

FUZZY LATTICE ORDERED M-NORMAL SUBGROUP

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Abstract- In this paper we introduce the notion of fuzzy lattice ordered m normal subgroups and investigated some of its basic properties. In this paper we defined fuzzy lattice ordered m-normal group, arbitrary intersection of fuzzy lattice ordered m-normal groups, m-fuzzy characteristic of fuzzy lattice ordered m group. We also studied the Cartesian product of arbitrary family of fuzzy lattice ordered m-normal groups.

Keywords: Lattice ordered group, Fuzzy lattice ordered m-group, Fuzzy lattice ordered m normal group, characteristic of fuzzy group, direct product.

INTRODUCTION

A fuzzy algebra has become an important branch of research. A. Rosenfeld 1971 [9] used the concept of fuzzy set theory due to Zadeh 1965 [5]. Since then the study of fuzzy algebraic substructures are important when viewed from a Lattice theoretic point of view. N. Ajmal and K.V. Thomas [1] initiated such types of study in the year 1994. It was latter independently established by N. Ajmal [1] that the set of all fuzzy normal subgroups of a group constitute a sub lattice of the lattice of all fuzzy sub groups of a given group and is Modular. Nanda[8] proposed the notion of fuzzy lattice using the concept of fuzzy partial ordering. More recently in the notion of set product is discussed in details and in the lattice theoretical aspects of fuzzy sub groups and fuzzy normal sub groups are explored. G.S.V. Satya Saibaba [3] initiate the study of L-fuzzy lattice ordered groups and introducing the notice of L-fuzzy sub l- groups. J.A. Goguen [4] replaced the valuation set [0,1] by means of a complete lattice in an attempt to make a generalized study of fuzzy set theory by studying L-fuzzy sets. A Solairaju and R. Nagarajan [11] introduced the concept of lattice valued Q-fuzzy sub-modules over near rings with respect to T-norms. DrM.Marudai & V. Rajendran[6] modified the definition of fuzzy lattice and introduce the notion of fuzzy lattice of groups and investigated some of its basic properties. Gu [12] introduced concept of fuzzy groups with operator. Then S. Subramanian, R Nagarajan & Chellappa [10] extended the concept to m fuzzy groups with operator. In this paper we introduce the notion of fuzzy lattice ordered m normal subgroups and investigated some of its basic properties. In this paper we defined fuzzy lattice ordered m-normal group, arbitrary intersection of fuzzy lattice ordered m-normal groups, m-fuzzy characteristic of fuzzy lattice ordered m group. We also studied the Cartesian product of arbitrary family of fuzzy lattice ordered m-normal groups.

SECTION-2 PRELIMINARIES

Definition 2.1: Fuzzy group- Let $\mu: X \rightarrow [0, 1]$ be a fuzzy set & $G \in \mathcal{P}(X) = \text{Set of all fuzzy sets on } X$. A fuzzy set μ on G is called a fuzzy group if i) $\mu(x y) \geq \min \{ \mu(x), \mu(y) \}$ ii) $\mu(x^{-1}) \geq \mu(x)$, for all $x, y \in G$.

Definition 2.2: Lattice ordered group - A lattice ordered group is a system (G, \cdot, \leq) if i) (G, \cdot) is a group ii) (G, \leq) is a lattice. iii) $x \leq y$ implies $a x b \leq a y b$ (compatibility)

For $a, b, x, y \in G$

Definition 2.3: Fuzzy lattice ordered group- Let $\mu: X \rightarrow [0, 1]$ be a fuzzy set & G is a lattice ordered set, $G \in \mathcal{P}(X)$. A function μ on G is said to be a fuzzy lattice ordered group if i) $\mu(x y) \geq \min \{ \mu(x), \mu(y) \}$ ii) $\mu(x^{-1}) \geq \mu(x)$ for all $x, y \in G$

Definition 2.4: M group- Let G be a group, M be any set if i) $m x \in G$. ii) $m(x y) = (m x) y = x m y$ for all $x, y \in G, m \in M$. Then G is called a m group.

Definition 2.5: Fuzzy m group- Let $\mu: X \rightarrow [0, 1]$ be a fuzzy set & G be a M group G . A fuzzy set on $G, G \in \mathcal{P}(X)$ is called a fuzzy m group if i) $\mu(m(x y)) \geq \min \{ \mu(m x), \mu(m y) \}$ ii) $\mu(m x^{-1}) \geq \mu(m x)$ for all $x, y \in G, m \in M$

Definition 2.6: A fuzzy lattice ordered m-group- $\mu: X$ to $[0, 1], G \in \mathcal{P}(X), M \subset X$.

