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FUZZY SOFT NEAR-RINGS

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Abstract- ***In this paper we have introduced the concept of soft near-rings and fuzzy soft near-rings. We have also investigated their basic properties.***

Key words: Soft set, fuzzy set, fuzzy soft set, near-ring ,soft near-ring, fuzzy soft near-ring.

I. INTRODUCTION

The real world is complex and complexity in the world arises from uncertainty. Knowledge, which describes the real world, is usually imprecise and vague. The mathematical tools available to represent the real world are not suitable to describe rigorous and precise information. Thus there is always some difference between vagueness of reality and its rigorousness of mathematical model.

In an attempt to bridge this gap fuzzy set theory was initiated by L.A. Zadeh in 1960's.Since a membership in a fuzzy set theory is a matter of degree, we can represent the gradual membership of an element of a set describing the fuzzy attributes like cold, hot, tall, short etc in a better way. Thus fuzzy sets are more capable of expressing the vague concepts of natural languages. Due to this, fuzzy sets are widely used in highly complicated real life demands. Fuzzy set theory is applicable in control theory, robotics and other complex engineering systems.

In 1965 Zadeh [26] published a seminal paper. In that paper he introduced a new theory whose objects were sets having the boundaries, which are not clearly defined. The fuzzy set was defined with the help of a membership function. We refer usual sets as crisp sets. Every object in the universal set may or may not belong to the crisp set under consideration. But in fuzzy set theory we can say that the object belongs to the crisp set up to certain limit. Generally we express the degree of membership and the degree of truth of any proposition by the real number in [0, 1]. These sets have a broad utility for expressing the gradual transition from membership to non-membership and conversely. By fuzzy set theory we can express vague concepts into natural language.

In 1971 Rosenfeld [22] introduced the concept of fuzzification of groups and defined fuzzy subgroups. Goguen [12] introduced L-Fuzzy Sets. Liu [15] defined the fuzzy ideals of a ring and discussed the operations on fuzzy ideals

Near-rings were first studied by Fittings in 1932. It is a generalization of a ring. If in a ring we ignore the commutativity of addition and one distributive law then we get a near-ring. G.Pilz [13], J.D.P.Meldrum [17] and many other researchers have contributed and are contributing the near-ring theory.

Most of our traditional tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character. However there are many complicated problems in economics, engineering, environment, social science, and medical science etc. that involve the data which is not always crisp. In these situations we cannot use classical methods because of various types of uncertainties present in these problems. Consequently Molodstov [18] initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties. A soft set can be considered as an approximate description of an object. Soft set theory has a rich potential for applications in several directions. Further in 2001 Maji ,Biswas and Roy [16] combined fuzzy set and soft set models and introduced the concept of fuzzy soft sets. To continue the investigation on fuzzy soft sets, Ahmad and Kharal [4] presented some more properties.The algebraic structure of soft sets has been studied increasingly in recent years. Also Aktas and Cagman [6] defined the notion of soft groups and derived some properties. Munazza Naz, Muhammad Shabir, Muhammad Irfan Ali [19] introduced the concept of fuzzy soft semigroup which is a generalization of soft semigroup and studied some properties.

In 2010 Acar, F.Koyuncu and B. Tanay [3] introduced the basic notions of soft rings, which are actually a parameterized family of subrings of a ring. By using t-norm the concept of fuzzy soft groups was introduced by Aygunoglu and Aygun [7] .The concept of fuzzy ring was introduced by Liu [15]. Y.Celik, C.Ekiz and Sultan Yamak [10] also introduced the notion of soft ring and soft ideal over a ring. Ashhan Sezgin, Akin Osman Atagun, Emin Aygun [23] investigated the properties of idealistic soft near-rings with respect to the near-ring mappings and proved that the structure is preserved under the near-ring epimorphisms. Tridiv Jyoti Neog and Dusmanta Kumar Sut [20] studied the union and intersection of fuzzy soft sets. Also Manoj Borah, Tridiv Jyoti Neog and Dusmanta Kumar [8] studied some operations on fuzzy soft sets. Jayanta Ghosh, Bivas Dinda and Samanta [11] studied the notion of fuzzy soft rings and fuzzy soft ideals.

2. Preliminaries:-

In this section, we first recall the basic definitions related to near-rings, soft sets, fuzzy soft sets which would be used in the sequel.

Definition (2.1): By a near-ring, we mean a non-empty set R with two binary operations '+' and '·' satisfying the following axioms:

1) (R, +) is a group,
2) (R, ·) is a semi-group,
3) $x\cdot (y + z) = x\cdot y + x\cdot z$ for all $x, y, z \in R$.

Precisely speaking, it is a left near-ring because it satisfies the left distributive law. We will use the word "near-ring" instead of "left near-ring".
We denote xy instead of x·y.
Note that x0=0 and x (-y) = -xy, but 0x ≠ 0 for x, y ∈ R.

Theorem: (2.2) [5]: A non-empty subset I of a near-ring R is a subnear-ring of R if and only if x-y, xy∈I for all x, y∈I.

