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EXISTENCE OF FUZZY MIXED INTEGRO-DIFFERENTIALEQUATION

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Abstract- ***The aim of the present paper is to establish the existence of solution of Fuzzy mixed integro-di erential equation.***

I. INTRODUCTION

Integro-differential equations play an important role in characterizing many social, physical, biological and engineering problems. The study of fuzzy integro-differential equation has gained importance in recent times. In [9, 11], Lakshmikantham et. al. and D. O' Regana et. al. assumed that even if only the initial value is fuzzy, the solution is a fuzzy function, and consequently the derivative in the integro-differential equation must be considered as fuzzy derivatives.

In this paper we study the following fuzzy mixed integro-differential equation

$$\dot{x}(t) = F\left(t, x(t), \int_0^t k(t,s)x(s)ds, \int_0^T h(t,s)x(s)ds\right) \quad (1.1)$$

$x(0) = x_0,\ t \in I = [0, T],$ (1.2)
where, $F : I \times E^n \times E^n \times E^n \to E^n$.

Many authors deal with existence, uniqueness and other properties of solution of special forms of (1.1) - (1.2), see [1, 2, 3, 13] and references cited therein. Recently, in [6] T. Donchev proved existence of special form of (1.1) - (1.2). The aim of present paper is to prove existence of solution of first order fuzzy mixed integro-differential equation subject to given fuzzy initial condition. The main tool employed in our analysis is fixed point theorem.

2. Basic concepts

The Fuzzy set space is denoted by $E^n = \{x/x : R^n \to [0,1];\ x$ satisfies conditions (1) *to* (4)$\}$

(1) x is normal i.e there exists $y_0 \in R^n$ such that $x(y_0) = 1$,

(2) x is fuzzy convex i.e. for any $y, z \in R^n$ *and* $0 \le \lambda \le 1$, $x(\lambda y + (1 - \lambda)z) \ge \min\{x(y), x(z)\}$,

(3) x is upper semicontinuous,

(4) $[x]^0 = cl\{y \in R^n : x(y) > 0\}$ is compact.

The set $[x]^\alpha = \{y \in R^n : x(y) \ge \alpha\}$ is called as α-level set of a $x\ \forall\ \alpha \in (0,1)$.
Let fuzzy zero is defined by,

$$\hat{\theta}(y) = \begin{cases} 0 & y \neq 0 \\ 1 & y = 0 \end{cases}$$

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Let $D : E^n \times E^n \to [0,\infty)$ defined as

$$D(x,y) = \sup_{0 \le \alpha \le 1} D_H([x]^\alpha, [y]^\alpha),$$

$$D_H(A, B) = \max\left\{\max_{a \in A}\min_{b \in B}|a-b|,\ \max_{b \in B}\min_{a \in A}|a-b|\right\},$$

is the Housdroff distance between the convex compact subset of R^n. Here D is a metric on E^n. From [7], E^n can be embedded as a closed convex cone in Banach space X, the embedding map $J : E^n \to X$ is isometric and isomorphic. The function $g : I \to E^n$ is said to be simple function if there exists a finite number of pairwise disjoint measurable subsets $I_1, I_2, I_3, \ldots, I_n$ of I with $I = \bigcup_{k=1}^{n} I_k$ such that $g(\cdot)$ is constant on every I_k. The map
$F : I \to E^n$ is said to be strongly measurable if there exists a sequence $\{F_m\}_{m=1}^{\infty}$ of simple functions $F_m : I \to E^n$ such that $D(F_m(t), F(t)) \to 0$ as $m \to \infty\ \forall t \in I$.

In the fuzzy set literature, the integral of fuzzy function is defined levelwise i.e. there exists $g(t) \in E^n$ such that $[g]^\alpha(t) = \int_0^t [F]^\alpha(s)ds$.

If $g(\cdot) : I \to E^n$ is strongly measurable and integrable then $J(g)(\cdot)$ is strongly measurable and Bochner integrable and $J\left(\int_0^t g(s)ds\right) = \int_0^t J(g)(s)ds\ \forall t \in I$.

From [7] we recall some properties of integrable fuzzy set valued mapping.

