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LAMINAR BOUNDARY LAYER FLOW OF NON-NEWTONIAN POWER LAW FLUID ALONG WALL OF CONVERGENT CHANNEL.

Dr. B.P. JADHAV
DEPARTMENT OF MATHEMATICS,
THE NEW COLLEGE, KOLHAPUR-416012
Mobile-+91 9765258838, email- bpj.jadhav@gmail.com

Abstract- ***Laminar Boundary Layer Flow of Non-Newtonian Power Law Fluid along Wall of Convergent Channel has been considered. The governing equations of continuity and momentum are transformed into ordinary differential equations using similarity transformations. The equations are solved by using method of successive approximations starting with zeroth approximation. For n=1 the results tallies with Corresponding results for Newtonian fluids. Velocity profiles have been drawn for different values of parameter n; shows the behavior of power law fluids. Local Skin Friction Coefficient $c_f^* = [-f_1''(0)]^n$ has been calculated.***

Keywords: - boundary layer, power law fluids, successive approximations, Convergent channel, velocity profiles, Skin friction.

I. INTRODUCTION

Boundary layer flows of viscous, incompressible fluid past semi infinite flat plate were studied by Blassius [1], Howarth [2]. The boundary layer flow of viscous fluid along the wall of convergent channel was first considered by Pohlhausen[3] . Sanyal [4] studied the two dimensional boundary layers along the wall of convergent channel with curved boundaries. The study of compressible boundary layers along the convergent channel was made by Singh[5].

The study of non –Newtonian viscoelastic fluid along the wall of convergent channel was made by Black and den[6], Nandlal Singh[7].Similarity solutions for power law fluid past wedge were considered by Kapur and Shrivastav[8],Lee and Ames[9].The solution to the wedge flow of power law fluid has been obtained by S.Roy[10],Tapas Ranjan Roy[11].In this paper we have extended the problem of Jadhav B.P. [12] to Non-Newtonian Power law fluid flow along the wall convergent channel..

Mathematical Analysis:-

Consider a steady, two dimensional flow of an electrically conducting, non-Newtonian power law fluid along the wall of the convergent channel .

The *x*-axis is taken along the direction of flow and *y*-axis normal to it. Along the wall of the channel the potential flow velocity U(*x*) is given by;

$$U(x) = -\frac{u_1}{x} \quad (u_1 > 0) \qquad \text{---------------} (1)$$

The governing boundary layer equations are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} - \gamma\frac{\partial}{\partial y}\left(-\frac{\partial u}{\partial y}\right)^n \qquad \text{---} (2)$$

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \qquad \text{------} (3)$$

With boundary conditions

$u = 0,\ v = 0$ at $y=0$ and $u = U(x)\ ,\ v = 0\ as\ y \to \infty$ ---- (4)

Introduce a stream function $\emptyset$ (*x, y)* such that $u = \frac{\partial\emptyset}{\partial y}$, $v = -\frac{\partial\emptyset}{\partial x}$ ----- (5)

Introducing similarity transformation

$$\eta = y\left(\frac{U^{2-n}}{\gamma x}\right)^{\frac{1}{n+1}} \quad , \qquad \emptyset(\eta) = (\gamma x U^{2n-1})^{\frac{1}{n+1}} f(\eta)$$

--------- (6)

Substituting the values in (6) to the equation (2) reduces to

$$n(f'')^{n-1}f''' + \left(\frac{2n-2}{n+1}\right) f f'' + 1 - f'^2 = 0 \text{ -------} (7)$$

The Boundary conditions are

$$f(0) = 0, f'(0) = 0, f'(\infty) = 1 \qquad \text{------------} (8)$$

Method of Solution:-

To solve the non-linear differential equation (7) under the boundary conditions (8), we use method of successive approximations starting with zeroth approximation.

For zeroth approximation, we assume

$$f(\eta) = \eta - \frac{1}{\beta} + \frac{1}{\beta}e^{-\beta\eta} \qquad \text{------------} (9)$$

Where β is arbitrary constant to be determined such that for the first approximation $f_1'(0) = 0$, i. e. β is real root of the equation

$$\beta^{n+1} + \frac{4}{n(n+1)(n-2)^3} - \frac{1}{n(n+1)(n-2)} = 0 \qquad \text{---------------} (10)$$

The different successive approximations can be obtained from

$$n f'''_i = (f''_{i-1})^{1-n}\left[\left(\frac{2-2n}{n+1}\right) f_{i-1} f''_{i-1} + f'^2_{i-1} - 1\right] \text{- } \mathbf{(11)}$$