Definition (2.3): An ideal I of a near-ring R is a subset of R such that

i) (I, +) is a normal subgroup of (R, +),
ii) RI ⊆ I and
iii) (r + i) s - rs ∈ I for all i∈ I and r,s ∈ R.

Note that if I satisfies (i) and (ii) then it is called a left ideal of R and if I satisfies (i) and (iii) then it is called a right ideal of R.
For the basic concepts in fuzzy set theory we refer G. Klir and Bo Yuan [34].Suppose U stands for the universal set and I for the unit interval of reals [0, 1].

Definition (2.4): A fuzzy set μ in U is a function μ: U → I. The set of all fuzzy sets of U is denoted by F (U).

Definition (2.5): Let μ be a fuzzy set in U and t ∈ [0, 1]. Then the crisp set μ_t = {x ∈U | μ (x) ≥ t} is called a level subset of μ.

Definition (2.6): The support of a fuzzy set μ, denoted by Supp (μ) is defined as Supp (μ) = {x ∈ U | μ (x) > 0}.

Definition (2.7): A fuzzy set μ is said to be normal if h (μ) = 1.
Otherwise, it is called subnormal.

Definition (2.8): Let μ and γ be two fuzzy subsets of U.

Then μ∘γ is a fuzzy subset of U defined as follows:

μ∘γ (z) = sup {min (μ (x), γ(y))},if z is expressed
as z= xy = 0 otherwise

Definition (2.9): Let (G, +) be a group and μ be a fuzzy set in G. Then μ is said to be a fuzzy subgroup if:

(i) μ(x+y) ≥ min {μ(x), μ(y)} for all x,y ∈G,
(ii) μ (-x) ≥ μ(x) for all x ∈G.

Definition (2.10): Let (G, +) be a group and μ be a fuzzy set in G. Then μ is said to be a normal fuzzy subgroup if:

(i) μ is a fuzzy subgroup of G,
(ii) μ(x) = μ(y+x-y) for all x, y ∈G.

Definition (2.11): A fuzzy set μ of a near-ring R is said to be a fuzzy subnear-ring of R if for all x,y ∈G,

(i) μ(x+y) ≥ min {μ(x), μ(y)} for all x,y ∈G,
(ii) μ (-x) ≥ μ(x) for all x ∈G,
(iii) μ (x y) ≥ min (μ (x), μ (y))

Definition (2.12): Let μ be a non-empty fuzzy set in a near-ring R. Then μ is a fuzzy ideal of R if

(i) μ(x+y) ≥ min {μ(x), μ(y)}for all x,y ∈R,
(ii) μ (-x) ≥ μ(x) for all x ∈R,
(iii) μ(x) = μ(y+x-y) for all x, y ∈R,
(iv) μ (xy) ≥ μ(y) for all x,y ∈R,
(v) μ {(x+i) y-xy} ≥ μ (i) for all x, y, i ∈R.

If μ satisfies (i), (ii), (iii) and (iv) then it is called a fuzzy left ideal of R and if it satisfies (i), (ii), (iii) and (v) then it is called a fuzzy right ideal of R.

For soft sets we refer [3,10,16,18]:-

Definition (2.13): Let U be an initial universe set and E be the set of parameters. Let A be a subset of E. Let P(U) denote the power set of U.A pair (F,A) is called a soft set over U, where F is a mapping given by F:A→ P(U).

For each x∈A, F(x) is the set of x-approximate elements of the soft set (F, A).A soft set (F,A) over U is also denoted by a triple (F,A,U).

Definition (2.14): A soft (F,A) over U is said to be null soft set denoted by $\tilde{\phi}$ if F(c) = ϕ for all c∈A.

Definition (2.15): A soft set (F,A) over U is said to be empty if A= ϕ.

We write (ϕ, ϕ) for empty soft set over U.

Definition (2.16): A soft (F,A) over U is said to be universal if A= E and F(c) = U for all c∈A.

We write (U, E) for universal soft set over U.

Definition (2.17): A soft (F,A) over U is said to be absolute soft set denoted by $\tilde{A}$ if F(c) = U for all c∈A.

Definition (2.18): For a soft set (F, A) the set,

Supp (F, A) = {x∈A|F(x) ≠ ϕ} is called the support of the soft set (F, A).

If Supp (F, A) ≠ ϕ then the soft set (F, A) is called non-null.

Definition (2.19): For two soft sets (F,A) and (G,B) over a common universe

U, we say that (F,A) is a soft subset of (G,B) if

(i) A ⊆ B and

(ii) F(a) ⊆ G(a) for all a∈ A.

It is denoted by (F,A) ⊆ (G,B)

(F,A) is said to be soft super set of (G,B),if (G,B) is a soft subset of (F,A).

Definition (2.20): Two soft sets (F,A) and (G,B) over a common universe U are said to be soft equal if (F,A) is soft subset of (G,B) and (G,B) is a soft subset of (F,A).

i.e. (F,A) ⊆ (G,B) and (G,B) ⊆ (F,A) .

It is denoted by (F,A) = (G,B).

