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ON DISTRIBUTION-FREE TESTS FOR ONE-SAMPLE LOCATION PROBLEM BASED ON SUBSAMPLE EXTREMA

Parameshwar V. Pandit[1] and Deepa R. Acharya[2]
[1]Department of Statistics, Bangalore University, Bangalore-560056
[2]Department of Statistics, Govt. Science College, Bangalore -560001
[1]panditpv12@gmail.com; [2]deeparacharya@yahoo.co.in

Abstract- ***One-sample location problem is one of fundamental problems in nonparametric inference. Wilcoxon signed rank test is the most popular test for location when the distribution of the underlying sample is from a symmetric continuous distribution. In this paper, we propose a distribution-free class of statistics for one-sample location problem based on U-statistic whose kernel depends on subsample extrema. The asymptotic distribution of the proposed class of test statistics is established using U-statistic theory. The performance of few members of the class is evaluated in terms of Pitman asymptotic relative efficiency relative to the Wilcoxon signed rank test, test proposed by Mehra, Prasad and Rao(1990), Shetty and Pandit(2000) and Rattihalli and Raghunath (2012). It is observed that the members of the proposed class of tests are better than the tests mentioned above for heavy and light tailed distributions.***

Keywords and Phrases: Asymptotic relative efficiency, One-sample location problem, Distribution-free, Symmetric distributions, U-statistics.

I. INTRODUCTION

The problem of testing for location in one sample setting is one of the fundamental problems studied in the area of nonparametric inference. This problem is addressed with the assumption that the underlined distribution of the sample is symmetric about the parameter.Hence the problem boils down to the problem of testing for point of symmetry. There are many distribution free tests available in the literature for the above problem. Sign and Wilcoxon-signed rank tests are the two well known nonparametric tests for the above problem. Madhav Rao(1990) proposed a test procedure based on subsamples median. Mehra, Prasad and Rao(1990) proposed a test procedure on a U-statistics whose kernel depends on a positive constant 'a' and subsamples of size two. Ahmad(1996) proposed Mann-Whitney-Wilcoxon type statistics for one sample location. A generalisation of test due to Mehra, Prasad and Rao(1990) was considered by Shetty andPandit(2000). Bandyopadhyay and Datta(2007) proposed an adaptive nonparametric tests for one sample location problem.Larocquea,Nevalainenb and Oja(2008) developed one sample location test for multilevel data.Recently Rattihalli and Raghunath (2012) proposed class of tests for one sample location problem based on subsample order statistics and medians.

It has been seen that sign test is a good choice when the underlying distribution of the sample is heavy tailed, where as Wilcoxon-sign rank test performs better for moderate tailed distributions.

In this paper, a class of distribution free test statistics based on U-statistics is proposed whose kernel is of order degree (k+3).Motivated by Ahmad(1996) the proposed statistics is based on subsample extremes. The expressions for expected value and asymptotic variance of the proposed statistics are derived for arbitrary k. The performance of few members of the proposed class is evaluated by means of asymptotic relative efficiency relative to t-test.

Section2 contains the new proposed class of tests. The distribution theory of the class of test statistics is presented in Section 3. Section 4 is devoted to the asymptotic relative efficiency comparisons. Some remarks and conclusions are given in Section 4.

1. Proposed class of test statistics

Let X_1, X_2,…,X_n be a random sample from an absolutely continuous distribution function $F_\theta(x)$=G(x-) where G(y)+G(-y)=1.The problem of testing is to test the hypothesis
H_o : against the alternative H_1 : .
We propose a test based on U-statisticsfor the above, which is given by,
where and *C* is the set of combination of integers, A is combinations of integers .

The test criterion is to reject H_0 in favour of H_1 for large values of . That is reject H_0 if $U_k < d_1$or $U_k > d_2$.

3. Distributional Properties of U_k

The mean of U_k is given by,under H_0.
The following theorem gives the asymptotic distribution of U_k.

Theorem 3.1: The asymptotic distribution of is normal with mean zero and variance , where), wherewhere and

Proof: Proof of the result is direct consequence of Hoeffding(1948).

