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CONGRUENCE OF MAGNETIC AND ELECTRIC FIELD LINES IN RIEMANN-CARTAN SPACETIME

V. K. Patil

Department of Mathematics,
Krantisinh Nana Patil College, Walwa-416 313
vinayakpatil.82@rediffmail.com

Abstract- ***By using the covariant techniques with full connection, we derived the nonlinear electromagnetic formulae and successively applied to the magnetohydrodynamics (MHD) approximations in Riemann-Cartan (RC) space-time. In this formalism, the torsion does not generate any different physical field but it occurs algebraic in character in covariant equations. It is proved that the magnetic field remains frozen-in with the highly conducting fluid and the electric field remains frozen-in with charged fluid provided that magnetic field vanishes in RC space-time. Finally, the derived electrodynamics formulae are successively applied to study the space-like congruences of magnetic and electric field lines. It is found that, the magnetic field lines have zero rotation in a perfectly conducting fluid if and only if the conduction current is orthogonal to the magnetic field.***

Keywords: Electromagnetic field, Space-like congruences, electrodynamics

I. INTRODUCTION

In RC space-time, we review the covariant approach to relativistic electrodynamics and then derive the set of constraints and propagation equations that describe the magnetic effect on the kinematics and dynamics of the medium. In particular, the torsion terms in the covariant equations are all algebraic in character; it does not give a different physical field. Therefore, we describe the covariant equations with mathematical transparency and physical clarity. Further, we consider the case of ideal MHD approximation and examine the kinematical implications in the evolution of magnetic field.

We consider the formalism of gauge invariance and minimal coupling to be compatible with the torsion. The electromagnetic field tensor has secure gauge invariance in the presence of torsion [1]. The Hojman-Rosenbaum-Ryan-Shepley (HRRS) dynamical theory of torsion preserves local gauge invariance of electrodynamics and makes minimal coupling compatible with torsion. The allowed torsion in this theory is completely determined by the gradient of a scalar function [2]. It is possible to propose a theory in which torsion and electromagnetism interact, without modifying the form of local gauge invariance, provided that a semi-minimal photon-torsion coupling is chosen on the ground of physical reasonableness [3]. A particle with a Coulombian electric field and a dipolar magnetic field has also a torsionic dipolar moment proportional to its magnetic moment and it generates a torsion field. Recently, HRRS theory is modified and made it consistent with experiment by introducing a massive component of the torsonic potential in the existence of both the electric and magnetic fields [4].

The magnetic fields are a widespread and significant component of the universe. To discuss the important role of magnetic field in RC space-time we extend our work to the ideal MHD approximation. A primordial magnetic field could produce global spin alignment near the big-bang in anisotropic cosmological models where the effects of spin and torsion may be important, and a strong magnetic field in a neutron star could cause spin alignment of the neutrons making up the fluid of a neutron star [5-7]. A primordial magnetic field produces global spin alignment and it is associated with the shear [8]. In the collapsed state of matter there will be orientation of the spin which is caused by a magnetic field. A cosmic magnetic field, if it exists, may have been sufficiently large in the past to cause spin alignment during the hottest stage of the evolution of universe [9].Therefore the role of the magnetic field vector may be significant for the further study in RC space-time.

The material of this paper is organized in the following manner: The Section 2 reveals the cursory account of the kinematics of time-like congruence in RC space-time of gravitation. In Section 3, the electrodynamics formulae are derived and extended to the ideal MHD approximation. In Section 4, the properties of congruence of magnetic and electric field lines are discussed. It is proved that, the comoving observer measures the electric strength which depends on the charge density of the fluid in the expansion or contraction of the electric field lines. Finally, some conclusions are drawn in Section 5.

§ (2): *Kinematical Descriptions in RC Space-time*

In the standard general relativity, the covariant approach leads to the kinematical variables of the fluid, its energy density and pressure and electromagnetic fields. Initially, the covariant techniques were introduced by Ellis [10] to study of electrodynamics fields and more recently it is reviewed in [11-13]. All these studies are relativistic and many authors have worked out with numerous applications. In RC space-time; we consider the covariant formalism leads to the kinematical variables with full connection of the fluid and the electromagnetic fields.

