,\delta}<0$ then

$$r_{\varepsilon_1,\delta_1}\square s_{\varepsilon,\delta}=(rs)_{-r\delta-s\delta_1-\frac{1}{2}\delta\delta_1,\,-r\varepsilon-s\varepsilon_1+\frac{1}{2}\varepsilon\varepsilon_1}$$

We say that two fuzzy numbers $a_{\varepsilon_1,\delta_1}$ and $b_{\varepsilon_2,\delta_2}$ are equal if a = b, $\varepsilon_1=\varepsilon_2$ and $\delta_1=\delta_2$. We call $0_{0,\delta}$ as positive zero and $0_{\varepsilon,0}$ as negative zero.

III. FULLY FUZZY LINEAR EQUATION AX +B = C

Let AX+ B = C be a fully fuzzy linear equation where A, X, B and C are triangular fuzzy numbers. Let $A=a_{\varepsilon_1,\delta_1}$, $X=x_{\varepsilon,\delta}$, $B=b_{\varepsilon_2,\delta_2}$ and $C=c_{\varepsilon_3,\delta_3}$. Then the equation becomes $a_{\varepsilon_1,\delta_1}x_{\varepsilon,\delta}+b_{\varepsilon_2,\delta_2}=c_{\varepsilon_3,\delta_3}$ Following propositions gives the conditions for existence of solution and the solutions of this equation.

Proposition 1:- The fully fuzzy linear equation, $a_{\varepsilon_1,\delta_1}x_{\varepsilon,\delta}+b_{\varepsilon_2,\delta_2}=c_{\varepsilon_3,\delta_3},\ a_{\varepsilon_1,\delta_1}>0$

has solution for $x_{\varepsilon,\delta}$

1. For c > b if $\frac{\varepsilon_1}{a}\le\frac{(\varepsilon_3-\varepsilon_2)}{c-b}$ and $\frac{\delta_1}{a}\le\frac{(\delta_3-\delta_2)}{c-b}$ then, positive solution is given by $x=\frac{c-b}{a},\varepsilon=\frac{a(\varepsilon_3-\varepsilon_2)-\varepsilon_1(c-b)}{a(a-\frac{1}{2}\varepsilon_1)}$ and $\delta=\frac{a(\delta_3-\delta_2)-\delta_1(c-b)}{a(a+\frac{1}{2}\delta_1)}$
2. For c < b if $\frac{\delta_1}{a}\le\frac{(\varepsilon_3-\varepsilon_2)}{c-b}$ and $\frac{\varepsilon_1}{a}\le\frac{(\delta_3-\delta_2)}{c-b}$ then, negative solution is given by $x=\frac{c-b}{a},\varepsilon=\frac{a(\varepsilon_3-\varepsilon_2)+\delta_1(c-b)}{a(a+\frac{1}{2}\delta_1)}$ and $\delta=\frac{a(\delta_3-\delta_2)-\varepsilon_1(c-b)}{a(a-\frac{1}{2}\varepsilon_1)}$
3. For c = b,

(a) if $\varepsilon_2=\varepsilon_3,\ \delta_3\ge\delta_2$ then, positive zero solution is given by $\varepsilon=0$ and $\delta=\frac{(\delta_3-\delta_2)}{(a+\frac{1}{2}\delta_1)}$.

(b) if $\varepsilon_2\le\varepsilon_3,\ \delta_3=\delta_2$ then, negative zero solution is given by $\varepsilon=\frac{(\varepsilon_3-\varepsilon_2)}{(a-\frac{1}{2}\varepsilon_1)}$ and $\delta=0$.

Proof: The given fully fuzzy linear equation is $a_{\varepsilon_1,\delta_1}.x_{\varepsilon,\delta}+b_{\varepsilon_2,\delta_2}=c_{\varepsilon_3,\delta_3}$. Since $a_{\varepsilon_1,\delta_1}>0$, we have $a-\varepsilon_1\ge 0$ and hence $(a-\frac{1}{2}\varepsilon_1)>0$.