Theorem 2.1. *Let $G, K : I \to E^n$ is integrable and $\lambda \in R$ then*

(i) $\int_I (G(t) + K(t))dt = \int_I G(t)dt + \int_I K(t)dt,$

(ii) $\int_I \lambda G(t)dt = \lambda \int_I G(t)\ dt$

(iii) $D(G, K)$ is integrable

(iv) $D\left(\int_I G(t)\,dt, \int_I K(t)\,dt\right)$

(v) $\leq \int_I \left(D(G(t)\ K(t)\right)dt.$

Definition 1. *A mapping $F : I \rightarrow E^n$ is said to be differentiable at $t \in I$ such that there exists $F(t) \in E^n$ and limits* $\lim_{h \to 0^+} \frac{F(t+h)-f(t)}{h}$ *and* $\lim_{h \to 0^+} \frac{F(t)-F(t-h)}{h}$ *exists and equal to F(t).*

At the end point of I we consider only one sided derivative. Note that E^n is not locally compact [12]. Consequently we need compactness type assumption to prove existance of solution [5].

Let Y be complete metric space with metric $\rho_y(\cdot,\cdot)$. The Housedroff measure of non-compactness $\beta : Y \rightarrow R$ for the bounded subset A of Y

$\beta(A) = \inf(d > 0\ /$ *A can be covered by finite many balls with radius $\leq d$)*
and Kuratowski measure of noncompactness $\rho : Y \rightarrow R$ for bounded subset A of Y is defined as.

$\rho(A) = \inf(d > 0\ /$ *A can be covered by finite many subset with diameter $\leq d$).*

diam (A) - $\sup_{a,b \in A} \rho_y(a, b)$. [11] $\rho(A) \leq \beta(A) \leq 2\rho(A)$.

Let $v(\cdot)$ represent the both $\rho(\cdot)$, $\beta(\cdot)$ then some properties of $v(\cdot)$ are listed below.

(i) $v(A) = 0$ iff A is precompact i.e $cl(A)$ is compact

(M) $v(A + B) = v(A) + v(B)$

(in) If $A \subset B$ then $v(A) \leq v(B)$

(iv) $v(A \cup B) = \max(v(A), v(B))$

(iv) $v(\cdot)$ is continuous w.r.t Hausdroff distance.

Theorem 2.2. [8] *Let X be separable Banach space and let $(g_n(\cdot))_{n=1}^{\infty}$ be integrably bounded sequence of measurable functions from I into X then $t \rightarrow \beta(g_n(t), n \geq 1)$ is measurable and* β

$$\left(\int_t^{t+h} \cup gi(s)\,ds\right) < \int_t^{t+h} \beta(\cup gi(s)) \quad where\ t, t+h \in I$$

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The map $t \rightarrow (\cup gi(t))$ is a set valued (multifunction).The integral is denned in Auman sence i.e. union of the values of integrals of all (strongly) measurable selections.

Remark 2.1. Since the embedding map $J : E^n \rightarrow X$ is isometry and isomorphism. It preserves diameter of any closed subset $\rho(A) = \rho(J(A))$ for any closed and bounded set $A \in E^n$.

Theorem 2.3. *let $\{f_n\}_{n=1}^{\infty}$ be a integrally bounded sequence of strongly measurable fuzzy functions defined from I into E^n then $t \rightarrow \rho(f_m(t), m \geq 1)$ is measurable and*

$$\rho\left(\int_a^b \cup f_m(s)\,ds\right) \leq 2\int_a^b \rho(\cup f_m(s))\,ds.$$

Theorem 2.4. *Let $u(t), f(t), g(t), h(t) \in C(I, R_+)$ and suppose for*
$t \in I, u(t) \leq c +$
$$\int_{t_0}^{t} f(t) + [u(s) + \int_{t_0}^{s} g(\sigma)u(\sigma)\,d\sigma + \int_{t_0}^{s} h(\sigma)u(\sigma)\,d\sigma]\,ds,$$
where
$c \geq 0$ is constant.
If $d = \int_{t_0}^{b} h(s)\exp\left(\int_{t_0}^{\sigma}[f(\tau) + g(\tau)]d\tau\right)d\sigma < 1$ *then*

$u(t) \leq \frac{c}{1-d}\exp\left(\int_{t_0}^{t}[f(s) + g(s)]ds\right)$

3. Main Results

In this section we state and prove the existence of solution of Fuzzy mixed integro-differential equation.