A comprehensive description of Riemann-Cartan kinematics (RCK) of time-like congruence is described in this section. In RCK, the connection Γ^a_{bc} is antisymmetric which is related to g_{ab} through $g_{ab/c} = 0$. Here the

covariant derivative with respect to the connection Γ^a_{bc} and Christoffel symbol $\{^a_{bc}\}$ are denoted by oblique line **/** (or the operator ∇_a) and by semicolon **;** (or by operator $\hat{\nabla}_a$) respectively. Because of the connection, the RCK includes the torsion term where the torsion $Q_{bc}{}^a$ is the antisymmetric part of connection and is defines as

$$Q_{bc}{}^a = \frac{1}{2}(\Gamma^a_{bc} - \Gamma^a_{cb}) \quad .(2.1)$$

It is well known that; the connection is expressed as the combination of Christoffel symbol and contorsion tensor $K_{bc}{}^a$ in the form

$$\Gamma^a_{bc} = \{^a_{bc}\} - K_{bc}{}^a \quad ,(2.2)$$

where $K_{bc}{}^a = -Q_{bc}{}^a + Q_c{}^a{}_b - Q^a{}_{bc}$.(2.3)

The covariant derivatives with respect to the connection and Christoffel symbol are related through the relation

$$A^a{}_{/b} = A^a{}_{;b} - K_{bc}{}^a A^c . \quad (2.4)$$

The signature (-, -, -, +) is used through out this paper. The covariant decomposition of 4-velocity u^a in RC space-time is given by [14]

$$u_{a/b} = \sigma_{ab} + \frac{\theta}{3}h_{ab} + \omega_{ab} + \dot{u}_a u_b + 2h_{ca}Q_{bd}{}^c u^d , \quad (2.5)$$

where $h_{ab} = g_{ab} - u_a u_b$ and $\dot{u}^a = u^a{}_{/b}u^b$. The vector u^a determines the time direction, and the tensor h_{ab} projects orthogonal to 4-velocity field into what is known as the observer's instantaneous rest space of the fluid. When dealing with multi-component medium, one needs to account for the velocity 'tilt' between the matter components and the fundamental observers. Here we will consider a single component fluid and we assume that the fundamental (comoving) observer moves with it. The observer's motion is characterized by the irreducible kinematical quantities which emerge from the covariant decomposition of u^a in equation (2.5).

The relative position vector $\delta_\perp x^a = h^a_b \delta x^b$ of associated world lines in a time-like congruence can be expressed as

$$h_{ab}(\delta_\perp x^b)^{\dot{}} = \upsilon_{ab}\delta_\perp x^b , \quad (2.6)$$

where $\upsilon_{ab} = h^c{}_a h^d{}_b u_{c/d} - 2h_{ca}Q_{bd}{}^c u^d$. (2.7)

The tensor υ_{ab} can be decompose in the kinematical quantities

$$\upsilon_{ab} = \sigma_{ab} + \frac{\theta}{3}h_{ab} + \omega_{ab} , \quad (2.8)$$

where υ_{ab} describes the relative motion of the neighboring observers (with the same 4-velocity). To discuss the Riemannian and non-Riemannian parts in the kinematical

quantities, by using the relation (2.2) we may express the tensor υ_{ab} as

$$\upsilon_{ab} = \hat{\upsilon}_{ab} + \Omega_{ab} , (2.9)$$

where $\hat{\upsilon}_{ab} = h^c{}_a h^d{}_b u_{c;d}$, (2.10) and

$$\Omega_{ab} = h^c{}_a h^d{}_b K_{tcd} u^t \quad .(2.11)$$

The antisymmetric space-like part ω_{ab} in equation (2.5) denotes the rotation of the flow having the expression

$$\omega_{ab} = \upsilon_{[ab]} = \hat{\omega}_{ab} + \Omega_{ab} \quad , (2.12)$$

The space-like symmetric traceless part σ_{ab} - the shear of the fluid and the expansion θ are defined as

$$\sigma_{ab} = \theta_{ab} - \frac{\theta}{3}h_{ab} = \hat{\sigma}_{ab} \quad , \qquad (2.13)$$

$$\theta = u^a{}_{/a} + Q_a u^a = \hat{\theta} \ , \ (2.14)$$

where the expansion tensor θ_{ab} is defined by $\theta_{ab} = \upsilon_{(ab)} = \hat{\theta}_{ab}$ and $Q_a = 2Q_{ab}{}^b$. Out of these kinematical quantities in RC space-time, only vorticity ω_{ab} contains changes triggered by torsion through the skew tensor Ω_{ab}. In particular, the vorticity vector ω^a in RC space-time and $\hat{\omega}^a$ in Riemann space-time are related through $\omega^a = \hat{\omega}^a + \Omega^a$, one defines the vorticity vector ω^a and the torsion vector Ω^a with the help of η^{abcd} as