Case 1: c > b then x > 0. We search for ε for which $x-\varepsilon\ge 0$. Therefore $x_{\varepsilon,\delta}$ becomes positive fuzzy number.

By multiplication and addition of positive epsilon-delta fuzzy numbers we get,

$$(ax+b)_{a\varepsilon+x\varepsilon_1-\frac{1}{2}\varepsilon\varepsilon_1+\varepsilon_2,\,a\delta+x\delta_1+\frac{1}{2}\delta\delta_1+\delta_2}=c_{\varepsilon_3,\delta_3}$$

By applying equality of epsilon-delta fuzzy numbers we have

$$x=\frac{c-b}{a},\ \varepsilon=\frac{a(\varepsilon_3-\varepsilon_2)-\varepsilon_1(c-b)}{a(a-\frac{1}{2}\varepsilon_1)}\ \text{and}$$
$$\delta=\frac{a(\delta_3-\delta_2)-\delta_1(c-b)}{a(a+\frac{1}{2}\delta_1)}\qquad\ldots(1)$$

Since ε, δ are non-negative real numbers, we must have $a(\varepsilon_3-\varepsilon_2)-\varepsilon_1(c-b)\ge 0$ and $a(\delta_3-\delta_2)-\delta_1(c-b)\ge 0$

$$\Rightarrow \frac{\varepsilon_1}{a}\le\frac{(\varepsilon_3-\varepsilon_2)}{c-b} \text{ and } \frac{\delta_1}{a}\le\frac{(\delta_3-\delta_2)}{c-b}.$$

Case 2: If c < b then x < 0. Therefore we search for δ for which $x+\delta\le 0$. By multiplication of positive and negative epsilon-delta fuzzy number we get,

$$(ax+b)_{a\varepsilon-x\delta_1+\frac{\varepsilon\delta_1}{2}+\varepsilon_2,\,a\delta-x\varepsilon_1-\frac{\delta\varepsilon_1}{2}+\delta_2}=c_{\varepsilon_3,\delta_3}$$

By applying equality of two fuzzy numbers we get,

$$x=\frac{c-b}{a},$$

$$\varepsilon=\frac{a(\varepsilon_3-\varepsilon_2)+\delta_1(c-b)}{a\left(a+\frac{1}{2}\delta_1\right)} \text{ and } \delta=\frac{a(\delta_3-\delta_2)+\varepsilon_1(c-b)}{a\left(a-\frac{1}{2}\varepsilon_1\right)} \qquad \ldots(2)$$

$$a(\varepsilon_3-\varepsilon_2)-\delta_1(c-b)\ge 0 \text{ and } a(\delta_3-\delta_2)-\varepsilon_1(c-b)\ge 0$$

$$\Rightarrow \frac{\delta_1}{a}\le\frac{(\varepsilon_2-\varepsilon_3)}{c-b} \text{ and } \frac{\varepsilon_1}{a}\le\frac{(\delta_2-\delta_3)}{c-b}$$

Since ε, δ are non-negative we must have

Case 3: b = c then x = 0, in this case we get two sub-cases which is defined as follow

(a) If $\varepsilon_2=\varepsilon_3$ and $\delta_3\ge\delta_2$ then from (1) we get positive zero solution $\varepsilon=0$, $\delta=\frac{(\delta_3-\delta_2)}{\left(a+\frac{1}{2}\delta_1\right)}$.

(b) If $\varepsilon_2\le\varepsilon_3$ and $\delta_3=\delta_2$ then from (2) we get negative zero solution $\varepsilon=\frac{(\varepsilon_3-\varepsilon_2)}{\left(a+\frac{1}{2}\delta_1\right)}$, $\delta=0$.