Definition (2.21) : The complement of a soft set **(**F,A) denoted by **(**F,A)^c is defined by **(**F,A)^c = (F^c, A) where F^c: A→ P(U) is a mapping given by F^c (e) = [F (e)]^c for all e∈ A.

Definition (2.22) : If **(**F, A) and (G, B) are two soft sets over a common universe U then "(F, A) AND (G,B)" is a soft set denoted by (F,A) ∧ (G,B) and is defined by (F,A)∧(G,B)= (H,A× B) where H(x, y)=F(x) ∩ G(y)for all (x, y)∈A× B.

Definition (2.23) : If (F,A) and (G,B) are two soft sets over a common universe U then "(F, A) OR (G,B)" is a soft denoted by (F,A) ∨ (G,B) and is defined by

(F,A) ∨ (G,B) = (H,A× B) where H(x, y)=F(x) ∪ G(y) for all (x, y)∈A× B.

Definition (2.24) : Let (F,A) and (G,B) be two soft sets in a soft class (U,E) with A∩B ≠ ϕ The intersection of two soft sets (F,A) and (G,B) over a common

universe U is the soft set (H,C) where C= A∩B , and H(c) = F(c) ∩ G (c) for all c∈ C

We write (F,A) $\tilde{\cap}$ (G,B) =(H,C).

Pei and Miao pointed out that generally F(c) or G (c) may not be identical. In order to avoid the degenerate case Ahmed and Kharal proposed that A∩B must be non-empty and thus revised the definition as follows which is known as the restricted intersection:

Definition (2.25) : Let (F, A) and (G, B) be two soft sets over a common universe U such that A∩B ≠ϕ. Then the restricted intersection of (F,A) and (G,B) is defined as (F,A) $\cap_R$ (G,B) = (H,C) where C = A∩B and for all c∈ C, H(c) = F(c)∩G (c).

It is denoted by (F,A) $\cap_R$ (G,B) = (H,C).

Definition (2.26) : The extended intersection of two soft sets (F,A) and (G,B) over a common universe U is defined to be the soft set (H,C),where C= A ∪B and for all c∈ C

H(c) = F(c) if c∈ A - B

= G (c) if c∈ B – A

= F(c) ∩ G (c) if c∈A ∩ B

This relation is denoted by (F,A) $\cap_\varepsilon$ (G,B) =(H,C) .

Definition (2.27) : The union or extended union of two soft sets (F,A) and (G,B) over a common universe U is defined to be the soft set (H,C),where C= A∪B and for all c∈ C

H(c) = F(c) if c∈ A - B

= G (c) if c∈ B – A

= F(c) ∪ G (c) if c ∈ A ∩ B

It is denoted by (F,A) $\cup_\varepsilon$ (G,B) = (H,C).

Definition (2.28) : The restricted union of two soft sets (F,A) and (G,B) over a common universe U is defined to be the soft set (H,C),whereC= A ∩ B and for all c∈ C , H(c) = F(c) ∪ G (c).

It is denoted by H(c) = F(c) $\cup_R$ G (c).

Definition: (2.29) : Let {(F_i, A_i) |i∈ I} be a nonempty family of soft sets over a common universe U. The extended intersection of these soft sets is defined to be soft set (G,B) such that B = $\bigcup_{i\in I} A_i$ for all x ∈ B, G (x) = $\bigcap_{i\in I(x)} F_i(x)$ for all x ∈ B where I(x) = {i∈ I|x ∈ A_i }.In this case we write $\cap_\varepsilon$ (F_i, A_i) = (G,B).

Definition: (2.30) : Let {(F_i, A_i) |i∈ I} be a nonempty family of soft sets over a common universe U. The restricted intersection of these soft sets is defined to be soft set (G,B) such that B = $\bigcap_{i\in I} A_i$, for all x ∈ B, G (x) = $\bigcap_{i\in I(x)} F_i(x)$ for all x ∈ B where I(x) = {i∈ I|x ∈ A_i }.In this case we write $\cap_R$ (F_i, A_i) = (G,B).

Definition (2.31): Let {(F_i, A_i) |i∈ I} be a nonempty family of soft sets over a common universe U. The extended union of these soft sets is defined to be soft set (G,B) such that B = $\bigcup_{i\in I} A_i$ and for all x∈ B, G (x)= $\bigcup_{i\in I(x)} F_i(x)$ where I(x)={i∈ I|x ∈ A_i }.

It is denoted by $\cup_\varepsilon$(F_i, A_i) = (G, B).

Definition (2.32) : Let {(F_i, A_i) |i∈ I} be a nonempty family of soft sets over a common universe U. The restricted union of these soft sets is defined to be soft set (G, B) such that B = $\bigcap_{i\in I} A_i$ for all x∈ B G (x)= $\bigcup_{i\in I(x)} F_i(x)$ where I(x) = {i∈ I|x ∈ A_i }.

It is denoted by $\cup_R$(F_i, A_i) = (G, B).