$$\omega^a = \frac{1}{2}\eta^{abcd}u_b\omega_{cd} , \ (2.15)$$

$$\Omega^a = \frac{1}{2}\eta^{abcd}u_b\Omega_{cd} \ , \ \Omega^2 = \Omega_a\Omega^a = \frac{1}{2}\Omega_{ab}\Omega^{ab} \ , \ (2.16)$$

where η^{abcd} represents the totally antisymmetric permutation tensor of the space-time. Further, the permutation tensor is a covariantly constant quantity, with

$$\eta^{abcd}\eta_{qrst} = 4!\delta^{[a}_q\delta^b_r\delta^c_s\delta^{d]}_t , \eta^{abcd}\eta_{arst} = 3!\delta^{[b}_r\delta^c_s\delta^{d]}_t$$

(2.17)

§ (3): *Electromagnetic field and torsion interaction*

The Maxwell's equations can be expressed covariantly on any differential manifold and which do not depend on the specific connections used. Maxwell's equations can be expressed covariantly with respect to the affine connections and are gauge invariant in the presence of torsion [1]. By assuming the Maxwell's electromagnetic field equations are valid in RC space-time, the electrodynamics formulae are derived and successively applied to the highly conductive fluid. It is shown that, the covariant equations isolate the magnetic effects on the kinematics and the dynamics of the medium in RC space-time.

The Maxwell's equations in RC space-time are given by

$$F^{ab}{}_{/b} + Q_b F^{ab} + Q_{bc}{}^{a} F^{bc} = J^{a},$$

$$J^{a}{}_{/a} + Q_a J^{a} = 0 \;, \quad (3.1)$$

and

$$F_{[ab/c]} - 2Q_{[ab}{}^{t} F_{c]t} = 0 \;, (3.2)$$

where J^a is the electric 4-current that sources the electromagnetic field. By operating η_{arst} on (3.2), and using the identity (2.17) we obtain

$$^{*}F^{ab}{}_{/b} + {}^{*}F^{bc} T_{bc}{}^{a} = 0 \;, (3.3)$$

where $T_{bc}{}^{a} = Q_{bc}{}^{a} + \delta^{a}{}_{[b} Q_{c]}$, (3.4)

is called the modified form of the torsion tensor. The Maxwell's field is covariantly characterized by the electromagnetic field tensor F^{ab} and it's dual $^{*}F^{ab}$. It can be split up into the 4-velocity vector u^a, the magnetic field vector H^a and electric field vector E^a as follows:

$$F^{ab} = u^a E^b - u^b E^a - \eta^{abcd} u_c H_d \;, \quad (3.5)$$

$$^{*}F^{ab} = u^a H^b - u^b H^a + \eta^{abcd} u_c E_d \;, (3.6)$$

where $E^a = -F^{ab} u_b$ and $H^a = \frac{1}{2}\eta^{abcd} u_b F_{cd}$.

The relation between the 4-cuurent vector J^a and the electric field vector E^a is determined by the Ohm's law as

$$J^a = q u^a + k E^a \;, \quad (3.7)$$

where $q = J^a u_a$ represents the measurable charge density and k is the electric conductivity of the medium. Thus, Ohm's law in the observer's frame is $I^a = kE^a$, and $I^a (= h^a_b J^b)$ is orthogonally projected conduction current. This form of Ohm's law corresponds to the ideal MHD approximation; the non-zero conduction currents are compatible with a vanishing electric field as long as the conductivity is infinite (i.e. for $k \to \infty$). Thus, at the limit of ideal MHD, electric field vanishes (i.e. $E^a = 0$) in the frame of the fluid [10]. On the other hand, zero conductivity implies that the conduction current vanishes, even when the electric field is non-zero.

Relative to fundamental observer, each one of the Maxwell's equations decomposes into a time-like and space-like component. The Maxwell's equations in RC space-time can be decomposed with respect u^a into the following constraints and propagation equations.