Proposition 2. The fully fuzzy linear equation $a_{\varepsilon_1,\delta_1}x_{\varepsilon,\delta}+b_{\varepsilon_2,\delta_2}=c_{\varepsilon_3,\delta_3}$, $a_{\varepsilon_1,\delta_1}<0$ has solution for $x_{\varepsilon,\delta}$ if

1. For c > b if $\frac{(\delta_2-\delta_3)}{c-b}\le\frac{\varepsilon_1}{a}$, $\frac{(\varepsilon_2-\varepsilon_3)}{c-b}\le\frac{\delta_1}{a}$, then negative solution is
$$x=\frac{c-b}{a},\varepsilon=\frac{a(\varepsilon_2-\varepsilon_3)-\delta_1(c-b)}{a\left(a+\frac{1}{2}\delta_1\right)} \text{ and } \delta=\frac{a(\delta_2-\delta_3)-\varepsilon_1(c-b)}{a\left(a-\frac{1}{2}\varepsilon_1\right)}$$
2. For c < b if $\frac{(\varepsilon_3-\varepsilon_2)}{c-b}\le\frac{\varepsilon_1}{a}$, $\frac{(\delta_3-\delta_2)}{c-b}\le\frac{\delta_1}{a}$,

then positive solution is

$$x=\frac{c-b}{a},\varepsilon=\frac{a(\delta_2-\delta_3)+\delta_1(c-b)}{a\left(a+\frac{1}{2}\delta_1\right)} \text{ and } \delta=\frac{a(\varepsilon_2-\varepsilon_3)+\varepsilon_1(c-b)}{a\left(a-\frac{1}{2}\varepsilon_1\right)}$$

3. For c = b

(a) If $\varepsilon_3=\varepsilon_2$, $\delta_2\le\delta_3$ then negative zero solution is $x=0,\varepsilon=\frac{(\delta_2-\delta_3)}{\left(a-\frac{1}{2}\varepsilon_1\right)}$ and $\delta=0$

(b) If $\varepsilon_2\ge\varepsilon_3$, $\delta_3=\delta_2$, then positive zero solution is $x=0,\varepsilon=0$ and $\delta=\frac{(\varepsilon_2-\varepsilon_3)}{\left(a-\frac{1}{2}\varepsilon_1\right)}$.

Proof : The proof is similar to the proof of Proposition 1. Now by using the above proposition we solve the equations. Also, it is shown that the solution obtained is a classical solution which satisfies the equation.

Example 1. Consider the equation

$$2_{1,2}\square x_{\varepsilon,\delta}+5_{2,2}=8_{5,6}$$

Here, $a_{\varepsilon_1,\delta_1}=2_{1,2}\ge 0$ and c > b then from case1 of proposition1 the necessary conditions

$$\frac{\varepsilon_1}{a}=\frac{1}{2}<\frac{(\varepsilon_3-\varepsilon_2)}{c-b}=1 \text{ and } \frac{\delta_1}{a}=1<\frac{(\delta_3-\delta_2)}{c-b}=\frac{4}{3}$$

hold. Then positive solution is

$$x=\frac{3}{2},\varepsilon=\frac{a(\varepsilon_3-\varepsilon_2)-\varepsilon_1(c-b)}{a\left(a-\frac{1}{2}\varepsilon_1\right)}=1 \text{ and}$$

$$\delta=\frac{a(\delta_3-\delta_2)-\delta_1(c-b)}{a\left(a+\frac{1}{2}\delta_1\right)}=\frac{1}{3}.$$ Thus we get required solution is $x_{\varepsilon,\delta}=\left(\frac{3}{2}\right)_{1,\frac{1}{3}}$.

