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S - DISTRIBUTIVE POSET

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Abstract- ***s - Distributivity in a partially ordered set (poset) is defined. Some properties of s - distributive poset are obtained. Mainly it is prove that poset P is a distributive if and only if P is s - distributive for each s □ P.***

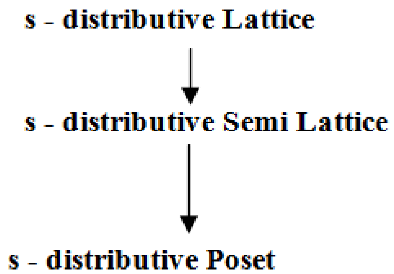
I. INTRODUCTION

In this paper we extend the concept of s - distributivity in a lattice to the concept of s - distributivity in a poset. Venkatanarasimhan [5] was the first who introduced and studied in detail pseudo complemented poset . As a generalization of pseudo complemented poset, Pawar and Dhamake [3] introduced 0 - distributive poset . This has eased our passage in introducing the notion of s - distributive poset . We define s - distributive poset as,

Definition :- Let P be a poset and s (s ≠1) be any fixed element in P. The poset P is called s - distributive if x , y_1, $y_2 \ldots y_n \in P$
(n finite) with $(x] \cap (y_i] \subseteq (s]$ for all i , $1 \le i \le n$ imply
$(x] \cap (y_1 \vee y_2 \vee \ldots \vee y_n] \subseteq (s]$ whenever $y_1 \vee y_2 \vee \ldots \vee y_n$ exists
in P.

It is interesting to note that a lattice L is s - distributive iff it is
s - distributive poset for any fixed element a (s ≠1) in P.

The following flow chart indicates a position of our concept in relation to the known concepts in lattice theory.



§ 1.2 Preliminaries: -

Throughout this chapter we shall concern with partially ordered set or poset . < P, ≤ > , denotes a partially order set with the ordering relation ' ≤ ' defined on non-empty set P . For a finite set
A = {a_1, a_2 , … , a_n}, the least upper bound (l. u. b) and greatest lower bound (g. l. b) of A , if exist, are denoted by $a_1 \vee a_2 \vee \ldots \vee a_n$

and $a_1 \wedge a_2 \wedge \ldots \wedge a_n$ respectively. The least and the greatest element of a poset , when they exist, are denoted by 0 and 1 respectively.

A non-empty subset A of P is called as a semi-ideal if $a \in A$, $b \le a$ implies $b \in A$. A semi-ideal A of P is called as an ideal if the least upper bound of any finite number of elements of A , whenever it exists, belongs to A . This definition of an ideal in a poset given by Venkatanarasimhan [5], is different from that introduced by Frink

[2] . An element a of poset P with 0 is said to be dense if $(a] \cap (b] = (0] \Leftrightarrow b = 0$ for $b \in P$. The set of all elements x of P such that $x \le a$ for some fixed a in P forms an ideal of P. It is called the principal ideal generated by a and is denoted by (a].

We need following lemmas.

Lemma 1.2.1 [5]:- The set I of all ideals of a poset with 0 is complete lattice under set inclusion as ordering relation.

Lemma 1.2.2 [5]:- In a poset finite join $a_1 \vee a_2 \vee \ldots \vee a_n$ exists iff
$(a_1] \vee (a_2] \vee \ldots \vee (a_n]$ is a principal ideal and in this case
$(a_1 \vee a_2 \vee \ldots \vee a_n] = (a_1] \vee (a_2] \vee \ldots \vee (a_n]$.

§ 1.3 s - distributive Poset :-

s - distributive semi lattice is first introduced by varlet [4] .

A semi lattice S with the fixed element s (s ≠ 1) is s - distributive, if for any $x \in P$, < x , s> is an ideal in S where < x , s > = { $y \in S \mid x \wedge y \le s$ }.

As a generalization of s - distributive semi lattice [4] we define
s - distributive poset .

Definition 1.3.1:- Let P be a poset and s (s ≠1) be any fixed element in P. The poset P is called s - distributive if x , y_1, $y_2 \ldots y_n \in P$ (n finite) with $(x] \cap (y_i] \subseteq (s]$ for all i , $1 \le i \le n$ imply $(x] \cap (y_1 \vee y_2 \vee \ldots \vee y_n] \subseteq (s]$ whenever $y_1 \vee y_2 \vee \ldots \vee y_n$ exists in P. As example of s - distributive poset we have

Example 1.3.2:- Let P = {0, a, b, c, d, e, f, g, h} and b and h ≠1 be any fixed element of P. The diagrammatic representation < P ≤ > as follows.

