MNK Publication

International Journal of Latest Research in Science and Technology **ISSN (Online):2278-5299**
Volume 4, Issue 6: Page No.69-70, November-December 2015 (Special Issue Paper)
http://www.mnkjournals.com/ijlrst.htm
Special Issue on International conference on Methematics Sciences-2015(MSC-2015)
Confernece Held at Sadguru Gadage Maharaj College, Karad ,Maharastra,(India)

SOME BILATERAL GENERATING INTEGRAL AND LAPLACE TRANSFORM OF MODIFIED LAGUERRE'S POLYNOMIALS $L_{a,b,c,n}(t)$.

P.G.Andhare*
*Department of Mathematics.
R.B.N.B.College, Shrirampur.
Dist.Ahmednagar.Pin-413709.

Abstract- ***Goyal*** [1,1983] ***has introduced the explicit of modified Laguerre polynomials*** $L_{a,b,c,n}(t)$ ***In present paper we haveobtained some bilateral generating integral and Laplace Transform of Modified Laguerre's polynomials*** $L_{a,b,c,n}(t)$ ***.Some particular case are also obtained.***

I. INTRODUCTION

Goyal [1] has introduced the explicit form of Modified Laguerre's Polynomials $L_{a,b,c,n}(t)$ as

$$L_{a,b,c,n}(t) = \frac{b^n (c)_n}{n!}\, {}_1F_1\left[-n; c; -\!-; \frac{at}{b}\right]$$

$$L_{a,b,c,n}(t) = \frac{b^n (c)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k \left(\frac{at}{b}\right)^k}{(c)_k \quad k!} \quad \ldots(1.1)$$

But we know

$$(-n)_k = \frac{(-1)^k . n!}{(n-k)!}, \qquad 0 \le k \le n$$

$$= 0 \,, k > n$$

$$L_{a,b,c,n}(t) = \frac{b^n (c)_n}{n!} \sum_{k=0}^{n} \frac{(-1)^k . n! \left(\frac{at}{b}\right)^k}{(c)_k (n-k)! \quad k!} \quad ..(1.2)$$

2 Lalacpe Transform of Modified Laguerre's Polynomials $L_{a,b,c,n}(t)$:

By using definition of Laplace Transform

$$L\{L_{a,b,c,n}(t)\} = \int_0^\infty \exp.(-st) L_{a,b,c,n}(t).dt. =$$

$$= \int_0^\infty \exp.(-st) \left\{ \frac{b^n (c)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k \left(\frac{at}{b}\right)^k}{(c)_k \quad k!} \right\} .dt$$

$$L\{L_{a,b,c,n}(t)\} = L\left\{ \frac{b^n (c)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k \left(\frac{at}{b}\right)^k}{(c)_k \quad k!} \right\} \quad \ldots.(2.1)$$

$$= L\left\{ \frac{b^n (c)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k \left(\frac{a}{b}\right)^k t^k}{(c)_k \quad k!} \right\}$$

$$= \frac{b^n (c)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k \left(\frac{a}{b}\right)^k}{(c)_k \quad k!} L\{t^k\}$$

$$= \frac{b^n (c)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k \left(\frac{a}{b}\right)^k}{(c)_k \quad k!} \left(\frac{k!}{s^{k+1}}\right)$$

$$= \frac{b^n (c)_n}{s.n!} \sum_{k=0}^{\infty} \frac{(-n)_k \left(\frac{a}{bs}\right)^k}{(c)_k}$$

We take c = 1

$$L\{L_{a,b,c,n}(t)\} = \frac{b^n (1)_n}{s.n!} \sum_{k=0}^{\infty} \frac{(-n)_k \left(\frac{a}{bs}\right)^k}{(1)_k}$$

$$L\{L_{a,b,c,n}(t)\} = \frac{b^n n!}{s.n!} \sum_{k=0}^{\infty} \frac{(-n)_k \left(\frac{a}{bs}\right)^k}{k!}$$

$$L\{L_{a,b,c,n}(t)\} = \frac{b^n}{s}\left(1-\frac{a}{bs}\right)^n \quad \text{….(2.2)}$$

3.Particular Case: If a=b =1,then equation (2.2) reduces to [4,P.216 Ex. (10)]

$$\int_0^{\infty} e^{-st} L_n(t).dt = \frac{1}{s}\left(1-\frac{1}{s}\right)^n$$

4.Some Bilateral Generating Integral: Pittaluga G.,Sacripante L.and Srivastava H.M[,P.150] obtained the following generating function

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(c)_n} L_{a,b,c,n}(x) t^n = (1-bt)^{-c} {}_1F_1\left[\lambda; c; —; -\frac{a.x.t}{1-bt}\right] \quad \text{…………(4.1)}$$

If c= λ in equation (4.1) we get

$$\sum_{n=0}^{\infty} L_{a,b,c,n}(x) t^n = (1-bt)^{-c} \exp\left\{-\frac{a.x.t}{1-bt}\right\} \quad \text{…..(4.2)}$$

The integral representation of the Hermite polynomial

[2] and[5,p.7Eq.2]

$$H_n(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left\{-v^2\right\} [2(x+iv)]^n .dv \quad \text{………(4.3)}$$

From (4.3) and (4.2)

$$\sum_{n=0}^{\infty} L_{a,b,c,n}(x).H_n(y) t^n = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left\{-v^2\right\}. \sum_{n=0}^{\infty} L_{a,b,c,n}(x) [2t(y+iv)]^n \, dv$$

$$= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left\{-v^2\right\}.[1-2bt(y+iv)]^{-c} \exp\left[\frac{-2axt(y+iv)}{1-2bt(y+iv)}\right].dv$$

……………………………………(4.4)

5.Particular Case:

If we put a = b = 1 and $c = \lambda + 1$, then we get equation (10) [5 , P.9].

$$\sum_{n=0}^{\infty} L^{(\lambda)}{}_n(x).H_n(y) t^n = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left\{-v^2\right\}.[1-2t(y+iv)]^{-1-\lambda} \exp\left[\frac{-2xt(y+iv)}{1-2t(y+iv)}\right].dv$$

REFERENCES

1. Goyal,G.K. (1983):Modified Laguerre Polynomials of VijnanaParishad Anusandhan Patrika 26 (1983) 263-266.,J.Math Anal. Appl,45,176-198
2. Lebedev, N. N. (1965) : Special Functions and Their Applications. Prentice-Hall, Englewood Cliffs, New Jersey.
3. Pittaluga G.,Sacripante L.and Srivastava H.M.: Some Generating Function of Laguerre and modified Laguerre Polynomials.Applied Mathematics and Computation 113 (2000),P. 141-160.
4. Rainville, E. D. (1960) : Special Functions. Macmillan, New York; Reprinted by . Chelsea Publ. Co., Bronx, New York, 1971.
5. Taha I.H.and Chatterea S.K.: Some unusual Bilateral Generating Integrals, Pure. Math.Manuscript Vol.3 (1984),P.7-9.