Verification. $2_{1,2}\square x_{\varepsilon,\delta}+5_{2,2}=8_{5,6}\Rightarrow 2_{1,2}\square\left(\frac{3}{2}\right)_{1,\frac{1}{3}}+5_{2,2}$

$$=\left(2\square\tfrac{3}{2}\right)_{2+\frac{3}{2}-\frac{1}{2},\frac{2}{3}+3+\frac{1}{3}}+5_{2,2}=(3)_{3,4}+5_{2,2}=(8)_{5,6}$$

Example 2. Consider the fuzzy equation

$$4_{3,4}\square x_{\varepsilon,\delta}+7_{2,3}=5_{4,5}.$$

Here, $a_{\varepsilon_1,\delta_1}=(4)_{3,4}>0$ and c < b then from case 2 of Proposition 1, necessary conditions

$$\frac{\varepsilon_1}{a}=\frac{3}{4}<1=\frac{(\delta_2-\delta_3)}{c-b} \text{ and } \frac{\delta_1}{a}=1=\frac{(\varepsilon_2-\varepsilon_3)}{c-b}$$ hold.

Then negative solution is

$$x=\frac{-1}{2},\delta=\frac{a(\delta_3-\delta_2)+\varepsilon_1(c-b)}{a\left(a-\frac{1}{2}\varepsilon_1\right)}=\frac{1}{5} \text{ and}$$

$$\varepsilon=\frac{a(\varepsilon_3-\varepsilon_2)+\delta_1(c-b)}{a\left(a+\frac{1}{2}\delta_1\right)}=0.$$

The required solution is $x_{\varepsilon,\delta}=\left(\frac{-1}{2}\right)_{0,\frac{1}{5}}$.

Verification:- $4_{3,4}\square x_{\varepsilon,\delta}+7_{2,3}\Rightarrow 4_{3,4}\square\left(\frac{-1}{2}\right)_{0,\frac{1}{5}}+7_{2,3}$

$$=(-2)_{0+2+0,\frac{4}{5}+\frac{3}{2}-\frac{3}{10}}+7_{2,3}=5_{4,5}$$

Example 3. Consider the fuzzy equation

$$(-3)_{1,1}\square x_{\varepsilon,\delta}+4_{3,2}=7_{5,4}.$$

In this equation, $a_{\varepsilon_1,\delta_1}=(-3)_{1,1}<0$ and c > b. Therefore by case1 of proposition2 necessary conditions

$$\frac{\varepsilon_1}{a}=\frac{-1}{3}\ge\frac{(\delta_2-\delta_3)}{c-b}=\frac{-2}{3} \text{ and } \frac{\delta_1}{a}=\frac{-1}{3}\ge\frac{(\varepsilon_2-\varepsilon_3)}{c-b}=\frac{-2}{3}$$

hold. Then negative solution is given by

$$x=-1,\varepsilon=\frac{a(\delta_2-\delta_3)-\varepsilon_1(c-b)}{a\left(a-\frac{1}{2}\varepsilon_1\right)}=\frac{2}{7}$$

and $\delta = \dfrac{a(\varepsilon_2 - \varepsilon_3) - \delta_1 (c-b)}{a(a + \frac{1}{2}\delta_1)} = \dfrac{2}{5}$,

thus required solution is $x_{\varepsilon,\delta} = (-1)_{\frac{2}{7},\frac{2}{5}}$.

Verification.

$$(-3)_{1,1} \square x_{\varepsilon,\delta} + 4_{3,2} \Rightarrow (-3)_{1,1} \square (-1)_{\frac{2}{7},\frac{2}{5}} + 4_{3,2} = 3_{2,2} + 4_{3,2} = 7_{5,4}$$

Example 4. Consider the fuzzy equation

$$(-4)_{1,1} \square x_{\varepsilon,\delta} + 7_{1,3} = 3_{3,5}.$$

Then evidently $a_{\varepsilon_1,\delta_1} = (-4)_{1,1} < 0$ and c < b. Therefore using case 2 of proposition 2 necessary conditions $\frac{\varepsilon_1}{a} = \frac{-1}{4} \geq \frac{(\varepsilon_3 - \varepsilon_2)}{c-b} = \frac{-1}{2}$ and $\frac{\delta_1}{a} = \frac{-1}{4} \geq \frac{(\delta_3 - \delta_2)}{c-b} = \frac{-1}{2}$ hold. Hence the positive solution is,

$x = \dfrac{(c-b)}{a} = 1, \varepsilon = \dfrac{a(\delta_2 - \delta_3) + \delta_1 (c-b)}{a(a + \frac{1}{2}\delta_1)} = \dfrac{2}{7}$ and

$\delta = \dfrac{a(\varepsilon_2 - \varepsilon_3) + \varepsilon_1 (c-b)}{a(a - \frac{1}{2}\varepsilon_1)} = \dfrac{2}{9}$.