For the first approximation, we have,

$$n f_1''' = [f''_0)^{1-n}\left[\left(\frac{2-2n}{n+1}\right) f_0 f''_0 + f'^2_0 - 1\right] \text{ --} (12)$$

Integrating (13) with boundary conditions (9), we obtain

$$f_1'(\eta) = (A_1 - A_2\eta)e^{(n-2)\beta\eta} - A_3 e^{(n-3)\beta\eta} + 1 \text{-- (13)}$$

Where, $A_1 = \frac{4}{n(n+1)(n-2)^3\beta^{n+1}}$, $A_2 = \frac{2(n-1)}{n(n+1)(n-2)^2\beta^{n}}$,

$$A_3 = \frac{1}{n(n+1)(n-3)\beta^{n+1}} \text{--------- (14)}$$

Values of β can be obtained for various values of power index *n*. For different values of β, power index *n*, values of $f_1'(\eta)$ can be obtained. Hence Velocity profiles can be drawn for various values of *n*.

For *n*=1 the results tallies with the corresponding results for Newtonian fluids.

For different values of *n*, skin friction coefficient $c_f^* = [f_1''(0)]^n$, various boundary layer parameters are obtained.

Boundary layer parameters:-

i) **Displacement Thickness**:-

The displacement thickness δ_1 is given by

$$\delta_1 = \int_0^\infty (1 - f_1') \, dy, \quad \frac{\delta_1}{x}(\mathrm{Re})^{\frac{1}{n+1}} = \int_0^\infty (1 - f_1') \, d\eta$$

$$\delta_1^* = \frac{\delta_1}{x}(\mathrm{Re})^{\frac{1}{n+1}} = \frac{A_1}{(n-2)\beta} + \frac{A_2}{(n-2)^2\beta^2} - \frac{A_3}{(n-3)\beta} \text{--- (15)}$$

ii) Momentum Thickness:-

The Momentum thickness δ_2 is given by

$$\delta_2 = \int_0^\infty f_1'(1 - f_1') \, dy, \quad \frac{\delta_2}{x}(\mathrm{Re})^{\frac{1}{n+1}} = \int_0^\infty f_1'(1 - f_1') \, d\eta$$

$$\delta_2^* = \frac{\delta_2}{x}(\mathrm{Re})^{\frac{1}{n+1}} = \frac{A_1^2}{2(n-2)\beta} + \frac{A_1 A_2}{2(n-2)^2\beta^2} + \frac{A_2^{\,2}}{4((n-3)^3\beta^3} - \frac{2A_1 A_3}{(2n-5)\beta} - \frac{2A_2 A_3}{(2n-5)^2\beta^2} + \frac{A_3^2}{2(n-3)\beta} + \frac{A_1}{(n-2)\beta} + \frac{A_2}{(n-2)^2\beta^2} - \frac{A_3}{(n-3)\beta}$$

----------- (16)

iii) Skin-friction coefficient (c_f^*):- Skin-friction coefficient c_f^* is given by

$$c_f^* = [f_1''(0)]^n = [A_1(n-2)\beta - A_2 - A_3(n-3)\beta]^n \textbf{ (17)}$$

Table -1

Boundary layer parameters for various values of *n*

n	δ_1^*	δ_2^*	$\frac{\delta_1^*}{\delta_2^*}$	$c_f^* = [f_1''(0)]^n$
0.5	0.5344	0.2465	2.168	1.2669
0.8	0.7131	0.3446	2.0694	1.2137
1.0	0.8099	0.3947	2.0519	1.1339
1.2	0.9079	0.4427	2.0508	1.0110
1.5	1.0861	0.5243	2.0715	0.7435
1.8	1.3581	0.6394	2.1240	0.4109

CONCLUSIONS:-

The values of arbitrary parameter β are calculated for different values of flow index *n* from the equation (10).for different values of *n* various boundary layer parameters δ_1^*, δ_2^*, $\frac{\delta_1^*}{\delta_2^*}$, c_f^* are obtained; and presented in the table -1.It has been observed that for *n*=1,the results agrees with the results obtained by Pohlhausen for Newtonian fluid.

As *n* increases the both boundary layer thicknesses increases, but skin -friction parameter decreases. As *n* increases then for dilatants fluids the ratio of thicknesses decreases, while for pseduoplastic fluids the ratio increases. The present method employed gives good agreements with the available results.

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