International Journal of Latest Research in Science and Technology ISSN (Online):2278-5299
Volume 4, Issue 6: Page No.121-125, November-December 2015 (special Issue Paper)
http://www.mnkjournals.com/ijlrst.htm
Special Issue on International conference on Methematics Sciences-2015(MSC-2015)
Confernece Held at Sadguru Gadage Maharaj College, Karad ,Maharastra,(India)

GENERALIZED CONFIDENCE INTERVALS FOR THE SCALE PARAMETER OF THE INVERTED EXPONENTIAL DISTRIBUTION

S.S.Godase[a], D. T. Shirke[b] and D.N.Kashid[b]
[a]Department of Statistics SGM College, Karad , 415124 ,INDIA (suwarna_godase@rediffmail.com)
[b]Department of Statistics Shivaji University, Kolhapur 416004 , INDIA (dtshirke@gmail.com)
[b]Department of Statistics Shivaji University, Kolhapur 416004 , INDIA (dnk_stats@unishivaji.ac.in)

Abstract- ***We consider a k-unit series system with life time of each unit following inverted exponential distribution with an unknown scale parameter. We provide two generalized confidence intervals for the scale parameter based on maximum likelihood estimator and modified maximum likelihood estimator respectively. The performance of proposed generalized confidence intervals is evaluated using extensive simulation work. The proposed confidence intervals found to perform well for small to moderate sample sizes. Further the proposed generalized confidence intervals perform better than asymptotic confidence interval for small sample sizes.***

Keywords: Series system, maximum likelihood estimator, modified maximum likelihood estimator, generalized confidence interval.

I. INTRODUCTION

There is a large amount of literature about the estimation of scale parameter of inverted exponential distribution using different approaches. Inverted exponential distribution is life time distribution which is used in the reliability discipline. The inverted exponential distribution (IED) has been discussed as a life time model by Lin et al (1989) in detail. They have obtained maximum likelihood estimators, confidence limits and uniformly minimum variance unbiased estimators for the parameter and reliability function with complete samples. Stefanski (1996) has discussed some basic properties of the IED.

We see from the literature review that there is more work on estimation of parameter of inverted exponential distribution as compared to interval estimation. The main purpose of this article is to develop a generalized pivot variable that is simple to use for interval estimation of the parameter in life time distribution of a series system. The concept of generalized p-value was introduced by Tsui and Weerahandi (1989) for hypothesis testing. Weerahandi (1993) extended the idea for constructing confidence interval. Weerahandi (1995) gives a detailed discussion along with numerous examples. The concept of generalized confidence intervals have turned out to be very satisfactory for obtaining confidence interval for many complex problems; see Weerahandi(1993,1995),

Krishnamoorthy and Mathew (2003), Guo and Krishnamoorthy (2005), Ng (2007), Ye and Wang (2008), Kurian et al.(2008).

In this paper, we consider the problem of setting generalized confidence interval (GCI) for the scale parameter, when lifetime distribution of a unit in a k-unit series system has inverted exponential distribution. Recently Potdar and Shirke (2014) explained reliability estimation of k-unit series system based on progressively censored data.

In section 2, we provide asymptotic confidence interval (ACI) based on maximum likelihood estimator (MLE) and modified maximum likelihood estimator (MMLE) for the scale parameter, when lifetime distribution of a unit in a k-unit series system has inverted exponential distribution. Tiku and Suresh (1992) obtained a new method of estimation for location and scale parameters by using MMLE. R.P.Suresh (2004) provides estimation of location and scale parameters in the two parameter exponential distribution using MMLE. In section 3, generalized confidence interval has been developed. In section 4, we study performance of both confidence intervals (ACI, GCI) using MLE as well as MMLE for k=2, 3 and for small sample sizes using simulation technique. The proposed GCIs are simple to compute and perform better in small sample sizes.