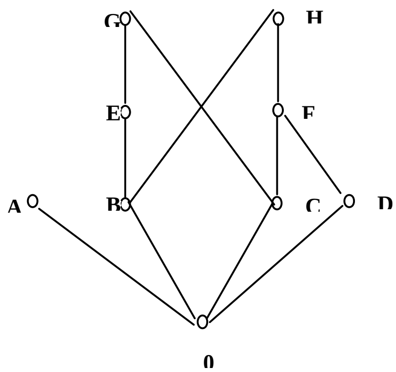


Fig.1.1

The poset represented in Fig. 1.1 is b-distributive but not f-distributive .

We furnished one more example of s - distributive poset . This poset contain infinite number of elements.

Example 1.3.3:- Let P = {0, a, b, c, d, e, f, …….,g } and $x \neq 1$ be any fixed element of P. The diagrammatic representation < P ≤ > as follows.

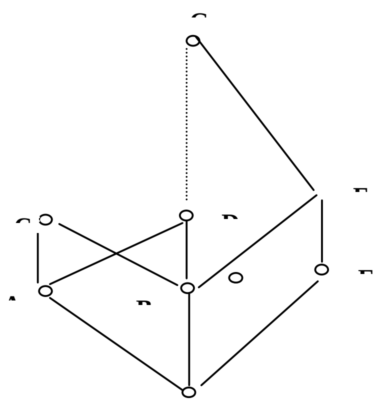


Fig. 1.2

The poset represented in Fig. 1.2 is x-distributive $\forall\, x \in P$.

An example of a poset which is only h -distributive.

Example 1.3.4:- Let P = {0, a, b, c, d, e, f, g, h} and $h \neq 1$ be any fixed element of P. The diagrammatic representation < P ≤ > as follows.

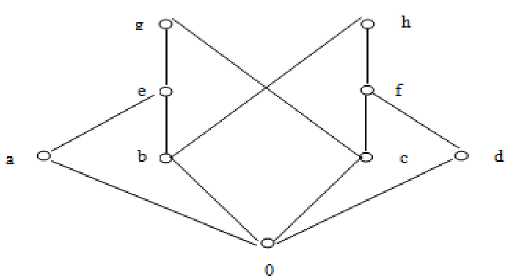


Fig. 1.3

The poset represented in Fig. 1. 3 is only h-distributive.□□□

Remark 1.3.5:- A subset of s - distributive poset containing the element a need not be s - distributive.

On the lines of [4] relative annihilators in poset are defined as follows.

Definition 1.3.6:- In a poset P containing the element s, b, we define

$< b, s > = \{x \in L \mid (x] \cap (b] \subseteq (s]\}$. $< b ,a >$ is called the relative annihilators of b in s .

By s-ideal (s-semi ideal) we mean an ideal (semi ideal) containing an element $s \in P$.

The necessary and sufficient condition for < b , s > to be s-ideal in the following theorem .

Theorem 1.3.7 :- Let P be a poset and s ($s \neq 1$) be any fixed element in P. For any $b \in P$, < b, s > is s-ideal in P if and only if P is s - distributive .

Proof:- Only if part .

Let < b, s > is s - ideal in P for any $b \in P$. To prove that P is s - distributive poset . Let y , $x_1, x_2, \ldots, x_n$ (n finite) $\in$ P with $(x_i] \cap (y] \subseteq (s]$ for all i, $1 \leq i \leq n$ and assume that $x_1 \vee x_2 \vee \ldots \vee x_n$ exists in P. But then $x_i \in < y, s >$ for all i, $1 \leq i \leq n$.

By assumption , < y, s > is an ideal .

Therefore $x_1 \vee x_2 \vee \ldots \vee x_n \in < y, s >$.

Thus $(y] \cap (x_1 \vee x_2 \vee \ldots \vee x_n] \subseteq (s]$. Hence P is s - distributive poset.

If Part.

Let L be s - distributive poset . Fix up any $b \in P$. To prove that <b, a> is a-ideal in P. Obviously <b, s> is a-semi ideal in P. Let $x_1, x_2, \ldots, x_n \in < b, s >$ (n finite) such that $x_1 \vee x_2 \vee \ldots \vee x_n$ exists. Then $x_i \in < b, s >$ implies

$(x_i] \cap (b] \subseteq (s]$ for all i, $1 \leq i \leq n$ as P is s - distributive poset we get (b] ∩

$(x_1 \vee x_2 \vee \ldots \vee x_n] \subseteq (s]$ .This implies $x_1 \vee x_2 \vee \ldots \vee x_n \in$ < b, s>.Hence < b, s > is s-ideal .

