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SOME REMARKS IN SEMI Q-LATTICE

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Abstract- ***In this paper, We define concepts semi q-lattice and down set, proper down set, prime down set, minimal down set for semi q-lattice. We prove Any prime down set of a semi q-lattice contains a minimal prime down set .***
Also we define filter in semi q-lattice and prove F be a non empty proper subset of a semi q-lattice S. Then F is a filter if and only if S-F is a prime down set .We prove F be a non empty sub set of a semi q-lattice S. Then F is a maximal filter if and only if S - F is a minimal prime down set .

Keywords - Semi q-lattice, Down Sets, Prime Down Sets, Filter

1. INTRODUCTION

Ivan Chajda [8] introduced the concept of a q-lattice and defined distributive q-lattice. After that G. C. Rao, P. Sundarayya, S. Kalesha vali, and Ravi Kumar Bandaru [3] defined ideals of distributive q-lattice , A. D. Lokhande, Ashok S Kulkarni [4] in paper 'Filter and Annihilator in Distributive q-lattices' defined Filter in a distributive q-lattice In paper [4] A. D. Lokhande, Ashok S Kulkarni defined annihilator in distributive q-lattice A and proved some properties.

In this paper We define concepts semi q-lattice and down set, proper down set, prime down set, minimal down set for semi q-lattice. We prove Any prime down set of a semi q-lattice contains a minimal prime down set We prove some remarks related to filter and maximal filter

2. PRELIMINARIES

Definition 2.1:[3]. An algebra $(A, \vee, \wedge)$ whose binary operations $\vee, \wedge$ satisfy the following is called a q-lattice.

(i) $a \vee b = b \vee a$; $a \wedge b = b \wedge a$ (commutativity)

(ii) $a \vee (b \vee c) = (a \vee b) \vee c$; $a \wedge (b \wedge c) = (a \wedge b) \wedge c$ (associatativity)

(iii) $a \vee (a \wedge b) = a \vee a$; $a \wedge (a \vee b) = a \wedge a$
(weak- absorption)

(iv) $a \vee b = a \vee (b \vee b)$; $a \wedge b = a \wedge (b \wedge b)$ (weak-idempotence)

(v) $a \vee a = a \wedge a$ (equalization)

3. semi q-lattice

Definition 3.1: S is a non empty set and $\wedge$ is a binary operation then $(S, \wedge)$ where binary operation $\wedge$ satisfy for all a, b, c $\in$ S the following

1) $a \wedge b = b \wedge a$ (Commutative)
2) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$ (Associative)
3) $a \wedge b = a \wedge (b \wedge b)$ (Weak idempotence)

Then $(S, \wedge)$ is called semi q-lattice

Definition 3.2:
A semi q- lattice S with 0 is called 0-distributive if for any a, b, c $\in$ S such that
$a \wedge b = 0 = a \wedge c$ implies $a \wedge d = 0$ for some d satisfying $d \wedge b = b \wedge b$ and
$d \wedge c = c \wedge c$

Definition 3.3:
Let S be a semi q- lattice. A non –empty subset D of S is called a down-set if $a \wedge a \in D$, $b \in S$ with $a \wedge b = b \wedge b$ implies that $b \wedge b \in D$.
A down-set D of S is called a proper down set if $D \neq S$.
Definition 3.3:
A prime down set is a proper down set P of S such that $a \wedge b \in P$ implies $a \in P$ or
$b \in P$.

Definition 3.4:
A prime down set P is called minimal if there is a prime down set Q such that $Q \subseteq P$, then $P = Q$.

Theorem 3.1: Any prime down set of a semi q-lattice contains a minimal prime down set .

Proof: Let S be a semi q-lattice with 0. Let P be a prime down set of S and let þ be the set of all prime down sets contained in P.
Therefore þ = { Q / Q is a prime down set contained in P }
Then þ is non empty since $P \in$ þ.
Let $\hat{C}$ be a chain in þ and let $M = \cap \{ X / X \in \hat{C} \}$
We claim that M is a prime down set.
As for all $a \wedge a \in P$ and for $0 \in S$
$a \wedge 0 = 0 \wedge 0 = 0$ implies $0 \in P$
Therefore 0 belongs to every down set
Implies $0 \in M$
Hence M is non empty.
To show M is a prime down set
i) let $a \wedge a \in M$, $b \in S$ such that $b \wedge a = b \wedge b$.
As $a \wedge a \in M$ implies $a \wedge a \in X$, $b \in S$ for all $X \in \hat{C}$ such that $b \wedge a = b \wedge b$.

Hence $b \wedge b \in X$ for all $X \in \hat{C}$ as X is a down set.

Thus $b \wedge b \in M$

Therefore M is a down set.

ii) Now let $x \wedge y \in M$ for some $x, y \in S$.

Then $x \wedge y \in X$ for all $X \in \hat{C}$.

Since X is a prime down set for all $X \in \hat{C}$,

we have either $x \in X$ or $y \in X$ for all $X \in \hat{C}$.

This implies that either $x \in M$ or $y \in M$.

Hence M is a prime down set.