2. Asymptotic Confidence Interval based on MLE and MMLE

Consider a k-unit series system with independent and identically distributed lifetimes of components. Let Y_1, $Y_2 \ldots Y_k$ be the lifetimes, where Y_i is the lifetime of i^{th} component namely inverted exponential distribution. Lifetime of the system is X=min (Y_1, $Y_2 \ldots Y_k$). The cdf of X is

$$F_X(x;\theta) = 1 - \left(1 - e^{-1/\theta x}\right)^k \quad ; \quad x \geq 0, \theta > 0 \quad (2.1)$$

The pdf of X is given by,

$$f_X(x,\theta) = \left(\frac{k}{\theta x^2}\right) e^{-1/\theta x}\left(1 - e^{-1/\theta x}\right)^{k-1} \quad ; \quad x \geq 0, \theta > 0$$

(2.2) = 0 ; otherwise

Here log likelihood of the sample is given by

$$L = n\log(k) - n\log(\theta) - 2\sum_{i=1}^{n} \log x_i - \frac{1}{\theta}\sum_{i=1}^{n}\left(\frac{1}{x_i}\right) + (k-1)\sum_{i=1}^{n} \log\left(1 - e^{-1/\theta x_i}\right)$$

The MLE of θ can be obtained by solving $\frac{dL}{d\theta} = 0$, where

$$\frac{dL}{d\theta} = -\frac{n}{\theta} + \frac{1}{\theta^2}\sum_{i=1}^{n}\frac{1}{x_i} - \frac{(k-1)}{\theta^2}\sum_{i=1}^{n}\frac{e^{-1/\theta x_i}}{x_i(1-e^{-1/\theta x_i})}$$
(2.3)

The solution can be obtained by Newton-Raphson Method by taking initial solution $\hat{\theta}_0 = \bar{X}$.Then Fisher Information is given by

$I(\theta) = E\left[-\frac{d^2L}{d\theta^2}\right]$

= (2.4)

By using asymptotic normal distribution of MLE, we construct confidence interval for θ. Let $\hat{\theta}_n$ is the MLE of θ. Therefore by Cramer (1946) $\hat{\theta}_n \sim AN\left(\theta, \sigma^2(\hat{\theta}_n)\right)$, where $\sigma^2(\hat{\theta}_n) = \frac{1}{I(\hat{\theta}_n)}$ be the asymptotic variance.

Therefore, 100(1-α) % asymptotic confidence interval for θ is given by

$$\left(\hat{\theta}_n - \tau_{\alpha/2}\sqrt{\hat{\sigma}^2(\hat{\theta}_n)}\ , \hat{\theta}_n + \tau_{\alpha/2}\sqrt{\hat{\sigma}^2(\hat{\theta}_n)}\right) \quad (2.5)$$

where $\tau_{\alpha/2}$ is the upper $100(\alpha/2)^{th}$ percentile of standard normal distribution.

In the following we discuss ACI based on MMLE on the lines of Tiku and Suresh (1992).

The likelihood equation is given by

$$\frac{dL}{d\theta} = -\frac{n}{\theta} + \sum_{i=1}^{n} z_i - \frac{(k-1)}{\theta}\sum_{i=1}^{n}\frac{z_i e^{-z_i}}{(1-e^{-z_i})} \quad (2.6)$$

where $z_i = \frac{1}{\theta x_i}$

The maximum likelihood equation (2.6) does not have explicit solution for θ. This is due to the fact that the term $g(z_i) = \frac{z_i e^{-z_i}}{(1-e^{-z_i})}$ is intractable. In this paper, we use the MML approach to derive approximate MLE for θ by linearizing the term $g(z_i) = \frac{z_i e^{-z_i}}{(1-e^{-z_i})}$ using Taylor series expansion around the quantile point of F with reference to Tiku et. Al. (1986), Tiku and Suresh (1992), R.P.Suresh (2004). The linearization is done in such a way that the derived MML estimators retain all the desirable asymptotic properties of the maximum likelihood estimators.

Here MMLE is

$$\hat{\theta} = \frac{\sum_{i=1}^{n}\frac{1}{x_i}(1-b(k-1))}{n(a(k-1)+1)}$$
(2.7)

where $a = \frac{\lambda_q e^{-\lambda_q}}{1-e^{-\lambda_q}} - \frac{\lambda_q e^{-\lambda_q}(1-\lambda_q-e^{-\lambda_q})}{(1-e^{-\lambda_q})^2}$, $b = \frac{e^{-\lambda_q}(1-\lambda_q-e^{-\lambda_q})}{(1-e^{-\lambda_q})^2}$.

For more details one may refer to Tiku and Suresh (1992) and Suresh (2004).

Lemma 2.1: Distribution of $\left(\frac{\hat{\theta}_n}{\theta}\right)$ and $\left(\frac{\hat{\theta}}{\theta}\right)$, both are free from θ.

Proof: The proof is similar to the one given by Gulati and Mi (2006).

While constructing generalized pivot this lemma can be used.