Thus required solution is $x_{\varepsilon,\delta} = (1)_{\frac{2}{7},\frac{2}{9}}$.

Verification.

$$(-4)_{1,1} \square x_{\varepsilon,\delta} + 7_{1,3} = 3_{3,5} \Rightarrow (-4)_{1,1} \square (1)_{\frac{2}{7},\frac{2}{9}} + 7_{1,3} = (3)_{2+1,2+3} = 3_{3,5}$$

4. Solution region

Fuzzy linear equation of the type F(X) = B where X and B are epsilon-delta fuzzy numbers may or may not have solution. But by allowing flexibility to the spreads of the coefficients it is possible to obtain a classical solution of such equations.

Example 5. Consider the fully fuzzy linear equation $2_{1,1} x_{\varepsilon,\delta} + (-2)_{1,1} = 4_{1,1}$.

Then clearly $a_{\varepsilon_1,\delta_1} = 2_{1,1} \geq 0$ and c > b . From case1 of proposition1 we must have

$\frac{\varepsilon_1}{a} \leq \frac{(\varepsilon_3 - \varepsilon_2)}{c-b}$ and $\frac{\delta_1}{a} \leq \frac{(\delta_3 - \delta_2)}{c-b}$ but

$\frac{\varepsilon_1}{a} = \frac{1}{2} \geq \frac{(\varepsilon_3 - \varepsilon_2)}{c-b} = 0$ and $\frac{\delta_1}{a} = \frac{1}{2} \geq \frac{\delta_3 - \delta_2}{c-b} = 0$

Hence the condition does not hold. Therefore solution of the equation does not exists.

In this example solution of the equation does not exists because the conditions in case1 of proposition1 fails. But by allowing flexibility to the spreads of the coefficients in given equation we can obtain the classical solution. Further the range of the values of the spreads gives a region in which the solution of the equation exists. We discuss the following example where the flexibility is given to the spreads of the coefficients. In some cases the flexibility concepts fails but in most of the cases we get a solution region where the solution of the equation exists.

The solution of the same equation is obtained by allowing flexibility to the spreads of the coefficients.

Case 1: Consider the equation $2_{1,1} x_{\varepsilon,\delta} + (-2)_{1,1} = 4_{1,1}$ We replace the fuzzy number $2_{1,1}$ by $2_{\varepsilon_1,\delta_1}$ that is flexibility on ε_1, δ_1 . Then we have $2_{\varepsilon_1,\delta_1} x_{\varepsilon,\delta} + (-2)_{1,1} = 4_{1,1}$. For existence of solution we must have

$\frac{\varepsilon_1}{a} \leq \frac{(\varepsilon_3 - \varepsilon_2)}{c-b}$ and $\frac{\delta_1}{a} \leq \frac{(\delta_3 - \delta_2)}{c-b} \Rightarrow \varepsilon_1 = 0$ and $\delta_1 = 0$,

and also we get $\varepsilon = 0$ and $\delta = 0$. In this case we get solution $x_{\varepsilon,\delta} = 3_{0,0}$.

Case 2: Similarly we can apply flexibility on ε_2 and δ_2 that is $2_{1,1} x_{\varepsilon,\delta} + (-2)_{\varepsilon_2,\delta_2} = 4_{1,1}$ For solution, we must have

$\frac{\varepsilon_1}{a} \leq \frac{(\varepsilon_3 - \varepsilon_2)}{c-b}$ and $\frac{\delta_1}{a} \leq \frac{(\delta_3 - \delta_2)}{c-b} \Rightarrow \varepsilon_2 \leq (-2)$ and $\delta_2 \leq (-2)$

This is not possible since ε_2, δ_2 are nonnegative real numbers. In this example we cannot impose flexibility on spreads ε_2, δ_2 .