Definition (2.33) : Let {(F_i, A_i) |i∈ I} be a nonempty family of soft sets over a common universe U. The AND soft set $\bigwedge_{i\in I}$ (F_i, A_i) of these soft sets is defined to be the soft set (H,B) such that B = $\prod_{i\in I} A_i$ and H (x) = $\bigcap_{i\in I(x)} F_i(x)$, for all x = $(x_i)_{i\in I}$ ∈ B.

For fuzzy soft sets we refer [4,7,8,9,11,18,19,20,24,25]:-

Definition: (2.34): Let U be an initial universe set and E be the set of parameters. Let A be a subset of E. A pair (F, A) is called a fuzzy soft set over U, where F is a mapping given by F: A→ I^U,where I^U denotes the collection of all fuzzy subsets of U.

i.e. For each a∈ A , F(a) = F_a: U→ I is a fuzzy set on U.

Definition: (2.35): Let U be an initial universe set and E be the set of parameters. Then the pair (U,E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a fuzzy soft class.

Definition (2.36): Let (F, A) be a fuzzy soft set .Then the set

Supp (F, A) ={x ∈A/F(x) = F_x ≠∅} is called the support of the fuzzy soft set (F,A).

A fuzzy soft set is called non-null if its support is not equal to the empty set.

Definition (2.37): The complement of a fuzzy soft set **(**F,A) denoted by (F,A)^c is defined by **(**F,A)^c = (F^c, A) where F^c: A→ I^U is a mapping given by

F^c (e) = [F (e)]^c for all e ∈ A.

Definition: (2.38): Let (F, A) and (G, B) be two fuzzy soft sets over a common universe U. Then (F, A) is called a fuzzy soft subset of (G, B) if

(i) $A \subseteq B$ and

(ii) $F(a) \subseteq G(a)$ for all $a \in A$.

It is denoted by $(F, A) \tilde{\subseteq} (G, B)$

Definition (2.39): If (F, A) and (G, B) are two fuzzy soft sets over a common universe U then "(F,A) AND (G,B)" is a fuzzy soft set denoted by $(F,A) \wedge (G,B)$ and is defined by $(F,A)\wedge(G,B) = (H,A\times B)$ where $H(a,b)= H_{a,b} = F_a \wedge G_b$

for all $(a, b) \in A \times B$.

Definition (2.40): If (F, A) and (G, B) are two fuzzy soft sets over a common universe U then "(F,A) OR (G,B)" is a fuzzy soft set denoted by $(F,A) \vee (G,B)$ and is defined by $(F,A) \vee (G,B) = (H,A\times B)$ where $H(a,b)= H_{a,b} = F_a \vee G_b$

for all $(a, b) \in A \times B$.

Definition (2.41): Let (F,A) and (G,B) be two fuzzy soft sets in a soft class (U,E). The intersection of two fuzzy soft sets (F,A) and (G,B) over a common universe U is the fuzzy soft set (H,C) where $C= A\cap B$, and H(c) = F(c) or G (c) for all $c \in C$.

We write $(F,A) \tilde{\cap} (G,B) =(H,C)$.

Ahmed and Kharal pointed out that generally F(c) or G (c) may not be identical. In order to avoid the degenerate case Ahmed and Kharal proposed that $A\cap B$ must be non-empty and thus revised the above definition as follows which is known as the restricted intersection.

Definition (2.42): Let (F, A) and (G, B) be two fuzzy soft sets over a common universe U such that $A\cap B \neq \phi$. Then the restricted intersection of (F, A) and (G, B) is the fuzzy soft set (H, C) where $C= A\cap B$ and for all $c \in C$,$H(c) = F(c) \wedge G(c)$.

It is denoted by $(H,C) = (F,A) \cap_R (G,B)$

Definition (2.43): The extended intersection two fuzzy soft sets (F,A) and (G,B) over a common universe U is the fuzzy soft set (H,C),where $C= A \cup B$ and for all $c \in C$,

$H(c) = F(c)$ if $c \in A - B$

$= G(c)$ if $c \in B - A$

$= F(c) \wedge G(c)$ if $c \in A \cap B$

We write $(H, C) = (F, A) \cap_e (G, B)$.

Definition (2.44): The union or extended union of two fuzzy soft sets (F, A) and (G, B) over a common universe U is the fuzzy soft set (H,C),where $C= A\cup B$ and for all $c \in C$,

$H(c) = F(c)$ if $c \in A - B$

$= G(c)$ if $c \in B - A$

$= F(c) \vee G(c)$ if $c \in A \cap B$

We write $(H,C) = (F, A) \cup_e (G, B)$.

Definition (2.45): If (F,A) and (G,B) are two fuzzy soft sets over a common universe U with $A \cap B \neq \emptyset$ then their restricted union is the fuzzy soft set (H, C) where $C= A\cap B$ for all $c \in C$, $H(c) = F(c) \vee G(c)$.

We write $(H,C) = (F,A) \cup_R (G,B)$.

Definition (2.46): Let $\{(F_i,A_i, U) / i\in I\}$be a family of fuzzy soft sets with $\bigcap_{i\in I} A_i \neq \phi$. The intersection of these fuzzy soft sets is a fuzzy soft set (H, C, U) where $C= \bigcap_{i\in I} A_i$ and

$H(c) = \bigwedge_{i\in I} f_i(c)$ for all $c \in C$.