3. Generalized Confidence Intervals

Suppose that $X=(X_1, X_2, \ldots, X_n)$ form a random sample from a distribution which depends on the parameters $\theta = (\psi, \nu)$ where ψ is the parameter of interest and ν^T is a vector of nuisance parameters. A generalized pivot $Q(X; x, \psi, \nu)$ where x is a observed value of X, for interval estimation defined by Weerahandi (1995), has the following properties:

i) $Q(X; x, \psi, \nu)$ has a distribution free of unknown parameters.

ii) The value of $Q(X; x, \psi, \nu)$ is ψ.

The percentiles of $Q(X; x, \psi, \nu)$ can then be used to obtain confidence intervals for θ. Such confidence intervals are referred to as generalized confidence intervals. For example, if $Q_{1-\alpha}$ denotes the $100_{1-\alpha}$th percentile of $Q(X; x, \psi, \nu)$, then $Q_{1-\alpha}$ is a generalized upper confidence limit for θ. A lower confidence limit or two-sided confidence limits can be similarly defined. Thus GCI is obtained by using a generalized pivot.

The generalized pivotal quantity based on $\hat{\theta}_n$ is

$$Q_i = \frac{\theta}{\hat{\theta}_i}\hat{\theta}_0 = \frac{\hat{\theta}_0}{\left(\hat{\theta}_i/\theta\right)} \quad i = 1, 2, \ldots N.$$

Obviously, the observed value of Q_i is θ. Moreover, the distribution of Q_i does not depend on unknown parameter. Therefore, Q_i is a generalized pivot for θ.

Computing algorithm

For a given data set $X_1, X_2 \ldots X_n$, the generalized confidence interval can be computed by the following steps. Here N is the number of simulations.

Algorithm to obtain GCI:

1. Input N, n, k, θ.
2. Generate independently and identically distributed observations (U1, U2,…,Un) from U(0,1).
3. For the given value of the parameter θ , set
4. $x_i = -\frac{1}{\theta \log(1-(1-U_i)^{1/k})}$ for i= 1,2,…,n.
5. Then (x1, x2,…, xn) is the required sample from the distribution of a k-unit series system with inverted exponential distribution as the component life distribution.
6. Compute MLE of θ ($\hat{\theta}_0$).
7. Generate N samples from F (.) (as given in (2.1)) by setting θ=1 and for each of the sample compute MLE (say $\hat{\theta}_i$).
8. Using $\hat{\theta}_0$ and $\hat{\theta}_i$, i=1,2,…,N
   $$Q_i = \frac{\theta}{\hat{\theta}_i}\hat{\theta}_0 = \frac{\hat{\theta}_0}{\left(\hat{\theta}_i/\theta\right)} \quad i = 1, 2, \ldots N.$$
9. compute
10. Arrange Qi in ascending order as Q [1] , Q[2], …, Q[N].
11. Compute GCI for θ as [Q([Nα/2]) , Q([N(1-α/2)])].
12. Extending above algorithm one can estimate coverage probability of the proposed GCI. Here [Q([Nα/2]) , Q([N(1-α/2)])] is a two-sided 100(1-α) percent GCI based on MLE.
13. In the above algorithm, we can replace MLE by MMLE and obtain GCI, based on MMLE.

1. Simulation study

We conduct extensive simulation experiments to evaluate performance of GCIs based on MLE and MMLE. We choose different values of θ, k, n and α. Results are tabulated in Table (1-6). Figures in the 1st row are based on MLE, while figures in the 2nd row are based on MMLE. From tables 1-6, we observe that simulated coverage of GCI does not differ significantly whether it can be computed from MLE as well as MMLE. However, large sample approach underestimates the coverage probabilities for most of the scenarios, especially when the sample size is small and (or) the parameter θ is large. Also the performance of the proposed GCI does not depend on θ. As the sample size is large, the two estimators (MLE, MMLE) are equally efficient.The results reported in this paper can be extended to other members of inverted scale family of distributions given by Potdar and Shirke (2013).

able 1. Mean coverage of ACI and GCI by using MLE and MMLE when θ=1, k=1.