Theorem 3.1. *Let in the domain $Q = \{(t,x,y,z) \in I \times E^n \times E^n \times E^n\}$ the following conditions hold*

(I) *Let $F: I \times E^n \times E^n \times E^n \rightarrow E^n$ is such that*
(i) $t \rightarrow F(t, x, y, z)$ is strongly measurable $\forall x, y, z \in E^n$,
(ii) $(x, y, z) \rightarrow F(t, x, y, z)$ *is continuous for all most all $t \in I$.*

(II) For all non-empty bounded subset A,B,C $\in E^n$ and $\lambda(\cdot) \in L^1(I, R_+)$,
$\rho(F(t, A, B, C)) \leq \lambda(t)(\rho(A) + \rho(B) + \rho(C))$.

(III) There exists $a(\cdot), b(\cdot) \in L^1(I, R_+)$ such that
$D(F(t, x, y, z), \hat{0}) \leq a(t) + b(t)(D(x, \hat{0}) + D(y, \hat{0}) + D(z, \hat{0}))$

forall $(t, x, y, z) \in Q$.
(IV) $K, H : \Delta = (t, s)\ ;\ 0 \leq s \leq t \leq a \rightarrow R_+$ is continuous function.
Then equation (1.1) - (1.2) has at least one solution on the interval I.
Proof. We will show that the solution of (1.1) - (1.2) is bounded.

$$D(x(t), \hat{\theta}) = D(x_0, \hat{\theta}) + D\left(\int_0^t F\left(s, x(s), \int_0^t k(t,s)x(s)ds, \int_0^T h(t,s)x(s)ds\right)ds, \hat{\theta}\right)$$

$$\leq D(x_0, \hat{\theta}) + \int_0^t D\left(F\left(s, x(s), \int_0^t k(t,s)\,x(s)\,ds, \int_0^T h(t,s)\,x(s)ds\right)ds, \hat{\theta}\right)$$

$$\leq D(x_0, \hat{\theta}) + \int_0^t \left\{ a(s) + b(s)\left[D(x_0, \hat{\theta}) + D\left(\int_0^t k(t,s)\,x(s)ds, \hat{\theta}\right) + D\left(\int_0^T h(t,s)\,x(s)ds, \hat{\theta}\right) \right]\right\} ds$$

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where,
$K_\Delta = \max_{(t,s)\in\Delta} |k(t,s)|$ and $H_\Delta = \max_{(t,s)\in\Delta} |h(t,s)|$.

Let $m(t) = D(x(t), \hat{\theta})$ hence we get

$$m(t) = m(0) + \int_0^t a(s) + b(s)\left[m(s) + K_\Delta \int_0^t m(\tau)d\tau + H_\Delta \int_0^T m(\tau)d\tau\right] ds. \qquad (3.1)$$

By theorem there exists $M_0 > 0$ such that
$m(t) = D(x(t), \hat{\theta}) \le M_0$.

Now, consider

$$D\left(\int_0^t k(t,s)x(s)ds, \hat{\theta}\right) \le \int_0^t D\left(k(t,s)\,x(s), \hat{\theta}\right)ds$$

$$\le K_\Delta \int_0^t D\left(x(s), \hat{\theta}\right)ds \le K_\Delta M_0 T = M_1,$$

$$D\left(\int_0^T h(t.s)x(s)\,ds. \hat{\theta}\right) \le \int_0^T D\left(h(t.s)\,x(s). \hat{\theta}\right)ds$$

$$\le H_\Delta \int_0^t D\left(x(s), \hat{\theta}\right)ds \le H_\Delta M_0 T = M_2,$$

Therefore,

$$D\left(F\left(t, x(t), \int_0^t k(t,s)x(s)ds, \int_0^T h(t,s)x(s)ds\right) \hat{\theta}\right) \le a(t) + Mb(t) = \mu(t),$$

where, $M = M_0 + M_1 + M_2$, $a(\cdot)$ *and* $b(\cdot) \in L^1(I, R_+)$ and $\mu(\cdot) \in L^1(I, R_+)$.