A function μ on G is said to be a fuzzy lattice ordered m-group if

- i) (G, \cdot) is a M-group.
- ii) (G, \cdot, \leq) is a lattice ordered group.
- iii) $\mu(m(x y)) \geq \min \{ \mu(m x), \mu(m y) \}$
- iv) $\mu(m(x^{-1})) \geq \mu(m x)$
- v) $\mu(m x v m y) \geq \min \{ \mu(m x), \mu(m y) \}$
- vi) $\mu(m x \wedge m y) \geq \min \{ \mu(m x), \mu(m y) \}$ For all $x, y \in G$

Definition 2.7: Fuzzy lattice ordered m-normal subgroup- A fuzzy lattice ordered m-group is said to be a fuzzy lattice ordered m-normal subgroup if

$$\mu((m x) (m y)) = \mu((m y) (m x)) \quad \text{for all } x, y \in G, m \in M$$

Definition 2.8: M-fuzzy characteristic of lattice ordered m-group

A fuzzy lattice ordered m group A is said to be a m-fuzzy characteristic of lattice ordered m-group if $\mu_A (m x) = \mu_A (f(m x))$ for all $x \in G, m \in M. f \in \text{Aut}(G)$

SECTION-3 BASIC PROPERTIES OF FUZZY LATTICE ORDERED M-NORMAL GROUP

Proposition 3.1: Intersection of two fuzzy lattice ordered m-normal subgroup is again fuzzy lattice ordered m-normal subgroup.

Proof- Let A and B be two fuzzy lattice ordered m-normal groups on G.

$$\begin{aligned} \mu_{A \cap B} ((mx)(my)) &= \mu_A ((mx)(my)) \wedge \mu_B ((mx)(my)) \\ &= \mu_A ((my)(mx)) \wedge \mu_B ((my)(mx)) \\ &= \mu_{A \cap B} ((my)(mx)) \end{aligned}$$

Proposition 3.2: If $\{A_i\}$ is a family of fuzzy lattice ordered m-normal subgroups of G then $\cap A_i$ is a fuzzy lattice ordered m-normal group of G where $\cap A_i = \{ mx, \wedge \mu_{A_i}(mx) / x \in G, m \in M \}$

Proof- $(\cap \mu_{A_i})((mx)(my)) = \wedge \mu_{A_i}((mx)(my))$
 $= \wedge \mu_{A_i}(my mx)$
 $= (\cap \mu_{A_i})(my mx)$

Proposition 3.3: If A is a m-fuzzy characteristic of fuzzy lattice ordered m group of G then A is fuzzy lattice ordered m-normal subgroup of G.

Proof- Let A be a m-fuzzy characteristic of fuzzy lattice ordered m group of G.

Consider the map $f : G \rightarrow G$ defined by $f(mx) = my mx (my)^{-1}$

Clearly $f \in \text{Aut}(G)$

Now $\mu_A((mx)(my)) = \mu_A(f(mx)(my))$
 $= \mu_A(my mx (my)^{-1})$
 $= \mu_A(my mx)$

Proposition 3.4: A fuzzy lattice ordered m-group is a normal if and only if A is constant on conjugate classes of G.

Proof- Let A be a fuzzy lattice ordered m-normal subgroup of G $\mu_A(m y)^{-1} (m x) (m y) = \mu_A((my)^{-1} m y m x) = \mu_A(m x)$
Hence A is constant on conjugate classes of G. Conversely $\mu_A(m x m y) = \mu_A(m x m y m x (m x)^{-1}) = \mu_A(m x (m y m x) (m x)^{-1})$

$$= \mu_A(my mx)$$

Therefore the fuzzy lattice ordered m-group is a normal.

Proposition 3.5: For fuzzy lattice ordered m normal group $\mu_A(m y)^{-1} (m x) (m y) = \mu_A(m y) (m x) (m y)^{-1}$

Proof- Let A be a fuzzy lattice ordered m-normal subgroup of G. $\mu_A(m y)^{-1} (m x) (m y) = \mu_A(m x)$

$$\begin{aligned} &= \mu_A((m x)(m y)(m y)^{-1}) \\ &= \mu_A(m y)(m x)(m y)^{-1} \end{aligned}$$

Proposition 3.6: A is normalized if and only if $\mu_A(me) = 1$

Proof- Let A is normalized.

$$\mu_A(mx) = 1 \text{ for some } x \in G$$

But $\mu_A(mx) \leq \mu_A(me)$ for all $x \in G$
 $1 \leq \mu_A(me)$

Therefore $\mu_A(me) = 1$

Conversely $\mu_A(me) = 1$

Hence A is normalized.

Proposition 3.7: If A and B are fuzzy lattice ordered m normal groups of group G_1 and G_2 respectively, then $A \times B$ is fuzzy lattice ordered m normal subgroup of $G_1 \times G_2$.

Proof- The direct product of fuzzy lattice ordered m groups is a fuzzy lattice ordered m-group.

Let A & B be fuzzy lattice ordered m normal groups of group G_1 and G_2 respectively.

Then $A \times B$ is a fuzzy lattice ordered m group.