Nominal covera	0.90		0.95		0.97		0.99	
	ACI	GCI	ACI	GCI	ACI	GCI	ACI	GCI
2	0.7631	0.9021	0.7924	0.9547	0.8096	0.9726	0.8436	0.9968
3	0.7973	0.9095	0.8262	0.9525	0.8436	0.9741	0.8797	0.9932
4	0.814	0.9032	0.8537	0.9519	0.8688	0.9719	0.8989	0.9956
5	0.8278	0.9001	0.8731	0.9539	0.8834	0.9775	0.9168	0.9958
6	0.8454	0.9064	0.8856	0.9564	0.898	0.9784	0.9291	0.9905
7	0.8533	0.9031	0.8841	0.9599	0.9003	0.9764	0.9345	0.9911
8	0.8621	0.9022	0.8913	0.9531	0.9152	0.9733	0.9369	0.9987
9	0.8633	0.9040	0.8955	0.9588	0.9075	0.9709	0.9399	0.9907
10	0.8647	0.9019	0.9032	0.9594	0.9208	0.9757	0.9471	0.9914
15	0.8829	0.9008	0.9211	0.9534	0.9275	0.9794	0.9596	0.9994
30	0.8869	0.9054	0.9345	0.9589	0.9497	0.9720	0.9736	0.9961
50	0.893	0.9087	0.9428	0.9564	0.9569	0.9790	0.9783	0.9985

Table2. Mean coverage of ACI and GCI when θ=1, k=2.

Nominal covera	0.90		0.95		0.97		0.99	
	ACI	GCI	ACI	GCI	ACI	GCI	ACI	GCI
2	0.6998	0.9025	0.7095	0.9521	0.8733	0.9703	0.9062	0.9935
3	0.7221	0.9016	0.7478	0.9531	0.8955	0.9714	0.9281	0.9964
4	0.7394	0.9002	0.782	0.9542	0.9097	0.9798	0.9381	0.9917
5	0.7543	0.9031	0.7831	0.9515	0.9198	0.9714	0.9471	0.9994
6	0.778	0.9008	0.8078	0.9529	0.9321	0.9825	0.9515	0.9933
7	0.7849	0.9014	0.8149	0.9564	0.935	0.9798	0.9511	0.9909
8	0.7938	0.9087	0.8331	0.9524	0.9368	0.9735	0.9572	0.9947
9	0.8021	0.9019	0.8373	0.9508	0.9393	0.9744	0.9604	0.9964
10	0.8017	0.9034	0.8423	0.9511	0.9407	0.9729	0.964	0.9919
15	0.8207	0.9064	0.8715	0.9509	0.9535	0.9767	0.9713	0.9935
30	0.8469	0.9074	0.9005	0.9534	0.9563	0.9714	0.9774	0.9960
50	0.9487	0.9108	0.9469	0.9524	0.9601	0.9731	0.9823	0.9976

Table 3. Mean coverage of ACI and GCI when θ=1, k=3.

Nominal covera	0.90		0.95		0.97		0.99	
	ACI	GCI	ACI	GCI	ACI	GCI	ACI	GCI
2	0.8478	0.9010	0.8833	0.9565	0.8889	0.9709	0.9264	0.9958
3	0.8649	0.9037	0.8992	0.9567	0.9155	0.9780	0.9414	0.9919
4	0.8726	0.9005	0.9082	0.9558	0.9303	0.9757	0.9567	0.9971
5	0.8823	0.9081	0.92 0.91	0.958	0.9333	0.9744	0.9578	0.9911
6	0.8828	0.9029	0.9268	0.9514	0.9429	0.9788	0.9631	0.9904
7	0.8905	0.9097	0.9257	0.9574	0.9391	0.9764	0.965	0.9920
8	0.8873	0.9011	0.926	0.9502	0.9479	0.9779	0.9707	0.9994
9	0.8858	0.9027	0.9333	0.9515	0.9515	0.9702	0.9699	0.9917
10	0.8916	0.9099	0.9377	0.9541	0.9448	0.9747	0.9724	0.9968
15	0.8913	0.9012	0.9409	0.9556	0.9562	0.9798	0.9817	0.9974
30	0.8945	0.9006	0.9477	0.9509	0.9626	0.9705	0.9838	0.9913
50	0.8956	0.9002	0.9475	0.9601	0.9641	0.970	0.9849	0.9991

Table 4. Mean coverage of ACI and GCI when θ=2, k=1.

Nominal								
2	0.76	0.90	0.79	0.95	0.81	0.97	0.84	0.99
3	0.79	0.90	0.82	0.95	0.84	0.97	0.87	0.99
4	0.82	0.90	0.85	0.95	0.86	0.97	0.90	0.99
5	0.83	0.90	0.86	0.95	0.89	0.97	0.91	0.99
6	0.84	0.90	0.88	0.95	0.89	0.97	0.92	0.99
7	0.85	0.90	0.88	0.95	0.90	0.97	0.93	0.99
8	0.85	0.90	0.89	0.95	0.90	0.97	0.93	0.99
9	0.86	0.90	0.89	0.95	0.91	0.97	0.94	0.99
10	0.86	0.90	0.90	0.95	0.91	0.97	0.94	0.99
15	0.88	0.90	0.91	0.95	0.92	0.97	0.95	0.99
30	0.89	0.90	0.93	0.95	0.95	0.97	0.96	0.99
50	0.89	0.90	0.94	0.95	0.96	0.97	0.98	0.99

Table 5. Mean coverage of ACI and GCI when θ=2, k=2.