It is denoted by $(H,C) = \bigcap_{i\in I} (F_i,A_i)$.

Definition (2.47): Let $\{(F_i,A_i, U) / i\in I\}$ be a family of fuzzy soft sets. The union of these fuzzy soft sets is a fuzzy soft set (H,C,U) where $C= \bigcup_{i\in I} A_i$ and

$H(c) = \bigvee_{i\in I} f_i(c)$ for all $c \in C$. It is denoted by $(H,C) = U\{(F_i,A_i) / i\in I\}$.

3. Fuzzy Soft Near-rings:-

Definition (3.1) [23]: Let $(R, +, \cdot)$ be a near-ring and E be the set of parameters and $A \subset E$. Let (F, A) be a non-null soft set over R. Then (F,A) is called a soft near-ring over R if and only if for each $a\in Supp(F,A)$, $F(a) = F_a$ is a subnear-ring of R.

i.e.(i) $x,y\in F_a \Rightarrow x-y\in F_a$

(ii) $x,y\in F_a \Rightarrow xy\in F_a$

Example (3.2): Let R = {a, b, c, d} be a non-empty set with two binary operations '+' and '·' defined as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	a	a	a
c	a	a	a	a
d	a	b	c	d

Then $(R, +, \cdot)$ is a near-ring.
Let (F,A) be a soft set over R, where A= R and $F: A\to P(R)$ is a set-valued function defined by $F(x) = \{y\in R / xy\in\{a,b\}\}$for all $x\in A$.
Then F(a) = F(b) = F(c) = R and F(d) = {a,b} are all subnear-rings of R.
Hence (F,A) is a soft near-ring over R.

Definition (3.3) [23]: Let (F,A) and (G,B) be soft near-rings over R. Then the soft near-ring (F,A) is called a soft subnear-ring of (G,B) if it satisfies:

(i) $A \subseteq B$

(ii) F(x) is a subnear-ring of G(x) for $x\in Supp(F, A)$.

At the outset we shall define fuzzy soft near-ring as follows:

Definition (3.4) Let (R, +, ·) be a near-ring and E be the set of parameters and A ⊂ E.

Let F be a mapping given by F: A→$[0,1]^R$ where $[0,1]^R$ is the collection of all fuzzy subsets of R. Then (F, A) is called fuzzy soft near-ring over R if and only if for each a∈A, the corresponding fuzzy subset F_a of R is a fuzzy sub near-ring of R. i.e. (i) F_a (x+y) ≥ min (F_a (x) , F_a (y)) ,

(ii) F_a (-x) ≥ min (F_a (x),

(iii) F_a (xy) ≥ min(F_a (x) , F_a (y)),

for all x,y ∈ R.

Example (3.5): Let R = {a, b, c, d} be a non-empty set with two binary operations '+' and '·' defined as follows:

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

.	a	b	c	d
a	a	a	a	a
b	a	b	c	d
c	a	a	a	a
d	a	b	c	d

Then (R, +, ·) is a near-ring.
Let A= {e_1, e_2, e_3} be the set of parameters.
Define a fuzzy soft set (F,A) on a near-ring R by

F	e_1	e_2	e_3
a	0.2	0.4	0.7
b	0.2	0.3	0.6
c	0.1	0.2	0.3
d	0.1	0.2	0.3

Then clearly the fuzzy soft set (F,A) is a fuzzy soft near-ring over a near-ring R.

Example (3.6): Since each soft set can be considered as a fuzzy soft set and since each characteristic function of a subnear-ring of a near-ring is a fuzzy subnear-ring, we may consider a soft near-ring as a fuzzy soft near-ring.

Theorem (3.7): Let (F, A) be a fuzzy soft set over a near-ring R. Then (F, A) is a fuzzy soft near-ring over R if and only if for each a∈A and x,y∈R the following conditions hold:

(i) F_a(x-y) ≥ min (F_a(x), F_a (y)).

(ii) F_a (xy) ≥ min (F_a(x), F_a (y)).

Proof: Let (F, A) be a fuzzy soft near-ring over R.

Let a ∈A and x, y∈ R. Then

(i) F_a(x-y) = F_a(x +(-y)) ≥ min (F_a(x), F_a (-y))

≥ min (F_a(x), F_a (y))

Since (F, A) is a fuzzy soft near-ring over R, the second condition holds.

Conversely let (F, A) be a fuzzy soft set over a near-ring R satisfying the given conditions.

Now consider F_a(0) = F_a(x-x) ≥ min (F_a(x), F_a (x)). ≥ F_a(x).

Thus F_a(0) ≥ F_a(x) for all x∈R.

Also F_a(-x) ≥ F_a(x) for all x∈R.

Therefore F_a(x-y) = F_a(x+(-y))

≥ min (F_a(x), F_a (-y))

≥ min (F_a(x), F_a (y)).

Hence the theorem.

Theorem (3.8): Let (F, A) and (G B) be two fuzzy soft near-rings over R.