Thus by applying the dual form of Zorms lemma to β , there is a minimal member of β.

Definition 3.5:

Let S be a semi q-lattice . A non empty subset F of S is called a filter if

i) $a, b \in F$ implies $a \wedge b \in F$

ii) $a \in S$, $b \wedge b \in F$ satisfying $a \wedge b = b \wedge b$ then $a \wedge a \in F$.

A filter F of a semi q-lattice S is called proper filter if $F \neq S$.

A maximal filter F of S is called proper filter if it is not contained in any other proper filter. This means if there is a proper filter G such that $F \subseteq G$ then $F = G$.

Theorem 3.2 : Let M be a proper filter of S with o. Then M is maximal if and only if for all $a \in S\text{-}M$, there is some $b \in M$ such that $a \wedge b = o$.

Proof: Let M be a proper filter of S with o.

suppose M is maximal and

for all b in M, $a \wedge b \neq 0$ where $a \notin M$ means $a \in S\text{-}M$

Consider the set

$M' = \{y \in S : y \wedge (a \wedge b) = a \wedge b,\ b \in M\}$

Now to show M' is a filter of S

(1) Let $x, y \in M'$ Implies $x \wedge (a \wedge b) = a \wedge b$ and $y \wedge (a \wedge b) = a \wedge b$

$$\begin{aligned} \text{Now } (x \wedge y) \wedge (a \wedge b) &= x \wedge (y \wedge (a \wedge b)) \\ &= x \wedge (a \wedge b) \\ &= (a \wedge b) \end{aligned}$$

Implies $(x \wedge y) \in M'$

(2) Now let $z \in S$, $y \wedge y \in M'$ such that $z \wedge y = y \wedge y$

means $z \in S$, $(y \wedge y) \wedge (a \wedge b) = a \wedge b$ such that $z \wedge y = y \wedge y$

Now consider

$$\begin{aligned} (z \wedge z) \wedge (a \wedge b) &= (a \wedge b) \wedge (z \wedge z) \\ &= (a \wedge b) \wedge z \\ &= z \wedge (a \wedge b) \quad \text{--------(i)} \end{aligned}$$

Also as $z \wedge y = y \wedge y$

Implies $(z \wedge y) \wedge (a \wedge b) = (y \wedge y) \wedge (a \wedge b)$

Implies $z \wedge (y \wedge (a \wedge b)) = (a \wedge b) \wedge (y \wedge y)$

Implies $z \wedge (a \wedge b) = (a \wedge b)$ as $y \wedge y \in M'$

Implies $z \wedge (a \wedge b) = (a \wedge b)$ -------------(ii)

from (i) and (ii) we write

$(z \wedge z) \wedge (a \wedge b) = (a \wedge b)$

Therefore $(z \wedge z) \in M'$

Therefore M' is a filter of S.

Now as $0 \wedge (a \wedge b) = 0$

Means as $0 \wedge (a \wedge b) \neq (a \wedge b)$ implies $0 \notin M'$

Hence M' is proper filter.

Now let $b \in M$ then $\exists$ $a \notin M$

Such that $a \wedge b \neq 0$

$$\begin{aligned} \text{Now } b \wedge (a \wedge b) &= (b \wedge a) \wedge b \\ &= (a \wedge b) \wedge b \\ &= a \wedge (b \wedge b) \\ &= a \wedge b \end{aligned}$$

Implies $b \in M'$

Thus $M \subseteq M'$

Which is contradiction of maximality of M

Hence for $a \in S\text{-}M$ there must exist some b in M such that $a \wedge b = 0$

Conversely let for any element $a \in S\text{-}M$ means for $a \notin m$ there exist an element $b \in M$ with $a \wedge b = 0$

To show that M is maximal :

Now, suppose M is not maximal then (as $0 \in S$),

there exist a maximal filter M' properly containing M means $M \subset M'$

Therefore for any element $a \in M' - M$, means for any $a \notin M$ there exist an element $b \in M$ such that $a \wedge b = 0$

Now as $a \in M'$, $b \in M'$ and as M' is filter imply that $a \wedge b \in M'$

Implies $a \wedge b = o \in M'$

Which is a contradiction Hence M must be a maximal filter.

Theorem 3.3: Let F be a non empty proper subset of a semi q-lattice S. Then F is a filter if and only if S-F is a prime down set .

Proof : Let F be a filter of a semi q-lattice S .

To show that S-F is a prime down set

Let $x \wedge x \in S\text{-}F$, $y \in S$ and $y \wedge x = y \wedge y$.

Then $x \wedge x \notin F$, $y \wedge x = y \wedge y$ and F is a filter

Implies $y \wedge y \notin F$.

This implies $y \wedge y \in S\text{-}F$.

Thus S-F is a down set .

Since F is a filter and $S\text{-}F \neq S$.

Thus S-F is a proper down set .

Now to prove S- F is a prime down set.

Let $a, b \in S$ such that $a \wedge b \in S - F$.

Then $a \wedge b \notin F$ and hence as F is a filter either $a \notin F$ or $b \notin F$.