Let $x = (x_1, x_2), y = (y_1, y_2) \in G_1 \times G_2$

$$\begin{aligned} \mu_{A \times B}(m x m y) &= \mu_{A \times B}(m x_1 m y_1, m x_2 m y_2) \\ &= \min \{ \mu_A(m x_1 m y_1), \mu_B(m x_2 m y_2) \} \\ &= \min \{ \mu_A(m y_1 m x_1), \mu_B(m y_2 m x_2) \} \\ &= \mu_{A \times B}(m y_1 m x_1, m y_2 m x_2) \\ &= \mu_{A \times B}(m y, m x) \end{aligned}$$

Proposition 3.8: If A_1, A_2, \dots, A_n are fuzzy lattice ordered m normal groups of group G_1, G_2, \dots, G_n respectively, then $A_1 \times A_2 \times \dots \times A_n$ is fuzzy lattice ordered m normal subgroup of $G_1 \times G_2 \times \dots \times G_n$.

Proof- The direct product of fuzzy lattice ordered m groups is a fuzzy lattice ordered m-group.

Let A_1, A_2, \dots, A_n are fuzzy lattice ordered m normal groups of group G_1, G_2, \dots, G_n respectively. Then $A_1 \times A_2 \times \dots \times A_n$ is a fuzzy lattice ordered m group.

Let $x = (x_1, x_2, \dots, x_n), y = (y_1, y_2, \dots, y_n) \in G_1 \times G_2 \times \dots \times G_n$

$$\begin{aligned} \mu_{A_1 \times A_2 \times \dots \times A_n}(m x m y) &= \mu_{A_1 \times A_2 \times \dots \times A_n}(m x_1 m y_1, m x_2 m y_2, \dots, m x_n m y_n) \\ &= \min \{ \mu_{A_1}(m x_1 m y_1), \mu_{A_2}(m x_2 m y_2), \dots, \mu_{A_n}(m x_n m y_n) \} \\ &= \min \{ \mu_{A_1}(m y_1 m x_1), \mu_{A_2}(m y_2 m x_2), \dots, \mu_{A_n}(m y_n m x_n) \} \\ &= \mu_{A_1 \times A_2 \times \dots \times A_n}(m y_1 m x_1, m y_2 m x_2, \dots, m y_n m x_n) \\ &= \mu_{A_1 \times A_2 \times \dots \times A_n}(m y, m x) \end{aligned}$$

Proposition 3.9: If a fuzzy lattice ordered m-normal group A is conjugate to fuzzy lattice ordered m-normal group P of G_1 & If a fuzzy lattice ordered m-normal group B is conjugate to fuzzy lattice ordered m-normal group Q of G_2 Then $A \times B$ is conjugate to $P \times Q$ of $G_1 \times G_2$

Proof- $\mu_{A \times B}((m y)^{-1} (m x_1, m x_2) m y) = \mu_{A \times B}((m y)^{-1} m x_1(m y), (m y)^{-1} m x_2(m y))$

$$\begin{aligned} &= \min (\mu_A((m y)^{-1} m x_1(m y)), \mu_B((m y)^{-1} m x_2(m y))) \\ &= \min (\mu_P(m x_1), \mu_Q(m x_2)) \end{aligned}$$

$$= \mu_{P \times Q}((m x_1, m x_2))$$

Therefore $A \times B$ is conjugate to $P \times Q$.

Proposition 3.10: If A and B are fuzzy lattice ordered m-groups of G_1 and G_2 respectively & $A \times B$ is a fuzzy lattice ordered m-normal group of $G_1 \times G_2$ Then the following are true.

i) If $\mu_A(mx) \leq \mu_B(me')$ then A is a fuzzy lattice ordered m-normal group of G_1

ii) If $\mu_B(mx) \leq \mu_A(me)$ then B is a fuzzy lattice ordered m - normal group of G_2

Proof- i) $\mu_{A \times B}((mx, me)(my, me')) = \mu_{A \times B}((mx my, me'))$
 $= \min \{ \mu_A(mx my), \mu_B(me') \}$
 $= \min \{ \mu_A(mxy), \mu_B(me') \}$
 $= \mu_A(mxy)$
 $= \mu_A(mxmy)$
 $\mu_{A \times B}((my, me')(mx, me')) = \mu_{A \times B}((my mx, me'))$
 $= \min \{ \mu_A(my mx), \mu_B(me') \}$
 $= \min \{ \mu_A(myx), \mu_B(me') \}$
 $= \mu_A(myx)$
 $= \mu_A(mymx)$
 Therefore $\mu_A(mxmy) = \mu_A(mymx)$

ii) $\mu_{A \times B}((me, mx)(me, my)) = \mu_{A \times B}((me, mx my))$
 $= \min \{ \mu_A(me), \mu_B(mx my) \}$
 $= \min \{ \mu_A(me), \mu_B(mxy) \}$
 $= \mu_B(mxy)$
 $= \mu_B(mxmy)$
 $\mu_{A \times B}((me, my)(me, mx)) = \mu_{A \times B}((me, mymx))$
 $= \min \{ \mu_A(me), \mu_B(mymx) \}$
 $= \min \{ \mu_A(me), \mu_B(myx) \}$
 $= \mu_B(myx)$
 $= \mu_B(mymx)$

Therefore $\mu_B(mx my) = \mu_B(my mx)$

Applications: Lattice structure has been found to be extremely important in the areas of quantum logic, Ergodic theory, Reynold's operations, Soft Computing, Communication system, Information analysis system, artificial intelligences and physical sciences.

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