If (F, A) ∧ (G, B) is non-null, then it is a fuzzy soft near-ring over R.

Proof: Let (F, A) and (G B) be two fuzzy soft near-rings over R.

Let (F, A) ∧ (G, B) = (H, A×B),

where H(a,b)= $H_{a,b} = F_a \cap G_b$ for all (a, b)∈A × B.

Since (H, A×B) non-null, there exists a pair (a, b)∈A × B such that

$H_{a,b} = F_a \wedge G_b \neq 0_R$.

Since F_a is a fuzzy subnear-ring of R for all a∈A and G_b is a fuzzy subnear-ring of R for all b∈B and since the intersection of two fuzzy subnear-rings of R is a subnear-ring of R, therefore

$H_{a,b} = F_a \cap G_b$ is a subnear-ring of R.

Hence (F, A) ∧ (G, B) = (H, A×B) is fuzzy soft near-rings over R.

Theorem (3.9): Let (F, A) and (G B) be two fuzzy soft near-rings over R.

If (F, A) $\cap_R$ (G B) is non-null, then it is a fuzzy soft near-ring over R.

Proof: Let (F, A) $\cap_R$(G B) = (H, C), where C= A∩ B.

Let c∈C.

Since (F, A) $\cap_R$ (G B) = (H, C), where C= A∩ B and H_c = F_c∩ G_c for all c∈C.

Since (H, C) non-null, there exists c∈C such that $H_c(x) \neq \phi$ for some x∈R.

Since intersection of two sub near-rings is a sub near-ring we see that F_c∩ G_c

is a fuzzy sub near-ring of R.

Hence (F, A) $\cap_R$ (G B) is a fuzzy soft near-ring over R.

Theorem (3.10): Let (F, A) and (G B) be two fuzzy soft near-rings over R.

Then the extended intersection (F, A) $\cap_\varepsilon$ (G B) is a fuzzy soft near-ring over R.

Proof: Let (F, A) and (G B) be two fuzzy soft near-rings over R.

Let (F, A) $\cap_\varepsilon$(G B) = (H, C), where C= A∪ B.

Then consider the following cases;

(i) If $c \in B-A$ then $H_c = G_c$ for all $c \in C$.

Since G_c is a fuzzy subnear-ring of R, H_c is a fuzzy subnear-ring of R.

(ii) If $c \in A-B$ then $H_c = F_c$ for all $c \in C$.

Since F_c is a fuzzy subnear-ring of R, H_c is a fuzzy subnear-ring of R.

(iii) If $c \in A \cap B$ then $H_c = F_c \cap G_c$ for all $c \in C$.

Since intersection of two fuzzy sub near-rings of R is a fuzzy sub near-ring of R we see that $F_c \cap G_c$ is a fuzzy sub near-ring of R

i.e. H_c is a fuzzy subnear-ring of R.

Thus in any case H_c is a fuzzy subnear-ring of R.

Hence (F, A) $\cap_\varepsilon$ (G B) = (H, C) is a fuzzy soft near-ring over R.

Theorem (3.11): Let (F, A) and (G B) be two fuzzy soft near-rings over R.

Then (F, A) $\cup_e$ (G, B) is a fuzzy soft near-ring over R if $A \cap B = \emptyset$.

Proof: Let (F, A) and (G B) be two fuzzy soft near-rings over R.

Let (F, A) $\cup_e$ (G, B) = (H, C) where $C = A \cup B$

If $c \in C$, since $A \cap B = \emptyset$ then either $c \in A$ or $c \in B$.

If $c \in A$ then $H_c = F_c$

If $c \in B$ then $H_c = G_c$

Since F_c , G_c are the fuzzy sub near-rings of R, therefore in both cases H_c is a fuzzy subnear-ring of R.

Hence (F, A) $\cup_e$ (G, B) = (H, C) is a fuzzy soft near-ring over R.

Theorem (3.12): Let $\{(F_i, A_i) | i \in I\}$ be a non-empty family of fuzzy soft near-rings over R. Then we have the following:

(i) If $\wedge\{(F_i, A_i) | i \in I\}$ is non-null ,then it is a fuzzy soft near-ring.

(ii) If $\cap \{(F_i, A_i) | i \in I\}$ is non-null, then it is a fuzzy soft near-ring.

(iii) If $\{ A_i | i \in I\}$ are pairwise disjoint and $\cup\{ (F_i, A_i) / i \in I\}$ is non-null ,then it is a

fuzzy soft near-ring.

Proof: (i) Let $\wedge\{(F_i, A_i) | i \in I\}$= (G, B) where B

$= A_1 \times A_2 \times \cdots \times A_n$

and $G_b = \wedge \{F_i, (b_i) | i \in I\}$ for all $b_i \in B$.

Suppose the fuzzy soft set (G,B) is non-null.

If b $\in$ Support (G,B), then $G_b = \wedge F_i, (b_i)$.

Since (F_i, A_i) is a fuzzy soft near-ring for all $i \in I$, then $F_i, (b_i)$ is a fuzzy sub near-ring of R.