Nominal covera	0.90		0.95		0.97		0.99	
	ACI	GCI	ACI	GCI	ACI	GCI	ACI	GCI
2	0.8275	0.9009	0.8541	0.9513	0.874	0.9768	0.905	0.9901
3	0.826	0.9012	0.8809	0.9550	0.8977	0.9789	0.914	0.9925
4	0.8454	0.9013	0.8927	0.9565	0.9163	0.9701	0.931	0.9989
5	0.8628	0.9041	0.9037	0.9540	0.909	0.9789	0.949	0.9947
6	0.8738	0.9005	0.9127	0.9528	0.942	0.9712	0.964	0.9905
7	0.8768	0.9006	0.9134	0.9586	0.928	0.9758	0.956	0.9915
8	0.8809	0.9047	0.9238	0.9510	0.949	0.9715	0.971	0.9934
9	0.8848	0.9015	0.9245	0.9573	0.944	0.9729	0.969	0.9987
10	0.8845	0.9085	0.925	0.9588	0.927	0.9709	0.968	0.9913
15	0.888	0.9024	0.9354	0.9514	0.942	0.9714	0.97 0.99	0.9924
30	0.8959	0.9025	0.9411	0.9550	0.956	0.9729	0.983	0.9941
50	0.8913	0.9015	0.9501	0.9526	0.961	0.9713	0.992	0.9907

CONCLUSION

Generalized confidence intervals are provided for the scale parameter of life time distribution of k-unit series system, when unit life time distribution is inverted exponential. The proposed confidence interval performs satisfactory for small to moderate sample sizes. These intervals are superior to the asymptotic confidence intervals.

Table 6. Mean coverage of ACI and GCI by using MLE when θ=2, k=3.

Nominal covera	0.90		0.95		0.97		0.99	
	ACI	GCI	ACI	GCI	ACI	GCI	ACI	GCI
2	0.8484	0.8999	0.8901	0.9548	0.9025	0.9761	0.9281	0.9987
3	0.8702	0.8915	0.902	0.9549	0.9187	0.9708	0.9448	0.9985
4	0.8816	0.9014	0.9139	0.9547	0.9279	0.9786	0.9538	0.9990
5	0.8826	0.9009	0.9193	0.9589	0.9344	0.9741	0.9615	0.9909
6	0.885	0.9021	0.9254	0.9558	0.9362	0.9789	0.9673	0.9994
7	0.8843	0.9089	0.9268	0.9504	0.9452	0.9714	0.9674	0.9904
8	0.8872	0.9030	0.9303	0.9568	0.9431	0.9735	0.9686	0.9970
9	0.8925	0.9029	0.9307	0.9504	0.9481	0.9771	0.9731	0.9988
10	0.8951	0.9065	0.9334	0.9587	0.9513	0.9700	0.9743	0.9905
15	0.8935	0.9054	0.9368	0.9549	0.9576	0.9756	0.979	0.9946
30	0.901	0.9047	0.9423	0.9506	0.9623	0.9778	0.9838	0.9919
50	0.8929	0.9068	0.9476	0.9555	0.9616	0.9777	0.9859	0.9928