Let $c = \int_0^T \mu(s)ds + 1$. Define

$\Omega = \{x(\cdot) \in c([0,T], E^n) / \sup_{t\in[0,T]} D(x(t), x_0) \le c\}$
Clearly Ω is closed, bounded and convex set.
We now define the operator $P : c\{[0,T], E^n\} \to c\{[0,T], E^n\}$ by

$$P[x(t)] = x_0 + \int_0^t F\left(s, x(s), \int_0^t k(t,s)x(s)ds, \int_0^T h(t,s)x(s)ds\right) ds, t \in I.$$

Now

$$D(P[x(t)], x_0) = D\left(\int_0^t F\left(s, x(s), \int_0^t k(t,s)x(s)ds, \int_0^T h(t,s)x(s)ds\right), \hat{\theta}\right)$$

$$\le \int_0^t D\left(F\left(s, x(s), \int_0^t k(t,s)ds, \int_0^T h(t,s)x(s)ds\right). \hat{\theta}\right)$$

$$\le \int_0^T \mu(s)ds < c.$$

Hence $P[x(t)] \in \Omega$ i.e $P[\Omega] \subset \Omega$ and $P(\Omega)$ is uniformly bounded. Now we will show that P is continuous operator on Ω Let $x_n(\cdot) \in \Omega$ such that $x_n(\cdot) \to x(\cdot)$ then

$$D(P[x_n(t)], P[x(t)]) = D\left(\int_0^t F\left(s, x_n(s), \int_0^t k(t,s)x_n(s)ds, \int_0^T h(t,s)x_n(s)ds\right)ds,\right.$$
$$\left.\int_0^t F\left(s, x(s), \int_0^t k(t,s)x(s)ds, \int_0^T h(t,s)x(s)ds\right)ds\right)$$

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Now

$$D\left(\int_0^t k(t,s)x_n(s)ds, \int_0^t k(t,s)x(s)ds\right) \le \int_0^t D\left(k(t,s)x_n(s), k(t,s)x(s)\right)ds$$

$$\le K_\Delta \int_0^t D\left(x_n(s), x(s)\right)ds \to 0 \text{ as } n \to \infty.$$

$$D\left(\int_0^T h(t,s)x_n(s)ds, \int_0^T h(t,s)x(s)ds\right) \le \int_0^T D\left(h(t,s)x_n(s), h(t,s)x(s)\right)ds$$

$$\le H_\Delta \int_0^t D\left(x_n(s), x(s)\right)ds \to 0 \text{ as } n \to \infty.$$

Thus by (7) it follows that $D(P[x_n(t)], P[x(t)]) \to 0$ as $n \to \infty$ uniformly on I. Hence *P* is continuous operator on Ω.

The function $t \to \int_0^t \mu(s)\, ds$ is uniformly continuous on closed set I i.e there exists $\eta > 0$ such that $\left|\int_0^t \mu(\tau)d\tau\right| \le \frac{\varepsilon}{2}\ \forall t, s \in I$, with $|t - s| < \eta$. Further for each m ≥ 1 we divide I into *m* subintervals $[t_i, t_{i+1}]$ with $t_i = \frac{iT}{m}$.

$$x_m(t) = \begin{cases} x_0 & \text{if } t \in [0, t_1] \\ P x_m(t - t_i) & \text{if } t \in [t_i, t_{i+1}] \end{cases}$$

Then $x_m \in \Omega$ for every $m \ge 1$. For $t \in [0, t_1]$

$$D(P[x_m(t)], x_m(t)) = D\left(\int_0^t F\left(s, x_m(s)\int_0^t k(t,s)x_m(s)ds, \int_0^T h(t,s)x_m(s)ds\right) ds, \hat{\theta}\right)$$

$$\le \int_0^{t_1} D\left(F\left(s, x_m(s), \int_0^t k(t,s)x_m(s)ds, \int_0^T h(t,s)x_m(s)ds\right), \hat{\theta}\right)ds$$