The asymptotic distribution under $H_0$is given in the following corollary 3.1.

Corollary 3.1: The asymptotic distribution of ,
under H_0, is normal with mean zero and variance given by) ,
where and .

In the following table 1, we tabulate asymptotic variance of , under H_0 for different values of k.

Table 1:Asymptotic null variance of

k	2	3	4	5	6	7	8
Variance	0.492857	0.586753	0.614718	0.612066	0.595542	0.573087	0.548568

4.Asymptotic relative efficiency

In this section we first obtain the Pitman asymptotic relative efficiency of ,with respect to the classical t-test .For this we compute the efficacy of ,given by

The efficacy of the classical t-test is $1/\sigma$, where .So without loss of generality, assuming ,the expression itself becomes the ARE (.

Here, we consider Sign test(S), Wilcoxon signed-rank test(W), test due to Mehra et.al.,(1990)(T_{a*}), test due to Shetty and Pandit(2000)($U_{a*}(4,2)$) and test due to Rattihalli and Raghunath(2012)($V_{a*}(3,1)$). The various values of ARE's eff(S),eff(W),eff(T_{a*}),eff(Ua*(4,2)) for various underlying probability distributions like Cauchy,Laplace, Logistic,Triangular, Parabolic,Uniform and the distribution with pdf are given in Table 2. Table 3 gives the AREs eff(U_k) for various values of k.

Table:2:The AREs of different tests relative to T-test

Density	eff(S)	eff(W)	eff(Ta*)	eff(Ua*(4,2))	eff(Va*(3,1))
Cauchy	0.4053	0.3040	0.4053	0.4252	0.6411
Laplace	2.0000	1.5000	2.0000	2.0000	2.0000
Logistic	0.8225	1.0966	1.0966	1.1364	1.0799
Triangular	0.6667	0.8889	0.8889	0.7965	0.9657
Parabolic	0.4500	0.8460	0.9360	0.8125	1.0829
Uniform	0.3333	1.3333	1.3333	0.6869	2.0021
No name	0	2.6667	10.6600	16.4563	14.3957

Table 3: AREs of U_k relative to T-test.

Density	k=2	k=3	k=4	k=5	k=6	k=7	k=8	Remark
Laplace	2.3712	2.1956	2.0018	1.8166	1.6507	1.5063	1.3226	↓ in k
Logistic	1.0680	1.0298	0.9830	0.9331	0.8838	0.8369	0.7930	↓ in k
Triangular	1.9218	1.9689	1.9987	2.0175	2.0296	2.0381	2.0442	↑ in k
Parabolic	1.0253	1.1003	1.1626	1.2138	1.2562	1.2919	1.3224	↑ in k
Uniform	1.5217	1.8406	2.1690	2.5007	2.8335	3.1667	3.5002	↑ in k
Noname	5.3002	7.0833	8.9906	10.9569	12.9491	14.9518	16.9578	↑ in k

5. Some Remarks and Conclusions:

1. A class of test statistics for one-sample location problem is considered in the paper assuming that the underlying distribution of the sample drawn is symmetric. The test proposed in this situation is testing for point of symmetry in one-sample problem.
2. The asymptotic variance of the few members, U_k (for k=2,3,4,5,6,7,8) of the class of test statistics are computed as a ready reference for the researchers.
3. The performance of the members of the proposed class is evaluated in terms of asymptotic relative efficiencies (AREs).
4. From table 2 and 3, it is observed that the performance of the proposed test is better than the tests existing in the literature for this problem for the distributions Laplace, Logistic, Triangular, Parabolic, Uniform and no name distributions. However, the performance behaviour is different for heavy tailed and light tailed distributions.
5. The performance of the test due to Rattihalli and Raghunath(2012) is better for Cauchy distribution.
6. For heavy tailed distributions such as Laplace and logistic distributions, the performance in terms of ARE decreases with k (that is with subsample size).
7. For light tailed distributions such as Triangular, Parabolic,Uniform and no name distributions, the performance in terms of ARE is increasing with k (that is with subsample size).

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