REFERENCES

1. Bhattacharya G.K. (1985) The asymptotics of maximum likelihood and related estimators based on Type II censored data, Journal of American Statistical Association, 80 ,398-404.
2. Cramer H. (1946) Mathematical Methods of Statistics, Princeton University Press, Prinnceton, N.J.
3. Guo H. and Krishnamoorthy K. (2005) Comparison between two quantiles: Normal and Exponential cases. Communications in Statistics, Simulation and computation, 34,243-252.
4. Jordan S. M. and Krishnamoorthy, K. (1996) Exact confidence intervals for the common mean of several normal populations. Biometrics, 52, 77-86.
5. Krishnamoorthy K., Mathew T., Ramchandran G. (2006) Generalized p-values and confidence intervals: A Novel approach for analyzing log normally distributed exposure data. Journal of Occupational and Environmental Hygiene, 3, 642-650.
6. Krishnamoorthy K., Mathew T (2003) Inferences on the means of lognormal distributions using generalized p-values and generalized confidence intervals. Journal of Statistical Planning and Inference, 115, 103-121.
7. Krishnamoorthy K. and Mathew T. (2004) One-Sided tolerance limits in balanced and unbalanced one-way random models based on generalized confidence limits. Technometrices, 46, 44-52.
8. Krishnamoorthy K., Mukherjee S. and Guo H. (2007) Inference on reliability intwo-parameter exponential stress-strength model. Metrika, vol. 65, 261 - 273.
9. Kumbhar, R. R. and Shirke, D. T., (2004) Tolerance limits for lifetime distribution of k-Unit parallel system, Journal of Statistical Computation and Simulation, 74, 201-213.
10. Kurian K. M., Mathew T. and Sebastian, G. (2008) Generalized confidence intervals for process capability indices in the one-way random model. Metrika, 67, 83-92.
11. Lin C.T., Duran B.S., Lewis T.O.(1989) Inverted gamma as life distribution, Microelectron Reliability, 29(4), 619-626.
12. Ng C.K. (2007) Performance of the three methods of the interval estimation of coefficient of variation. *Interstat.*

13. Potdar K.G., Shirke D.T. (2013) Inference for the parameters of generalized inverted family of distributions. Probstat Forum, 06, 18-28.
14. Potdar K.G., Shirke D.T. (2014) Reliability estimation of k-unit series system based on progressively censored data, Electronic journal of applied statistical analysis, 07,228-253.
15. Suresh R.P (2004) Estimation of location and scale parameters in a two parameter exponential distribution from a censored sample, Statistical Methods, 6(1),82-89.
16. Suresh R.P (1997) On approximate likelihood estimators in censored normal samples. Gujarat Statistical Review, 24, 21-28.
17. Tian L.L., Cappelleri J.C. (2004) A new approach for interval estimation and hypothesis testing of a certain intraclass correlation coefficient: the generalized variable method. Statistics in Medicine, 23, 2125 –2135.
18. Tiku M.L. (1967) Estimating the mean and standard deviation from a censored sample. Biometrica, 54, 155-165.
19. Tiku M.L. (1968) Estimating the parameters of normal and logistic distribution from censored samples, Australian Journal of Statistic, 10, 64-74.
20. Tiku M.L., Suresh R.P. (1992) A new method of estimation for location and scale parameters, Journal of Statistical Planning and Inference, 30, 281-292
21. Tiku M.L., Tan W.Y., Balkrishnan N. (1986) Robust Inference, Marvel Delker, Inc, New York.
22. Stefanski L. A. (1996) A note on the arithmetic-geometric-harmonic means inequalities. The American Statistician, 50(3), 246-247.
23. Singh S.K., Singh U., Kumar D.(2012) Bayes estimators of the reliability function and parameter of inverted exponential distribution using informative and non-informative priors, Journal of Statistical Computation and Simulation, DOI 10,1080/00949655.2012.690156.
24. Tian L.L., Cappelleri J.C. (2004) A new approach for interval estimation and hypothesis testing of a certain intraclass correlation coefficient: the generalized variable method. Statistics in Medicine, 23, 2125 –2135.
25. Tsui K., Weerahandi S. (1989) Generalized p-values in significance testing of hypotheses in the presence of nuisance parameters Journal of American Statistical Association, 84,602–607.
26. R.P.Suresh (2004) Estimation of location and scale parameters in a two parameter exponential distribution from a censored sample, 6(1), 82-89.
27. Verrill S. and Johnson R.A. (2007) Confidence bounds and hypothesis tests for normal distribution coefficients of variation. Communications in Statistics, 36, 2187-2206.
28. Weerahandi S. (1993) Generalized confidence intervals. Journal of American Statistical Association 88,899–905.
29. Weerahandi S. (1995) Exact Statistical methods for Data Analysis. Springer,New York.
30. Weerahandi S., Johnson R. A. (1992) Testing reliability in a stress-strength model when X and Y are normally distributed. Technometrics, 34, 83–91.
31. Yu P. L.H., Sun Y. and Sinha B. K. (1999) On exact confidence intervals for the common mean of several normal populations. Journal of Statistical Planning and Inference. 81, 263-277.
32. Ye R.D., and Wang S.G. (2008) Generalized inferences on the common mean in the MANOVA models. Communications in statistics theory and methods, 37, 2291-2303.