$$\le \int_0^{t_1} \mu(s)ds.$$

And for $t \in [t_i, t_{i+1}]$, $t - t_i \le \frac{T}{m}$ and hence

$$D(P[x_m(t)], x_m(t)) = D\left(P[x_m(t)], P[x_m(t \quad t_i)]\right)$$

$$\le D\left(\int_{t-t_i}^t F\left(s, x_m(s)\int_0^t k(t,s)x_m(s)ds, \int_0^T h(t,s)x_m(s)ds\right) ds, \hat{\theta}\right)$$

$$\le \int_{t-\frac{T}{m}}^{t_1} D\left(F\left(s, x_m(s), \int_0^t k(t,s)x_m(s)ds, \int_0^T h(t,s)x_m(s)ds\right), \hat{\theta}\right)ds$$

$$\le \int_{t-\frac{T}{m}}^{t_1} \mu(s)ds.$$

Therefore $\lim_{m\to\infty} D(P[x_m(t)], x_m(t)) = 0$ on $[0, T]$. Let $A = \{x_m(\cdot) \backslash m \ge 1\}$
Now we claim that *A* is equicontinuous on [0,T]. If $t, s \in [0, \frac{T}{m}]$ then $D(x_m(t), x_m(s)) = 0$ if $0 \le s \le \frac{T}{m} \le t \le T$ then

$$D(x_m(t), x_m(s)) = D\left(x_0 + \int_0^{t-\frac{T}{m}} F\left(c, x_m(c), \int_0^t k(t,\sigma)x_m(\sigma)d\sigma, \int_0^T h(s,\sigma)x_m(\sigma)d\sigma\right), x_0\right)$$

$$\le \int_0^{t-\frac{T}{m}} D\left(F\left(\sigma, x_m(\sigma), \int_0^t k(t,\sigma)x_m(\sigma)d\sigma, \int_0^T h(s,\sigma)x_m(\sigma)d\sigma\right), \hat{\theta}\right)$$

$$\le \int_0^{t-\frac{T}{m}} \mu(\sigma)d\sigma \le \int_0^t \mu(\sigma)d\sigma \le \frac{\varepsilon}{2} \quad \text{for } |t-s| < \eta$$

And if $\frac{T}{m} \le s \le t \le T$ then $D(x_m(t), x_m(s)) \le \frac{\varepsilon}{2}$ for $|t-s| < \varepsilon$.

Hence A is equicontinuous on [0, T]. Now we will prove that A is precompact for each $t \in [0,T]$. We have

$$\rho(A(t)) \le \rho\left(\int_0^{t-\frac{T}{m}} F\left(s, A(s), \int_0^t k(t,s)A(s)ds, \int_0^T h(t,s)A(s)ds\right)\right)$$

$$+\rho\left(\int_{t-\frac{T}{m}}^t F\left(s, A(s), \int_0^t k(t,s)A(s)ds, \int_0^T h(t,s)A(s)ds\right)\right)$$

Given $\varepsilon > 0$ we define $m(\varepsilon) > 0$ such that $\int_{t-\frac{T}{m}}^t \mu(s)\,ds < \frac{\varepsilon}{2}$ $\forall t \in [0,T]$ and $m \ge m(\varepsilon)$

Therefore.

$$\rho\left(\int_{t-\frac{T}{m}}^t F\left(s, A(s), \int_0^t k(t,s)A(s)ds, \int_0^T h(t,s)A(s)ds\right)\right)$$

$$= \rho\left(\int_{t-\frac{T}{m}}^t F\left(s, x_m(s), \int_0^t k(t,s)\,x_m(s)ds \int_0^T h(t,s)\,x_m(s)ds\right) ds; m \ge (\varepsilon)\right)$$

$$\le 2\left(\int_{t-\frac{T}{m}}^t \mu(s)ds\right) < \varepsilon$$

$$\therefore \rho(A(t)) \le \rho\left(\int_0^t F\left(s, A(s), \int_0^t k(t,s)A(s)ds, \int_0^T h(t,s)A(s)ds\right)ds\right)$$

$$\le 2\int_0^t \rho\left(F\left(s, A(s), \int_0^t k(t,s)A(s)ds, \int_0^T h(t,s)A(s)ds\right)\right)ds$$

$$\le 2\int_0^t \lambda(s)\left[\rho(A(s)) + \rho\left(\int_0^t k(t,s)A(s)ds\right) + \rho\left(\int_0^T h(t,s)A(s)ds\right)\right]ds$$

