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A NOTE ON NUMBER OF BINAY SEQUENCES ENDING BY A TYPICAL RUN

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Abstract- ***Binary sequences have many applications in statistics. In this article we give a simpler method than that of Feller (1972) to find number of sequences for which the run of 1's of length r (> 0) occurs for first time as the right end tail subsequence. We have shown that this is a valid rule and the expression for mean time is obtained.***

Key words: Random length; Mean length; Bernoulli trials; Sequential rule.

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I. INTRODUCTION

Binary sequences and a typical run:Consider a binary sequence of 0 and 1 of random length N. For N = k, in all there are 2^k sequences. Of these we are interested to find number of sequences for which the run of 1's of length r (> 0) occurs for first time as the right tail subsequence. Collection of all such sequences is denoted by a set B_k. Note that B_i's are disjoint. Here we have $B_1 = B_2 = \ldots = B_{r-1} = \phi$ and B_r, a set containing sequence of 1's of length r. If stopped according to this rule by taking sequences of length N = k (> r) then the subsequence of 1's of length r do not appear in the first (k-r-1) places and '0' must appear in $(k-r)^{th}$ place together with '1' appearing in each of the $(k-r+1)^{th}$, $(k-r+2)^{th}$, …, $(k-1)^{th}$ and k^{th} places. For a path leading to a sequence in B_k, $k \geq r$, the right tail subsequence must start with 0 followed by 1's of length r and the left tail subsequence of length k-r-1 (if any) should *not* contain 1's of length r. For $k \geq r$, total number of sequences of length k with right tail subsequence starting with 0 followed by 1's of length r is 2^{k-r-1}. But for a path leading to a sequence in B_k, we should not have subsequence containing 1's of length r in the left tail of length k-r-1. We know that as any such sequence containing 1's of length r is of the form "subsequence containing 1's of length r occurs for the first time by the end of the left tail subsequence of length i ($\leq$ k) and followed by any subsequence of length k-r-1-i". Let $|B_i|$, cardinality of B_i, denote the total number of sequences of length i for which subsequence containing 1's of length r occurs only at the end. Hence the total number of sequences with the left tail of length k-r-1 containing at least one subsequence of 1's of length r is $\sum_{i=1}^{k-r-1} |B_i|\, 2^{k-r-1-i} = \sum_{i=r}^{k-r-1} |B_i|\, 2^{k-r-1-i}$. Thus $|B_k| = 2^{k-r-1} - \sum_{i=r}^{k-r-1} |B_i|\, 2^{k-r-1-i}$. For example, in case of r = 4, the boundary regions are $B_1 = B_2 = B_3 = \phi$; $B_4 = \{(1111)\}$; $B_5 = \{(01111)\}$; B_6 has a 2 sequences; B_7 has 4 sequences; B_8 has 8 sequences; B_9 has $16 - \sum_{i=4}^{4} |B_i|\, 2^{4-i} = 16 - |B_4|\, 2^0 = 16 - 1 = 15$ sequences; B_{10} has $32 - \sum_{i=4}^{5} |B_i|\, 2^{5-i} = 32 - [|B_4|\, 2^1 + |B_5|\, 2^0] = 32 - 3 = 29$ sequences and so on (for details one may refer to [2]).

2. Valid Rule:

Let the probability of occurrence of 1 and 0 are respectively p and q such that p+q = 1. In the following, we will show that N is a proper random variable and obtain the mean length (time) E(N) of the rule (that is expected number of sequences in which the subsequence of 1's of length r occurred for first time as the right end tail subsequence) by simpler method than that of [1, page 324].

If $P_k = P(N = k)$ then we have, $P_r = p^r$; $P_{r+1} = qp^r$; $P_{r+2} = q^2 p^r + qp^{r+1} = qp^r$ and so on. In general for $k \geq r+1$, let E be the event that subsequence containing 1's of length r occurs only at the end in the sequences of length k. Then we have,

P_k = P(Subsequence of 1's of length r has occurred for the first time in the sequences of length k)

= P(Subsequence of 1's of length r does not occur in any of the previous sequences of length (k- r-1) and E occurred at end in the sequences of length k)

= P(N > k-r-1) P(N = r), since by independence

$= [1 - P(N \leq k\text{-}r\text{-}1)]\, qp^r$

$= [1 - (P_r + P_{r+1} + \ldots + P_{k-r-1})]\, qp^r$ (2.1)

This implies $P_r + P_{r+1} + \ldots + P_{k-r-1} = 1 - P_k/qp^r$ for all $k \geq r+1$. Hence we have,

$$P(N < \infty) = \sum_{k=1}^{\infty} P(N = k) = \lim_{k \to \infty} (P_r + P_{r+1} + \ldots + P_{k-r-1})$$

$= 1 - (qp^r)^{-1} \lim_{k \to \infty} P_k$ To find $\lim_{k \to \infty} P_k$ we proceed as below. From (2.1) we have,

$P_{k+1} - P_k = -qp^r P_{k-r}$ for all $k \geq r+1$.(2.2)

Since $P_{k+1} - P_k < 0$, $\{P_k\}$ is bounded and decreasing. Hence $\lim_{k \to \infty} P_k$ exists. Let $\lim_{k \to \infty} P_k = A$ (constant). Then from

(2.2) by taking limit as $k \to \infty$ we get,

$$A - A = -qp^3 A$$

$$\Rightarrow A = 0$$

$$\Rightarrow \lim_{k \to \infty} P_k = 0.$$

Hence $P(N < \infty) = 1$.

Thus the stopping rule is valid that is N is a proper r.v.

Mean Time:

Further from (2.2) we have, $P_k - P_{k+1} = qp^r P_{k-r}$ for $k = r+1, \ldots$

$$\Rightarrow \sum_{k=r+1}^{\infty} \{kP_k - (k+1)P_{k+1} + P_{k+1}\} = \sum_{k=r}^{\infty} qp^r\{(k-r)P_{k-r} + rP_{k-r}\}$$

$$\Rightarrow [E(N) - rP_r] - [E(N) - rP_r - (r+1)P_{r+1}] + [1 - P_r - P_{r+1}] = qp^r[E(N) + r]$$

$$\Rightarrow 1 - P_r + rP_{r+1} = qp^r[E(N) + r]$$

$$\Rightarrow 1 - p^r + rqp^r = rqp^r + qp^r E(N)$$

$$\Rightarrow E(N) = \frac{1 - p^r}{qp^r} \text{ for } r > 0. \quad (2.3)$$

Feller (1972) has obtained the same E(N) by using the probability generating function of stopping r.v N.

Applications in Statistics: Binary sequences are useful in sequential procedures. The above result is directly applicable to the sequential rule "Stop as soon as r successive 1's are observed" associated with a sequence of independent identical Bernoulli trials.

References

1. Feller William (Reprint, 1972). *An Introduction to Probability Theory and its Application Vol. I.*, Wiley Eastern Pvt. Ltd., New Delhi, India.
2. Patil H. S. (2009). *Sequential Procedures for Some Non-Regular and Regular Families of Distributions.* Ph. D Thesis (unpublished), Shivaji University, Kolhapur.