Hence (G,B) is a fuzzy sub near-ring of R for all b $\in$ Support (G,B).

Hence if $\wedge\{(F_i, A_i) | i \in I\}$ is non-null ,then it is a fuzzy soft near-ring.

(ii) Let $\cap\{(F_i, A_i) | i \in I\}$ = (G,B),where $B = \cap A_i$ and $G_b = \wedge \{F_i, (b_i) | i \in I\}$.

Suppose the fuzzy soft set (G,B) is non-null.

If b $\in$ Support (G,B), then $G_b = \wedge F_i, (b_i)$ is non-null.

Since (F_i, A_i) is a fuzzy soft near-ring for all $i \in I$, then $F_i, (b_i)$ is a fuzzy sub near-ring of R for all $i \in I$.

Hence (G,B) is a fuzzy sub near-ring of R for all b $\in$ Support (G,B).

Thus $\cap \{(F_i, A_i) | i \in I\}$ is a fuzzy soft near-ring.

(iii) Similar.

Definition (3.13) Let (F, A) be a fuzzy soft set. For each $\alpha \in (0,1]$,the soft set

(F, A)$^\alpha$ = (F^α, A) is called α-level soft set of (F,A) where

$(F_a)^\alpha = \{x \in R | F_a(x) \geq \alpha\}$ for each $a \in A$.

Clearly **(**F, A)$^\alpha$ is a soft set.

Theorem (3.14) Let (F,A) be a fuzzy soft set. Then (F,A) is a fuzzy soft near-ring if and only if for all $a \in A$ and for arbitrary $\alpha \in (0,1]$ with $(F_a)^\alpha \neq \phi$, the α-level soft set **(**F,A)$^\alpha$ is a soft near-ring.

Proof: Let (F,A) be a fuzzy soft near-ring.

Then for $a \in A$, F_a is a fuzzy subnear-ring of R.

Let $x,y \in (F_a)^\alpha$ for arbitrary $\alpha \in (0,1]$ with $(F_a)^\alpha \neq \phi$.

Then $F_a(x) \geq \alpha$ and $F_a(y) \geq \alpha$.

Therefore $F_a(x-y) \geq F_a(x) \wedge F_a(y) \geq \alpha$.

Hence $x-y \in (F_a)^\alpha$.

Also $F_a(xy) \geq F_a(x) \wedge F_a(y) \geq \alpha$.

Hence $xy \in (F_a)^\alpha$.

Thus the α-level soft set **(**F, A)$^\alpha$ is a soft near-ring.

Conversely let α-level soft set **(**F, A)$^\alpha$ is a soft near-ring.

Let $\alpha = F_a(x) \wedge F_a(y)$ for all $a \in A$ and $x,y \in R$.

Then $x , y \in (F_a)^\alpha$.

Since $(F_a)^\alpha$ is a sub near-ring of R, $x-y \in (F_a)^\alpha$ and $xy \in (F_a)^\alpha$.

Therefore $F_a(x-y) \geq F_a(x) \wedge F_a(y) \geq \alpha$ and $F_a(xy) \geq F_a(x) \wedge F_a(y) \geq \alpha$.

Hence F_a is a sub near-ring of R.

Thus (F, A) be a fuzzy soft near-ring of R.

4. Idealistic Fuzzy Soft Near-rings:-Definition (4.1) [23]: Let (F.A) be a soft near-ring over R. A non-null soft set (G,I) over R is called a soft left (resp. right) ideal of (F,A) if it satisfies the following conditions :

(i) I ⊂ A

(ii) G_x is a left ideal of F_x (resp. right) for all

x∈ Supp (G, I).

If (G, I) is both soft left and soft right ideal of (F,A) then (G, I) is called a soft ideal of (F.A).

Definition (4.2) [23]: Let (F.A) be a soft near-ring over R. If F_x is left (resp. right, ideal) ideal of R for all x∈ Supp (F, A), then (F, A) is called a left idealistic (resp. right idealistic, idealistic) soft near-ring over R.

Now we shall define the idealistic fuzzy soft near-ring:

Definition (4.3): Let (F.A) be a fuzzy soft near-ring over R. Then (F.A) is said to be an idealistic fuzzy soft near-ring if F_a is a fuzzy ideal of R for all a∈ Supp (F, A).

i.e. (i) F_a (x+y) ≥ min { F_a (x), F_a (y)} for all x,y ∈R,

(ii) F_a (-x) ≥ F_a (x) for all x ∈R,

(iii) F_n (x) = F_n (y+x-y) for all x, y ∈R,

(iv) F_a (xy) ≥ F_a (y) for all x,y ∈R,

(v) F_a {(x+i) y-xy} ≥ F_a (i) for all x, y, i ∈R.

If F_a satisfies (i), (ii), (iii) and (iv) then it is called left idealistic fuzzy soft near-ring of R and if it satisfies (i), (ii), (iii) and (v) then it is called right idealistic fuzzy soft near-ring of R.