Now

$$\rho\left(\int_0^t K(s,t)A(s)\,ds\right) \le \rho\left(\int_0^t k(s,t)x_m(s)ds, m \ge 1\right)$$

$$\le 2\int_0^t \rho\left(k(s,t)x_m(s), m \ge 1\right) ds$$

$$\le 2\int_0^t K_\Delta \rho\left(x_m(s), m \ge 1\right) ds$$

$$\le 2\int_0^t K_\Delta \rho(A(s))\,ds.$$

And

$$\int_0^t \rho\left(\int_0^t k(t,s)A(S)\,ds\right) \le \int_0^t 2\int_0^t K_\Delta \rho(A(\tau))d\tau \le 2\int_0^t\int_0^t K_\Delta \rho(A(\tau))d\tau ds$$

$$\le 2\int_0^t K_\Delta (t-\tau)\,\rho(A(\tau))d\tau$$

$$\le K_\Delta T \int_0^t \rho(A(\tau))d\tau.$$

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Similarly

$$\rho\left(\int_0^T h(s,t)A(s)ds\right) \le \rho\left(\int_0^T h(s,t)\,x_m(s)ds, m \ge 1\right)$$

$$\le 2\int_0^T \rho\left(h(s,t)\,x_m(s)\; m \ge 1\right)ds$$

$$\le 2\int_0^T H_\Delta \rho\left(x_m(s) m \ge 1\right)ds$$

$$\le 2\int_0^T H_\Delta \rho(A(s))ds. \text{And}$$

$$\int_0^t \rho\left(\int_0^T h(t,s)A(s)ds\right)ds \le \int_0^t 2\int_0^T H_\Delta \rho(A(\tau))d\tau ds$$

$$\le 2\int_0^t \int_\tau^T H_\Delta \rho(A(\tau))ds d\tau$$

$$\le \int_0^t H_\Delta \left(t-\tau)\,\rho(A(\tau))\right)d\tau$$

$$\le H_\Delta T \int_0^t \rho(A(\tau))d\tau$$

Therefore we obtain

$$\rho(A(t)) \le 2\int_0^t \lambda(s)[\rho(A(s)) + K_\Delta T\rho(A(s)) + H_\Delta T\rho(A(s))]ds.$$

Due to Gronwall inequality

$$\rho(A(t)) \le R\int_0^t \rho(A(s))ds,$$

$$\text{where } R = \exp\left(2(1 + K_\Delta T + H_\Delta)T\int_0^T \lambda(t)\,dt\right)$$

$$\therefore \rho(A(t)) \le \int_0^t \rho(A(s))ds.$$

Therefore $\rho(A(t)) = 0$ and hence A(t) is preconpact for every $t \in [0, T]$. Since A is equicontinuous and precompact hence Arzela-Ascoli theorem hold in this case. Thus the sequence $\{x_n(t)\}_{n=1}^{\infty}$ converges uniformly on [0,T] to a continuous function $x(\cdot) \in \Omega$. Due to triangle inequality

$D(P[x(t)],x(t)) \le D(P[x(t)],P[x_n(t)]) + D(P[x_n(t)],x_n(t)) + D(x_n(t),x(t)) \to 0.$

Hence we have $P[x(t)] = x(t)$ for all $t \in [0,T]$, i.e. $x(t)$ is solution of (1.1) - (1.2).

REFERENCES

1. K. Balachadran and P. Prakash; Existance of solution of nonlinear fuzzy Voltera-fredholm integral equation, *Indian J. Pure Appl. Math.,* 33 (2002) ,329-343.
2. P. Balasubramaniam and M. Chandrasekaran; Existance of solution of non-linear Integra-differential equation with nonlocal boundary condition in Banach space, *Atti. sere Mat. fis. univ. mode.ua XLVI,* (1998), 1-13.
3. P. Balasubrameniam and S. Muralisankar; Existence and uniqueness of fuzzy solution for the non-linear fuzzy integro-differeiitial equation, *Appl. Math, left.,* 14 (2001), 455-462.
4. J. Banas and B. Rzepka; An application of measure of Asymptotic stability, *Appl. Math, lett,*16 (2003),1-6.