Case 3: We apply flexibility on ε_3, δ_3. Consider the equation $2_{1,1} x_{\varepsilon,\delta} + (-2)_{1,1} = 4_{\varepsilon_3,\delta_3}$ For solution we must have

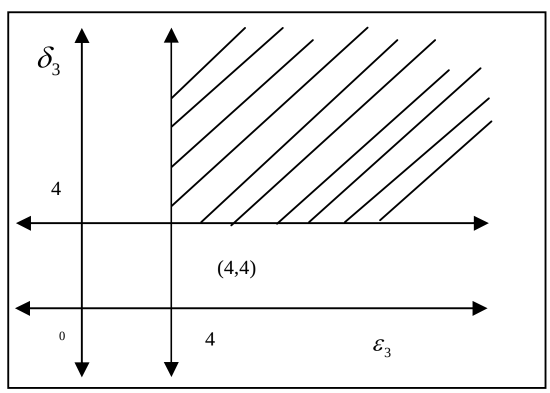
$\frac{\varepsilon_1}{a} \leq \frac{(\varepsilon_3 - \varepsilon_2)}{c-b}$ and $\frac{\delta_1}{a} \leq \frac{(\delta_3 - \delta_2)}{c-b} \Rightarrow 4 \leq \varepsilon_3$ and $4 \leq \delta_3$

Then we get

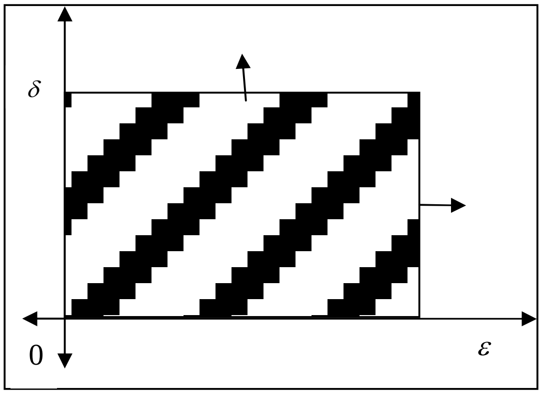
$\varepsilon = \dfrac{\varepsilon_3 - (x\varepsilon_1 + \varepsilon_2)}{(a - \frac{1}{2}\varepsilon_1)} = \dfrac{2\varepsilon_3 - 8}{3}$ and $\delta = \dfrac{\delta_3 - (x\delta_1 + \delta_2)}{(a + \frac{1}{2}\delta_1)} = \dfrac{2\delta_3 - 8}{5}$

and the minimal solution is $\varepsilon = 0, \delta = 0$ By translation of equation we get $\varepsilon_3 = \frac{1}{2}(3\varepsilon + 8)$ and $\delta_3 = \frac{1}{2}(5\delta + 8)$.

Graph for ε_3, δ_3 is given below,



The graph of $\varepsilon_3 - \delta_3$



Graph of $\varepsilon - \delta$

The above graph indicates the region of the possible values of ε, δ for which the given equation has solution. The values of ε, δ depends upon the choice of ε_3, δ_3 or vice-versa.

Case 4: In this case we can apply the flexibility on ε_1 and δ_2. Then the equation becomes

$2_{\varepsilon_1,1} x_{\varepsilon,\delta} + (-2)_{1,\delta_2} = 4_{1,1}$. For solution we must have

$\frac{\varepsilon_1}{a} \le \frac{(\varepsilon_3 - \varepsilon_2)}{c-b}$ and $\frac{\delta_1}{a} \le \frac{(\delta_3 - \delta_2)}{c-b}$ which implies

$\varepsilon = 0$ and $\delta_2 \le -2$. This case is fail because distance is never negative.