Theorem (4.4): Let (F.A) be a fuzzy soft set over a near-ring R and B⊆A. If (F.A) is an idealistic fuzzy soft near-ring then (F,B) is an idealistic fuzzy soft near-ring over R, provided that it is non-null.

Proof: Let (F.A) be a non-null fuzzy soft set and B⊆A.

Let a∈ Supp (F.B).

Then a∈ Supp (F,B) implies that a∈ Supp (F, A).

Let (F.A) be an idealistic fuzzy soft near-ring over R.

Then F_a is a fuzzy ideal of R for all a∈ Supp (F, A).

But since B⊆A, a∈ Supp (F.A) implies that a∈ Supp (F,B).

Hence F_a is a fuzzy ideal of R for all a∈ Supp (F,B).

Therefore (F, B) is an idealistic fuzzy soft near-ring over R.

Theorem (4.5): Let (F.A) and (G,B) be two idealistic fuzzy soft near-rings over a near-ring R. Then the restricted intersection of (F,A) and (G,B) is an idealistic fuzzy soft near-ring over R if it is non-null.

Proof: Let (F.A) and (G,B) be two idealistic fuzzy soft near-rings over a near-ring R.

Let (F,A) $\cap_R$ (G,B) = (H,C) where C = A∩B and for all c∈ C, H(c) = F(c)∩G (c).

Suppose (H,C) is non-null.

Then there exists a∈ Supp (H,C) such that $H_a = F_a \wedge G_a \neq 0_R$.

Since F_a and G_a are fuzzy ideals of R, it follows that H_a is fuzzy ideal of R for all

a∈ Supp (H,C).

Hence (F,A) $\cap_R$ (G,B) = (H,C) is an idealistic fuzzy soft near-ring over R.

Theorem (4.6): Let (F.A) and (G,B) be two idealistic fuzzy soft near-rings over a near-ring R. Then the extended intersection of (F,A) and (G,B) is an idealistic fuzzy soft near-ring over R if it is non-null.

Proof: Let (F.A) and (G,B) be two idealistic fuzzy soft near-rings over a near-ring R.

Let c ∈ Supp (H,C).

If c ∈ A – B then H(c) = F(c) = F_c and since F_c is fuzzy ideal of R ,therefore H(c) = H_c an idealistic fuzzy soft near-ring over R.

Similarly if c∈B – A then H(c) = G(c) = G_c and since G_c is fuzzy ideal of R therefore H(c) = H_c an idealistic fuzzy soft near-ring over R.

Again if c∈A ∩ B then H(c) = F(c) ∩G (c).

Since intersection of two fuzzy ideals of R is a fuzzy ideal of R, therefore

H(c) = F(c) ∩G (c) is a fuzzy ideal of R.

Hence (H, C) = (F, A) $\cap_e$ (G, B) is an idealistic fuzzy soft near-ring over R if it is non-null.

Theorem (4.7): Let (F.A) and (G,B) be two idealistic fuzzy soft near-rings over a near-ring R. If A and B are disjoint then the extended union (F, A) $\cup_e$ (G, B) is an idealistic fuzzy soft near-ring over R.

Proof: Let (F.A) and (G,B) be two idealistic fuzzy soft near-rings over a near-ring R.

Let A and B be disjoint.

If A ∩ B = ϕ then for a ∈ Supp (H,C) we have either a∈ A – B or a ∈ B – A.

If a ∈ A – B, then $H_a = F_a$ is a fuzzy ideal of R and so H_a is an idealistic fuzzy soft near-ring.

If a ∈B-A, then $H_a = G_a$ is a fuzzy ideal of R and so H_a is an idealistic fuzzy soft near-ring.

Thus for all a ∈Supp (H, C), H_a is a fuzzy ideal of R.

Hence the extended union (F, A) $\cup_e$ (G, B) is an idealistic fuzzy soft near-ring over R.

Note: If A and B are not disjoint then the theorem is not true since the union of two fuzzy ideals of a near-ring R is not a fuzzy ideal of a near-ring R

Theorem (4.8): Let (F.A) and (G,B) be two idealistic fuzzy soft near-rings over a near-ring R. Then (F,A)∧(G,B)= (H,A× B) is an idealistic fuzzy soft near-ring over R if it is non-null.

Proof: Let (F.A) and (G,B) be two idealistic fuzzy soft near-rings over a near-ring R.

Let (F,A)∧(G,B)= (H,A× B) where $H_{(a,b)} = F_a \wedge G_b$ for all (a, b)∈A× B.

Suppose ((H,A× B) is non-null fuzzy soft set.

If (a,b) ∈Supp (H, A× B) , then $H_{(a,b)} = F_a \wedge G_b \neq 0_R$.

Since (F.A) and (G,B) are two idealistic fuzzy soft near-rings over a near-ring R, we conclude that F_a and G_b are fuzzy ideals of R.

Since intersection of two fuzzy ideals of

R is a fuzzy ideals of R.

Therefore $H_{(a,b)}$ is a fuzzy ideal of

R for all (a,b) $\in$ Supp (H, A× B).

Thus (F, A)∧(G,B)= (H,A× B) is an idealistic fuzzy soft near-ring over R.

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