This implies either $a \in S - F$ or $b \in S\text{-}F$.

Therefore, S - F is prime down set.

Conversely, Let S - F be a prime down set

To prove that F is a filter

Let $x, y \in F$. Then clearly, $x, y \notin S - F$

Hence $x \wedge y \notin S - F$ as S- F is a prime down set .

Thus $x \wedge y \in F$.

Now let $y \in S$, $x \wedge x \in F$ and $x \wedge y = x \wedge x$.

Then $x \wedge x \notin S - F$ and $x \wedge y = x \wedge x$

Since S - F is a down set ,

we have $y \wedge y \notin S - F$.

Hence $y \wedge y \in F$.

This implies F is a filter of S.

Theorem 3.4: Let F be a non empty sub set of a semi q-lattice S. Then F is a maximal filter if and only if S - F is a minimal prime down set .

Proof: Let F be a maximal filter of a semi q-lattice
and S - F is not a minimal prime down set .
Then there exists a prime down set I such that $I \subseteq S - F$
Which implies $F \subseteq S - I$
Which contradict to the maximality of F .
Hence S - F is a minimal prime down set .

Conversely, Let S - F be a minimal prime down set
and F is not a maximal filter.
Thus there exists a proper filter G such that $F \subseteq G$
Implies $S - G \subseteq S - F$
which contradict the minimality of S -F.
Hence F is a maximal filter of S .

Definition 3.6: A semi q- lattice S is called directed above if for all x , y ∈ S there exists z ∈ S such that $z \wedge x = x \wedge x$ and $z \wedge y = y \wedge y$.

Theorem 3.5: Let S be a directed above semi q-lattice with 0. If S is not 0 -distributive , then the set
$F = \{ x \in S / x \wedge (a \wedge y) = (a \wedge y) \neq 0$ for all $y \wedge b = b \wedge b$ and $y \wedge c = c \wedge c \}$
Where a ,b , c ∈ S such that $a \wedge b = a \wedge c = 0$, is a proper filter.

Proof: As S is not 0-distributive , there are a , b, c ∈ S such that $a \wedge b = a \wedge c = 0$ and $a \wedge d \neq 0$ for all $d \wedge b = b \wedge b$, $d \wedge c = c \wedge c$
As $a \wedge (a \wedge y) = (a \wedge y)$
Implies $a \in F$. Hence F is non empty.
Also as $0 \wedge (a \wedge y) = 0 \neq (a \wedge y)$
Implies $0 \notin F$
Now to show F is a filter
Let $x, y \in F$
Implies $x \wedge (a \wedge y) = (a \wedge y)$ and $y \wedge (a \wedge y) = (a \wedge y)$
For all $y \wedge b = b \wedge b$, $y \wedge c = c \wedge c$
Now consider

(1) $(x \wedge (a \wedge y)) \wedge (y \wedge (a \wedge y)) = (a \wedge y) \wedge (a \wedge y)$
Implies $((x \wedge (a \wedge y)) \wedge y) \wedge (a \wedge y) = (a \wedge y)$
Implies $(x \wedge (y \wedge (a \wedge y))) \wedge (a \wedge y) = (a \wedge y)$
Implies $(x \wedge y) \wedge ((a \wedge y) \wedge (a \wedge y)) = (a \wedge y)$
Implies $(x \wedge y) \wedge (a \wedge y) = (a \wedge y)$
Implies $x \wedge y \in F$

(2) Let $x \wedge x \in F$, $z \in S$ with $z \wedge x = x \wedge x$ then to show $z \wedge z \in F$
As $x \wedge x \in F$, $z \in S$ with $z \wedge x = x \wedge x$
Implies $(x \wedge x) \wedge (a \wedge y) = (a \wedge y)$ for all $y \wedge b = b \wedge b$, $y \wedge c = c \wedge c$

And $z \in S$ with $z \wedge x = x \wedge x$

To show $z \wedge z \in F$
We have to show $(z \wedge z) \wedge (a \wedge y) = (a \wedge y)$ for all $y \wedge b = b \wedge b$, $y \wedge c = c \wedge c$
AS $(z \wedge z) \wedge (a \wedge y) = (a \wedge y) \wedge (z \wedge z)$
$= ((a \wedge y) \wedge z)$
$= (z \wedge (a \wedge y))$
We show $z \wedge (a \wedge y) = a \wedge y$ for all $y \wedge b = b \wedge b$, $y \wedge c = c \wedge c$
And $z \in S$ with $z \wedge x = x \wedge x$
Consider $(z \wedge x) = (x \wedge x)$
$(z \wedge x) \wedge (a \wedge y) = (x \wedge x) \wedge (a \wedge y)$
$(z \wedge x) \wedge (a \wedge y) = x \wedge (x \wedge (a \wedge y))$
$z \wedge (x \wedge (a \wedge y)) = x \wedge (x \wedge (a \wedge y))$
$z \wedge (a \wedge y) = x \wedge (a \wedge y)$
$z \wedge (a \wedge y) = a \wedge y$
Implies $z \in F$
And since $0 \notin F$
Therefore F is a proper filter.

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