3.1 Divergence equations for H^a and E^a

The Maxwell's equations (3.3) with (3.6) can be decomposed with u^a

$$H^{a}{}_{/b} h^{b}{}_{a} = -(Q_a - T_a) H^a + 2(\omega^a - \Omega^a) E_a \;, \quad (3.8)$$

where $T_a = 2Q_{abc} u^b u^c$ is the space-like vector. However, if we define the covariant operator $\tilde{\nabla}_a = \nabla_a + Q_a - T_a (= \hat{\nabla}_a - T_a)$ and the orthogonally projected operator $D_a H^a = h^b_a \tilde{\nabla}_b H^a$ then equation (3.8) becomes

$$D_a H^a = 2(\omega^a - \Omega^a) E_a \,. (3.9)$$

Similarly, for the electric field vector E^a, the Maxwell's equations (3.1) and (3.5) can be decomposed with u^a as

$$E^{a}{}_{/b} h^{b}{}_{a} = -(Q_a - T_a) E^a + q - 2(\omega^a - \Omega^a) H_a \,. \quad (3.10)$$

This can be written as

$$D_a E^a = q - 2(\omega^a - \Omega^a) H_a \,. (3.11)$$

The effects contain in equations (3.9) and (3.11) which are triggered by the divergence term as well as the relative motion of the observer carried by the kinematical quantity $2(\omega^a - \Omega^a)$. The right hand side of the equation (3.9) shows how the interaction between the torsion and the vorticity along the electric field lines affects the change on the magnetic field. But the term Ω^a, in particular, is a response of rotation to the twisting of force lines only and it does not represent any physical field. Therefore, the physical meaning of this side is clear: the torsion term Ω^a induces a change in ω^a which does not affect the change in motion of the observer. The term $2(\omega^a - \Omega^a) E_a$ in equation (3.9) acts as an effective magnetic charge caused by the relatively moving E-field, while in equation (3.11), the effective electric charge is caused by the relative motion of the magnetic field.

3.2 Propagation equation for H^a and E^a:

Contracting equation (3.3) with $h^c{}_a$, we find

$$h^a{}_b \dot{H}^b = u^a{}_b H^b - \theta H^a + 2h^a{}_b Q_{cd}{}^{b} u^c H^d + \mathrm{I}^a(E) \,, (3.12)$$

where $\mathrm{I}^a(E)$ is a space-like vector and it can be expressed after simplification as

$$\mathrm{I}^a(E) = \eta^{abcd} u_b \{(\dot{u}_c - T_c) E_d + E_{c/d}\} + \eta^{abcd} u_b K_{dtc} E^t \,. (3.13)$$

The torsion terms on the right-hand side of these equations are all algebraic in character. If torsion vanishes then it reduces to Ellis's results for Riemann space-time [10]. In addition, we define the curl operator in RC space-time for any vector v^a according to $curl v^a = \eta^{abcd} u_d h^e{}_b \tilde{\nabla}_e v_c$. Using this curl operator and with equation (2.5), equations (3.12) and (3.13) may be expressed in the form

,

$$\dot{H}_a = (\sigma_{ab} + \omega_{ab} - \frac{2}{3}\theta h_{ab}) H^b + \eta_{abcd} u^b \dot{u}^c E^d + curl E_a \quad (3.14)$$

Similarly, for the electric field vector E^a, contracting equation (3.1) with $h^c{}_a$ we get

$$h^a{}_b\dot{E}^b = u^a{}_{/b}E^b - \theta E^a + 2h^a{}_b Q_{cd}{}^b u^c E^d - \mathrm{I}^a(H) - I^a, \quad (3.15)$$

where

$$\mathrm{I}^a(H) = \eta^{abcd}u_b\{(\dot{u}_c - T_c)H_d + H_{c/d}\} + \eta^{abcd}u_b K_{dtc}H^t \quad (3.16)$$

We also write the propagation equation for the electric field in the kinematical form as

$$\dot{E}_a = (\sigma_{ab} + \omega_{ab} - \frac{2}{3}\theta h_{ab})E^b - \eta_{abcd}u^b\dot{u}^c H^d - curlH_a - I_a, \quad (3.17)$$

We note that, in addition of the usual 'curl' term, the effects contain in equations (3.14) and (3.17) are caused by the relative motion of neighbouring observers. Those effects are carried by the kinematical quantities which are defined with respect to the full connections on the right hand side of the equations. It is also note that there is no torsion contribution. This means that torsion does not explicitly affect the magnetic evolution, although the effect of torsion in the other constraint and propagation equations is taken into account. The acceleration term in those equations also reflects the fact that the RC space-time is treated as a single entity.