A poset Q is called distributive poset if $(a] \cap (b] \subseteq (c]$ (a, b, c $\in$ Q) implies the existence of x , y in Q , $x \geq a$, $y \geq b$ such that $(x] \cap (y] = (c]$.

Now we establish the relation between distributive poset and s - distributive poset.

Theorem 6.3.8:- A poset P is distributive if and only if P is s - distributive for each s ($\neq 1$) in P.

Proof:- Only if part .

Let P be distributive poset. Assume $x_1, x_2, \ldots x_n \in P$ (n finite) such that $\vee_{i=1}^{n} x_i$ exists in P. Let $b \in P$. Obviously , $\vee_{i=1}^{n} (x_i] \cap (b] \supseteq (x_i] \cap (b]$ for all i, $1 \leq i \leq n$.

Let $(x_i] \cap (b] \subseteq (s]$ for all i, $1 \leq i \leq n$. If no such a exists in P then obviously, $\vee_{i=1}^{n} (x_i] \cap (b] = \vee_{i=1}^{n} (x_i \cap b]$.

As P is distributive and $(x_i] \cap (b] \subseteq (s]$, we get

$(s] = (y_1] \cap (b_1]$ for some $y_1 \geq x_1$ and $b_1 \geq b$.

Now $y_1 \geq s$ and $(s] \supseteq (x_2] \cap (b]$ implies $(y_1] \supseteq (x_2] \cap (b]$. Hence by distributivity of P we get $(y_1] = (y_2] \cap (b_2]$ for some $y_2 \geq x_2$ and $b_2 \geq b$. Continuing like this we get an element y_{n-1} in P such that $(y_{n-1}] \supseteq (x_n] \cap (b]$. Applying the distributivity of P again , we get $(y_{n-1}] = (y_n] \cap (b_n]$ where $y_n \geq x_n$ and $b_n \geq b$. Now $y_n \geq y_{n-1}$ and $y_{n-1} \geq x_{n-1}$ imply $y_n \geq x_{n-1}$. As $y_n \geq x_{n-1}$ and $x_{n-1} \geq x_{n-2}$, by transitivity $y_n \geq x_{n-2}$. Continuing like this we get $y_n \geq x_r$ for each r, $1 \leq r \leq n-1$.

As $y_n \geq x_i$ for each i , $1 \leq i \leq n$ we get $y_n \geq \vee^n_{i=1} x_i$ ………………(I)

Again

$(y_{n-1}] \cap (b] = [(y_n] \cap (b_n]] \cap (b]$,
as $(y_{n-1}] = (y_n] \cap (b_n]$ and $b_n \geq b$ .Thus $(y_{n-1}] \cap (b] = (y_n] \cap [(b_n] \cap (b]] = (y_n] \cap (b]$. Similarly we get , $(y_{n-2}] \cap (b] = [(y_{n-1}] \cap (b_{n-1}]] \cap (b]$, as $(y_{n-2}] = (y_{n-1}] \cap (b_{n-1}]$ and $b_{n-1} \geq b$.
$(y_{n-2}] \cap (b] = (y_{n-1}] \cap [(b_{n-1}] \cap (b]] = (y_{n-1}] \cap (b]$.

Continuing in this way we get , $(y_1] \cap (b] = (y_2] \cap (b] = \ldots = (y_{n-1}] \cap (b] = (y_n] \cap (b]$……(II)

As $(a] = (y_1] \cap (b]$, $(a] \cap (b] = (y_1] \cap (b] \cap (b] = (y_1] \cap (b]$ Hence by (II), $(a] \cap (b] = (y_n] \cap (b]$. Thus $(a] \supseteq (y_n] \cap (b]$ .But by (I), $(a] \supseteq \vee^n_{i=1} (x_i] \cap (b]$. Hence P is a - distributive. The proof if part being obvious we omit it .

In support of result of Theorem 1.3.8 we give the following example.

Example 6.3.9:- Let P = {0, a, b, c, d, e, f, 1}. The diagrammatic representation < P ≤ > as follows.

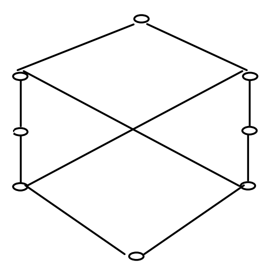


Fig.1.4

The poset shown in Fig . 1.4 is distributive poset and hence s - distributive poset for each s (≠1) in P.

Remark 1.3.10:- Every s - distributive poset need not be distributive.

For this consider the poset P which describe in the example 6.3. This poset is h-distributive but not distributive.

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