Case 5: Flexibility on ε_2 and δ_1 that is

$$2_{1,\delta_1} x_{\varepsilon,\delta} + (-2)_{\varepsilon_2,1} = 4_{1,1}$$

Case 6: Flexibility on ε_3 and δ_2 that is

$$2_{1,1} x_{\varepsilon,\delta} + (-2)_{1,\delta_2} = 4_{\varepsilon_3,1}$$

Case 7: Flexibility on ε_2 and δ_3 that is

$$2_{1,1} x_{\varepsilon,\delta} + (-2)_{\varepsilon_2,1} = 4_{1,\delta_3}$$

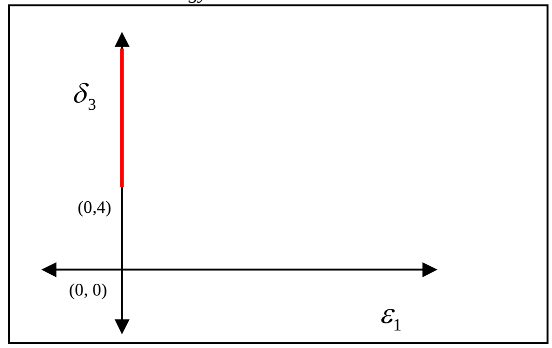
Above all three cases are fails because ε_2 and δ_2 are occur in negative which is not possible.

Case 8: Applying flexibility on ε_1 and δ_3 we have the equation $2_{\varepsilon_1,1} x_{\varepsilon,\delta} + (-2)_{1,1} = 4_{1,\delta_3}$.

Then $\frac{\varepsilon_1}{a} \le \frac{(\varepsilon_3 - \varepsilon_2)}{c-b} \Rightarrow \varepsilon_1 = 0$ and $\frac{\delta_1}{a} \le \frac{(\delta_3 - \delta_2)}{c-b} \Rightarrow \delta_3 \ge 4$ holds and hence

$$\varepsilon = \frac{\varepsilon_3 - (x\varepsilon_1 + \varepsilon_2)}{\left(a - \frac{1}{2}\varepsilon_1\right)} = 0 \text{ and } \delta = \frac{\delta_3 - (x\delta_1 + \delta_2)}{\left(a + \frac{1}{2}\delta_1\right)} = \frac{2\delta_3 - 8}{5}$$

The minimal solution is $\varepsilon_1 = 0, \delta_3 = 4$. The graph of ε_1, δ_3 is given below.



Graph of $\varepsilon_1 - \delta_3$

Case 9: Flexibility on ε_3 and δ_1 gives

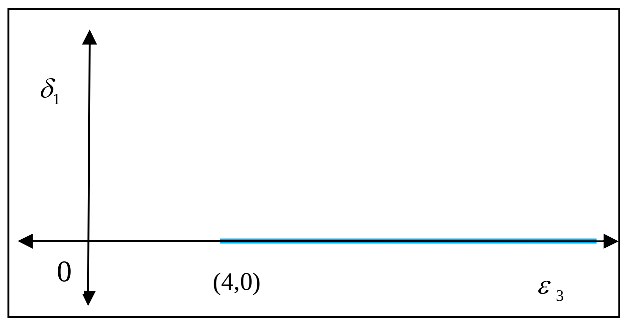
$2_{1,\delta_1} x_{\varepsilon,\delta} + (-2)_{1,1} = 4_{\varepsilon_3,1}$ For solution we have

$\frac{\varepsilon_1}{a} \le \frac{(\varepsilon_3 - \varepsilon_2)}{c-b} \Rightarrow \varepsilon_3 \ge 4$ and $\frac{\delta_1}{a} \le \frac{(\delta_3 - \delta_2)}{c-b} \Rightarrow \delta_1 = 0$

The conditions holds therefore solution is

$$\varepsilon = \frac{\varepsilon_3 - (x\varepsilon_1 + \varepsilon_2)}{\left(a - \frac{1}{2}\varepsilon_1\right)} = \frac{2\varepsilon_3 - 8}{3} \text{ and } \delta = \frac{\delta_3 - (x\delta_1 + \delta_2)}{\left(a + \frac{1}{2}\delta_1\right)} = 0,$$

Minimal solution is $\varepsilon = 0, \delta = 0$ and graph of ε_3, δ_1 is



Graph of $\varepsilon_3 - \delta_1$

IV. CONCLUSION

Condition for solution of the equation AX+B=C are obtained where the coefficients and the variable X are epsilon-delta fuzzy number. Whenever the conditions fails by applying flexibility to the spreads we can obtain solution region where the solution of the equation exists.