In a relativistic analogous way, instead of the last term in equation (3.12), the later part compare with equations (2.6-2.7) ensure that

$$h_{ab}(l^3H^b)^{\dot{}} = \upsilon_{ab}(l^3H^b) \quad (3.18)$$

where $3(\dot{l}/l) = \theta(= \hat{\theta})$. Hence $\delta_\perp x^a = l^3H^a$ is relative position vector connecting the same particles at all times. This guarantees that the magnetic field lines remain frozen-in with the matter if and only if $I^a(E) = 0$ in RC space-time. On the other hand the equation (3.15) describes that, the electric field lines are frozen-in with the matter if and only if $I^a(H) + I^a = 0$. If Ohm's law $I^a = kE^a$ is satisfied then, zero electric conductivity implies that conduction current vanishes, even when the electric field is non-zero. Further, in a charged fluid with vanishing magnetic field and in which Ohm's law is valid, commoving observers can be used all along a congruence of electric field lines, and the electric field lines are frozen-in with this fluid.

3.3 Ideal magnetohydrodynamics

In MHD, consider a space-time filled with a single barotropic fluid of infinite electric conductivity. The Ohm's law $I^a = kE^a$ in the frame of fundamental observers guarantees that the electric field vanishes even if the conduction current is finite. In the absence of electric field, the Maxwell's equations reduce to a single propagation formula, namely the covariant magnetic induction equation and three constraint equations as follows

$$\dot{H}_a = (\sigma_{ab} + \omega_{ab} - \frac{2}{3}\theta\, h_{ab})H^b, \quad (3.19)$$

$$curlH_a = -I^a - \eta_{abcd}u^b\dot{u}^c H^d, \quad (3.20)$$

$$2(\omega^a - \Omega^a)H_a = q \quad (3.21) \qquad D_aH^a = 0 \quad (3.22)$$

In the magnetic induction equation (3.19), the torsion does not affect explicitly on magnetic evolution. Therefore, the relative motion of the neighbouring observer carried out by kinematical quantities guarantees that the magnetic field lines are frozen-in with the fluid in MHD. The expression (3.20) provides a direct relation between the conduction current and magnetic field, which is responsible for keeping the magnetic field lines frozen-in with the matter. The equation (3.21) shows that, when $(\omega^a - \Omega^a)H_a \neq 0$, the rotating neighbouring observer will measure a non-zero charge density where the torsion term will measure only the rotational degrees of the freedom of the rotating observer. This result also holds in Riemann space-time if $\hat{\omega}^a H_a \neq 0$ [15]. The equation (3.22) represents that the magnetic field lines are closed, based on the operator D_a in RC space-time.

Finally, to study of space-like congruences of electric and magnetic field lines, the space-like vectors $\mathrm{I}^a(E)$ and $\mathrm{I}^a(H)$ play an important role. From equations (3.13) and (3.16), we obtain the mathematical identities as follows:

$$E_{[s/t]} = \dot{u}_{[t}E_{s]} + T_{[s}E_{t]} + u^rE_{r/[t}u_{s]} + u_{[t}\dot{E}_{s]}$$

$$+ Q_{ste}E^e + 2u_{[s}Q_{t]re}E^eu^r - \frac{1}{2}\eta_{stre}u^r\mathrm{I}^e(E), \quad (3.23)$$

$$H_{[s/t]} = \dot{u}_{[t}H_{s]} + T_{[s}H_{t]} + u^rH_{r/[t}u_{s]} + u_{[t}\dot{H}_{s]}$$

$$+ Q_{ste}H^e + 2u_{[s}Q_{t]re}H^eu^r - \frac{1}{2}\eta_{stre}u^r\mathrm{I}^e(H). \quad (3.24)$$

These identities are used in the latter section to calculate the expressions for rotation tensor of the congruence of electric and magnetic field lines.

§ (4): *Congruences of magnetic and electric field lines*

4.1 Theory of Space-like Congruence

The theory of space-like congruence in Riemann space-time was first introduced by Greenberg [16]. Further, the theory of space-like congruence in RC space-time is developed by us in our previous paper [17]. Here we list some results from our earlier paper for developing the space-like congruences in electrodynamics and mention the appropriate physical interpretations.

In Riemann space-time, the connecting vector δx^a of two particles on neighboring curves with respect to the space-like vector h^a satisfies

$$\underset{h}{\pounds}\,\delta x^a = 0. \quad (4.1)$$

As the Lie derivative is connection independed, it holds also in RC space-time. The Lie derivative of connecting vector with respect to h^a is in the form

$$\underset{h}{\pounds}\delta x^a = \delta x^a{}_{/c} h^c - h^a{}_{/c}\delta x^c - 2Q_{cd}{}^a \delta x^d h^c = 0 \,. \quad (4.2)$$

To observe the deformation of the curves of the space-like congruence, we now introduced an observer at a point P on the curve with 4-velocity w^a such that $w^a w_a = 1$ and $w^a h_a = 0$. For a given space-like vector h^a, there is not a unique time-like unit vector w^a. The vector $u^a = w^a + \lambda^a$ is another time-like unit vector orthogonal to h^a where λ^a satisfies the conditions

$\lambda^a h_a = 0$ and $\lambda_a \lambda^a + 2\lambda_a w^a = 0$.(4.3)

This freedom of choice of an observer is essential to observe the deformations of the curves of the congruence. The observer erects a screen orthogonal to curve at point P, so that the congruence of curves passes perpendicularly through the screen at P. Because the connecting vector δx^a need not lie on the screen at P, we introduced the projection tensor

$$P_{ab} = g_{ab} - w_a w_b + h_a h_b \quad .(4.4)$$

Since $P_{ab} w^b = P_{ab} h^b = 0$, the orthogonal connecting vector is given by $(\delta_\perp x^a) = P^a{}_b \delta x^b$.

With the aid of (4.2), a direct calculation gives

$$P^a{}_b (\delta_\perp x^b)^* = A^a{}_b (\delta_\perp x^b) + B^a{}_b \delta x^b \,, (4.5)$$

where $\overset{*}{A^a} = A^a{}_{/b} h^b$. In the equation (4.5), the operator A_{ab} and the additional term B_{ab} are defined as

$$A_{ab} = P^c{}_a P^d{}_b h_{c/d} - P^c{}_a P^d{}_b 2Q_{dtc} h^t \quad , \quad (4.6)$$

$$B^{ad} = P^a{}_b (\overset{o}{h^b} - \overset{*}{w^b} - 2Q_{ct}{}^b h^t w^c) w^d \quad , \quad (4.7)$$

where $\overset{o}{h_a} = h^a{}_{/b} w^b$ and $h_{a/b} h^a = 0$.

The presence of term B_{ab} is crucial. Except at the given point P, the motion of the observers employed along the curve has still to be specified. To observe the resulting deformation for the second observer, a transport law for the vector w^a must be specified as

$$\overset{*}{w^a} = \overset{o}{h^a} - \overset{o}{h^b} w_b w^a + \overset{*}{h_b} w^b h^a - 2Q_{ct}{}^a h^t w^c + 2Q_{ct}{}^b h^t w^c w_b w^a - 2Q_{ct}{}^b h^t w^c h_b h^a \,. \quad (4.8)$$

Further the kinematics of the space-like congruence in Riemann and RC space-time are related through the relation (2.3)

$$A_{ab} = \tilde{A}_{ab} + \Omega_{ab}(h) \quad (4.9)$$

where $\tilde{A}_{ab} = P^c{}_a P^d{}_b h_{c;d}$, (4.10)

$$\Omega_{ab}(h) = P^c_a P^d_b K_{tcd} h^t \,. \quad (4.11)$$

We decompose A_{ab} into its irreducible parts as follows:

$$A_{ab} = \Re_{ab} + \frac{1}{2}\Theta P_{ab} + \Im_{ab} \;, \quad (4.12)$$

where $\Re_{ab} = A_{[ab]} = \tilde{\Re}_{ab} + \Omega_{ab}(h)$, (4.13)

$$\Theta = A^a{}_a = h^a{}_{/a} - h_{a/b} w^a w^b + (Q_a - T_a) h^a = \tilde{\Theta} \,, \quad (4.14)$$

$$\Im_{ab} = A_{(ab)} - \frac{1}{2} A^c{}_c P_{ab} = \tilde{\Im}_{ab} \,. \quad (4.15)$$

Clearly $\Re_{ab} w^a = 0$,$\Re_{ab} h^a = 0$,

$$\Im_{ab} w^a = 0 \,, \Im_{ab} h^a = 0 \,. \quad (4.16)$$

The quantities $\Re_{ab}$, $\Im_{ab}$ and Θ are called, the rotation tensor, the shear tensor and expansion of the congruence as measured by w^a. The reader is referred to equations (4.13)-(4.15) for only a comparison between the kinematics of space-like congruence in Riemann and Riemann-Cartan space-time. Of the kinematical quantities, torsion therefore alters only rotation through the skew tensor $\Omega_{ab}(h)$. Here, we note that, the rotation $\Re_{ab} = \tilde{\Re}_{ab} + \Omega_{ab}(h)$ consists of two parts. One part $\tilde{\Re}_{ab}$ is the Riemannian part which measure the rotation of the curves whereas the other part $\Omega_{ab}(h)$ proportional to $K_{bc}{}^a$ describes the rotational degrees of freedom [18]. Therefore, the additional source term $\Omega_{ab}(h)$ in the expression is unaffected the rotational motion of the curves. The antisymmetry of rotation tensor $\Re_{ab}$ and torsion tensor $\Omega_{ab}(h)$ implies that we can define the rotation vector $\Re^a$ and torsion vector $\Omega^a(h)$ by means of the alternating tensor η^{abcd}