REFERENCES

1. S. Abbasbandy, R. Ezzati, LUDecomposition method for solving fuzzy system of linear equations, Applied mathematics and computation,172, (2006) 633-643.
2. A.I. Ban, Triangular and parametric approximations of fuzzy numbers- Revisited, Fuzzy Sets and Systems, 160, (2009) 3027-3047.
3. A.I. Ban, Triangular and parametric approximations of fuzzy numbers, Fuzzy Sets and Systems,160 ,(2009) 3048-3058.
4. M. S. Bapat, S. N. Yadav, P. N. Kamble, Triangular approximations of fuzzy numbers, International Journal of Statistika and Mathematika,7(3), (2013) 63-66.
5. S. M. Bargir, M. S. Bapat, Fully fuzzy linear equation with one variable, National conference on fuzzy mathematics and its application at Sadguru Gadage Maharaj College, Karad. ISBN-978-93-5196-779-8 (2014) 111-117.

6. J.J.Buckley, Y.Qu, Solving fuzzy equations: A new solution concept, Fuzzy Set and System, 39,(1991) 291-301.
7. J.J.Buckley, Y.Qu, Solving linear and quadratic fuzzy equation, Fuzzy Set and System,38 (1990) 43-59.
8. J.J.Buckley, Solving fuzzy equation, Fuzzy set and system, 50(1992) 01-14.
9. J.J.Buckley, Leonarld. J Jowers, Monte carlo methods in fuzzy optimization, Springer-Verlag Berlin Heidlberg, 2008.
10. D. Dubois, H. Prade, Operations on fuzzy numbers, International Journal of Systems Science,9 (6) (1978) 613-626.
11. R.Ezzati, Solving fully fuzzy linear system of equations in general form, International scientific publications and consulting services, 2012 (2012) 11.
12. P. Grzegorzewski, Trapezoidal approximation of fuzzy numbers persevering the expected interval-Algorithms and properties, Fuzzy Sets and Systems 159 (2008) 1354-1364.
13. Dug Hun Hong, On solving Fuzzy equation, Korean Journal Computational And Application, 8 (1) (2001) 213-223.
14. R.Horcik, Solution of a system of linear equation for with fuzzy number, Fuzzy Set System,159 (2008) 1788-1810.
15. Reza kargar, A proposed method for solving fuzzy system of linear equation, Hindawi Publication Corporation, 6 (2004).
16. G. J. Klir, and T. A. Folger, Fuzzy Sets, Uncertainty, and Information, Prentice-Hall, Englewood Cliffs, NJ, 1988.
17. Amit Kumar, A New approach for solving fully fuzzy linear system, Hindawi Publication Corporation 8 (2011).
18. Nasseri Mikaeilvand, Signed decomposition of fully fuzzy linear system, Application and applied mathematics 3 (2008) 77-88.
19. M. Mizumoto and K. Tanaka, Algebraic properties of fuzzy numbers, in M.M. Gupta, R.K. Regade and R.R Yager, eds., Advance in fuzzy set theory and application (1982).
20. S. H.Nasseri, M. Sohrabi, E. Ardil, Solving Fully Fuzzy Linear System By Use Of A Certain Decomposition Of The Coefficient Matrix, International Journal of Mathematics, 2 (7) (2008).
21. R.R. Yager, On the lack of inverse in fuzzy arithmetic, Fuzzy Sets and System 4 (1980) 73-82.