$$\Re^a = \frac{1}{2}\eta^{abcd} w_b \Re_{cd} \;, \quad (4.17)$$

$$\Omega^a(h) = \frac{1}{2}\eta^{abcd} w_b \Omega_{cd}(h) \quad .(4.18)$$

Rotation vector together with torsion vector determines the axis of rotation which is perpendicular to the screen and parallel to h^a. They represent only the direction of the axis and remains unaffected the rotational motion. In the present article, our analysis applies to all situations we use the kinematical quantities of the space-like congruence with full connection.

4.2 congruence of magnetic field lines

In an electromagnetic field, the magnetic and electric field 4-vectors are defined in terms of u^a. A space-like vector field gives rise to a space-like congruence of curves and is orthogonal to 4-velocity vector field u^a everywhere. To measure the deformation of the curves, a comoving observer, $w^a = u^a$, can be employed at any point P on a curve of congruence. Since $H^a u_a = 0$, we can always choose a comoving observer at any point P on a magnetic field line, although the other observer employed along the field line will not be comoving unless the magnetic field lines are frozen-in with the fluid. As proved in Section 3, for the magnetic field lines are frozen-in with the fluid, we may employ a comoving observer and observe the deformation of the field lines.

Because of the antisymmetry in the mathematical identities (3.23) and (3.24) we cannot calculate shear $\Im_{ab}$ and expansion Θ of the congruence of magnetic and electric field lines. But an expression for the rotation of congruence of magnetic and electric field lines can be obtained from those mathematical identities.

With the aid of equation (4.6) and $H^a = Hh^a$, the expression for rotation tensor of a congruence of magnetic field lines is

$$\Re_{ab} = A_{[ab]}$$

$$= \frac{1}{H} P^c{}_a P^d{}_b H_{[c/d]} - \frac{1}{H} P^c{}_a P^d{}_b [Q_{dtc} - Q_{ctd}] H^t . \text{ (4.19)}$$

By using the mathematical identity (3.24) for the magnetic field vector H^a, the expression (4.19) gives

$$\Re_{ab} = -\frac{1}{2H} P^c{}_a P^d{}_b \eta_{cdst} u^s \mathrm{I}^t(H) + \Omega_{ab}(h) . \text{ (4.20)}$$

For a comoving observer at a point P the projection tensor is $P_{ab} = g_{ab} - u_a u_b + h_a h_b$. The rotation and torsion vectors are constructed for a commoving observer along the magnetic field lines by using (4.17) and (4.18) as follows:

$$\Re^a = \frac{1}{2}\eta^{abcd} u_b \Re_{cd} \text{ and } \Omega^a(h) = \frac{1}{2}\eta^{abcd} u_b \Omega_{cd}(h) \text{ (4.21)}$$

It follows directly from (4.20) and (4.21) by means of the permutation tensor η^{abcd} that

$$\Re^a = \frac{1}{2H}(h_b \mathrm{I}^b(H))h^a + \Omega^a(h) \text{ .4.22)}$$

The dynamics is introduced through Maxwell's equation (3.15) in terms of E^a and conduction current I^a. In ideal MHD, the Maxwell's equation (3.15) reduces into $\mathrm{I}^a(H) = -I^a$ and the equations (4.20) and (4.22) read

$$\Re_{ab} = \frac{1}{2H} P^c{}_a P^d{}_b \eta_{cdst} u^s I^t + \Omega_{ab}(h) \text{ ,4.23)}$$

$$\Re^a = -\frac{1}{2H}(h_b I^b)h^a + \Omega^a(h) \text{ .(4.24)}$$

These equations provide the relation between rotation of magnetic field lines and the conduction current. Since the torsion term $\Omega_{ab}(h)$ is directly coupled with the rotation tensor $\Re_{ab}$ and the conduction currents are unaffected by torsion, then the magnetic field lines have zero rotation in a perfectly conducting fluid measured by u^a if and only if $H_a I^a = 0$.

The space-like vector $\mathrm{I}^a(H)$ from the propagation equation (3.16) yields only $H_{[c/d]}$ and not $H_{(c/d)}$. But the expression for expansion Θ can be evaluated by using Maxwell's equation (3.8). It follows from (4.14) with $H^a = Hh^a$ for a comoving observer that

$$\Theta = \frac{2}{H}(\omega^a - \Omega^a)E_a - (\ln H)_{/\alpha} h^\alpha \text{ . (4.25)}$$

The right hand side of the equation (4.25) shows that, the rotating comoving observer along the electric field lines will measure the magnetic strength (pressure) in the expansion or contraction of the fluid. It should also be noted that there are no effects due to torsion to measure of magnetic strength in the expansion or contraction of the fluid. In ideal MHD approximation, the above equation reduces to

$$\Theta = -(\ln H)_{/\alpha} h^\alpha . \text{ (4.26)}$$

Hence, the magnetic strength either dilutes with the expansion or increases with the contraction of the fluid. We also note that, the magnetic strength of magnetic field is conserved if and only if the congruence of magnetic field lines is expansion free in RC space-time. This result also holds in the case of MHD approximation in Riemann space-time [19].

4.3 Congruence of electric field lines

For a comoving observer, we consider a congruence of electric field lines in an electromagnetic field. To study the congruence of electric field lines, the roles played by the Maxwell's equations for H^a in congruence of magnetic field lines are interchanged by the electric field vector E^a.

The rotation of a congruence of electric field lines can be measured by a comoving observer at a point P and can be obtained from the equation (4.6) with $E^a = Eh^a$:

$$\Re_{ab} = \frac{1}{E} P^c{}_a P^d{}_b E_{[c/d]} - \frac{1}{E} P^c{}_a P^d{}_b [Q_{dtc} - Q_{ctd}] E^t \text{ .(4.27)}$$

Similarly, the rotation vector $\Re^a$ of the congruence of electric field lines measured by a comoving observer is given by

$$\Re_{ab} = -\frac{1}{2E} P^c{}_a P^d{}_b \eta_{cdst} u^s \mathrm{I}^t(E) + \Omega_{ab}(h) \text{ (4.28)}$$

$$\Re^a = \frac{1}{2E}(h_b \mathrm{I}^b(E))h^a + \Omega^a(h) . \text{ (4.29)}$$

We consider a charged fluid with vanishing magnetic field as measured by u^a; then from (3.12), we have $\mathrm{I}^a(E)=0$. Consequently, the electric field lines have zero rotation as measured by comoving observer at point P along the curve in RC space-time.

The expression for Θ can be evaluated by using Maxwell's equation (3.10). It follows from (4.14) with $E^a = Eh^a$ for a comoving observer that

$$\Theta = \frac{1}{E}\{q - 2(\omega^a - \Omega^a)H_a\} - (\ln E)_{/\alpha} h^\alpha . \quad (4.30)$$

The electric strength measure by the comoving observer along the magnetic field line depends on the charge density of the fluid in the expansion or contraction of the electric field lines. If we consider a charged fluid with vanishing magnetic field, then equation (4.30) reduces to

$$\Theta = q - (\ln E)_{/\alpha} h^\alpha . \quad (4.31)$$

If the fluid lines are expansion free then the electric strength is proportional to the charged density of the fluid.

§ (6): CONCLUSIONS

The covariant techniques have been applied by many authors for the study of electromagnetic field in Riemann theory. These techniques were originally applied within the Newtonian framework before extended to general relativistic electrodynamics. In this article, we successively used the covariant techniques with full connection for the study of electromagnetic field in RC space-time. We developed the nonlinear electrodynamics formulae and the same is extended in ideal MHD approximations. By taking the advantage of a relative motion treatment, we examined the kinematical implications on the evolution of magnetic and electric fields. It is shown that, the role of torsion in the fluid kinematics is neither affected on the relative motion of the neighbouring observer nor in the evolution of magnetic and electric fields. With the consideration of full MHD equations, the magnetic field lines are frozen-in with the fluid in RC space-time.

The main aim of our work is to develop the space-like congruences of magnetic and electric field lines by using the electrodynamics formulae. The effect caused by torsion is only on the rotation of magnetic field lines, but not on its shear and deformation. Moreover, it should be noted that, there are no effects due to torsion with the measure of magnetic strength in the expansion or contraction of the fluid. For a charged fluid with vanishing magnetic field, it is proved that the electric field lines have zero rotation in RC space-time.

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